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THE PROGRESSIVE  
ARITHMETIC

W.F. NICHOLS

PART THREE

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# THE PROGRESSIVE ARITHMETIC

## PART III

BY

WILBUR F. NICHOLS, M. A.

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## PREFACE.

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**THIS** is the third in a series of three books.

The plan of development pursued in the previous books of this series is here continued, giving a review of the subjects previously presented, and a study of those topics that have not been presented in the other books. This is followed by a condensed summary, or concise, topical review of all subjects. The latter feature will readily commend itself to those schools where the whole subject of arithmetic is carefully reviewed during the last year of the grammar school.

Throughout the series, principles are taught rather than rules, and the work is made practical. The shorter and simpler methods in daily use in the different trades and those used by merchants and bankers are introduced.

The aim has been to furnish material sufficient for the demands of any school. This means that the problems are not so difficult as to discourage the majority of the pupils, or so easy as to render unnecessary steady, thoughtful work on their part. Any teacher using these arithmetics should omit from the regular lesson any topic or example which seems unnecessary or too difficult for his class.

The lessons upon the different geometrical forms, which have been an important part of the previous books, are here continued and extended to include the simpler principles of concrete and constructive geometry which make the transition to theoretical geometry easier for the pupils.



The first principles of algebra are introduced, and throughout the book there are many problems that should be solved by means of algebra. Wherever this method of solution is the simpler, teachers should encourage the pupils to use it. Putting into algebraic language the conditions of an arithmetical problem forms an easy transition from the concrete language of arithmetic to the more abstract language of algebra.

While this book contains much new matter, the author has drawn largely from his "Graded Lessons in Arithmetic," a series which has been favorably received, consisting of a separate book for each school year.

The author wishes here to express his indebtedness to the many teachers, supervisors, and superintendents who have tested the problems, criticized the work, and given many helpful suggestions.

W. F. NICHOLS.

NEW HAVEN, April 15, 1903.

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# PROGRESSIVE ARITHMETIC.

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## PART THREE.

### MISCELLANEOUS REVIEW.

1. Find  $33\frac{1}{3}\%$  of \$28.80.  $6\frac{1}{4}\%$  of \$25.60.
2. \$174.04 is 95% of how many dollars?
3. Find the gain per cent when the cost is \$600, and the selling-price \$618.
4. Find the cost when the selling-price is \$73.84, and the loss 80%.
5. A man owns a rectangular lot 132 ft. long and 110 ft. wide. A fence runs from the northeast to the southwest corner. How many square feet in each part?
6. My garden is 100 ft. long and 25 ft. wide. How many boards, each 12 ft. long, 6 in wide, will it take to build a tight board fence 6 ft. high across one end and two sides of this garden?
7. The selling-price is \$175. Find the gain per cent, if the cost is \$150.
8. Make and solve an example illustrating how to find the number of cords in a pile of wood.
9. Add: Two thousand and four thousandths; three hundred-thousandths; six millionths; two million forty-five; ten thousandths.

Cancellation is a method of shortening the work in problems involving only multiplication and division. Dividing any one of a series of factors by any number divides their product by that number. Dividing dividend and divisor by the same number does not change the quotient.

1. Divide  $4 \times 3 \times 6 \times 12$  by  $2 \times 3 \times 4 \times 6$ .

$\frac{4 \times \cancel{3} \times \cancel{6} \times 12}{2 \times \cancel{3} \times \cancel{4} \times \cancel{6}}$  This is an example in division in which the dividend and divisor are partially factored. We shorten the operation by canceling the 3's; i.e., by dividing both dividend and divisor by three. We further shorten the work by canceling the 4's and 6's. The dividend is now 12 and the divisor 2; hence the result is 6.

2. Multiply 44 by 8 and the product by 7; then divide it by  $11 \times 16 \times 7$ .

3. Divide  $40 \times 18 \times 13 \times 8$  by  $10 \times 13 \times 16$ .
4. Divide  $15 \times 4 \times 8 \times 9$  by  $30 \times 2 \times 6 \times 12$ .
5. Divide  $108 \times 17 \times 9 \times 4$  by  $27 \times 3 \times 16 \times 17$ .
6. Divide  $5 \times 25 \times 874$  by  $2 \times 437 \times 5 \times 5 \times 5$ .
7. Divide  $376 \times 14 \times 21$  by  $7 \times 7 \times 16 \times 3$ .
8. Divide  $10 \times 5 \times 25$  by  $2 \times 5 \times 5 \times 5$ .
9. Divide  $120 \times 4 \times 9$  by  $3 \times 40 \times 4 \times 3$ .
10. Divide  $60 \times 3 \times 7 \times 21$  by  $20 \times 14 \times 3 \times 9$ .
11. Divide  $15 \times 18 \times 21 \times 25$  by  $9 \times 3 \times 7 \times 15$ .
12. Divide  $44 \times 8 \times 7$  by  $11 \times 16 \times 7$ .
13. Divide  $15 \times 4 \times 8 \times 9$  by  $30 \times 2 \times 6 \times 12$ .
14. Divide  $5 \times 25 \times 874$  by  $2 \times 437 \times 5 \times 5 \times 5$ .
15. Divide  $11 \times 39 \times 14 \times 96$  by  $44 \times 18 \times 26 \times 14$ .
16. Divide  $125 \times 60 \times 24 \times 42$  by  $25 \times 120 \times 36 \times 5$ .
17. Divide  $36 \times 21 \times 14$  by  $27 \times 7 \times 6$ .
18. Divide  $125 \times 105 \times 11$  by  $35 \times 33 \times 5 \times 5$ .
19. Divide  $54 \times 3 \times 4 \times 15$  by  $18 \times 12 \times 10$ .
20. Divide  $25 \times 160 \times 13 \times 90$  by  $51 \times 30 \times 8 \times 15$ .
21. Divide  $400 \times 125 \times 64 \times 72$  by  $36 \times 75 \times 32 \times 25$ .

1. Divide  $\frac{3}{4}$  by  $\frac{3}{4}$ .

$$(a) \quad \frac{3}{4} = \frac{1^8}{1^2} \quad \frac{1^8}{1^2} \div \frac{1^2}{1^2} = 8 \div 9 = \frac{8}{9}.$$

$$\frac{3}{4} = \frac{1^2}{1^2}$$

For explanation see Part II., page 190.

$$(b) \quad \frac{3}{4} \div 1 = \frac{3}{4}.$$

$$\frac{3}{4} \div \frac{1}{4} = 4 \times \frac{3}{4}.$$

$$\frac{3}{4} \div \frac{3}{4} = \frac{1}{3} \text{ of } 4 \times \frac{3}{4}, \text{ or}$$

$$\frac{3}{4} \times \frac{4}{3} = \frac{8}{9}.$$

$\frac{3}{4}$  divided by 1 is  $\frac{3}{4}$ .  $\frac{3}{4}$  divided by  $\frac{1}{4}$  must be 4 times  $\frac{3}{4}$ ; hence  $\frac{3}{4}$  divided by  $\frac{1}{4}$  must be  $\frac{1}{3}$  of 4 times  $\frac{3}{4}$ , or  $\frac{1}{3} \times \frac{3}{4}$ . Therefore, dividing  $\frac{3}{4}$  by  $\frac{3}{4}$  is the same as multiplying  $\frac{3}{4}$  by  $\frac{4}{3}$ , the divisor inverted.

NOTE. — In multiplying or dividing fractions, cancel when possible.

Divide;

$$\begin{array}{llll} 2. \quad \frac{6}{8} \text{ by } \frac{3}{4} & \frac{11}{12} \text{ by } \frac{5}{6} & \frac{1^8}{1^3} \text{ by } \frac{9}{30} & \frac{3}{4} \text{ by } \frac{1^4}{1^5} \\ 3. \quad \frac{1^5}{1^2} \text{ by } \frac{3}{4} & \frac{1^6}{1^2} \text{ by } \frac{3}{4} & \frac{3}{8} \text{ by } \frac{2^4}{2^7} & \frac{1^4}{1^5} \text{ by } \frac{3}{4} \\ 4. \quad \frac{1^8}{30} \text{ by } \frac{2}{4} & \frac{3}{8} \text{ by } \frac{1^7}{1^2} & \frac{1}{8} \text{ by } \frac{3}{4} & \frac{6}{8} \text{ by } \frac{3}{4} \end{array}$$

5. Learn: To divide a fraction by a fraction, divide by the numerator of the divisor, and then multiply by its denominator, or invert the divisor and proceed as in multiplication of fractions.

6.  $\frac{3}{8} \div \frac{4}{7}$  may be written  $\frac{\frac{3}{8}}{\frac{4}{7}}$ . When written in this form it is called a complex fraction.

7. A Complex Fraction is a fraction having a mixed number or a fraction for one or both of its terms.

$$8. \text{ Simplify: } \frac{12\frac{1}{2}}{37\frac{1}{2}} \quad \frac{2\frac{1}{2}}{1\frac{3}{4}} \quad \frac{1\frac{1}{2}}{2\frac{1}{3}} \quad \frac{34\frac{1}{2}}{7\frac{3}{4}} \quad \frac{37\frac{2}{3}}{6\frac{2}{3}} \quad \frac{9\frac{3}{4}}{4\frac{1}{2}}$$

$$9. \text{ Simplify: } \frac{17\frac{3}{4}}{51\frac{1}{2}} \quad \frac{28\frac{1}{11}}{41\frac{8}{11}} \quad \frac{11\frac{3}{4}}{3\frac{1}{2}} \quad \frac{12\frac{7}{9}}{51\frac{2}{8}} \quad \frac{26\frac{3}{8}}{81\frac{3}{4}}$$

Divide:

$$\begin{array}{llll} 10. \quad \frac{1}{8} \text{ by } \frac{3}{4} & \frac{1^7}{1^6} \text{ by } \frac{3}{4} & \frac{7}{8} \text{ by } \frac{3}{4} & \frac{1^7}{1^2} \text{ by } \frac{3}{8} \\ 11. \quad \frac{1^8}{1^5} \text{ by } \frac{3}{4} & \frac{11}{10} \text{ by } \frac{1^6}{1^6} & \frac{1^6}{1^8} \text{ by } \frac{3}{4} & \frac{3}{4} \text{ by } \frac{3}{8} \\ 12. \quad \frac{11}{12} \text{ by } \frac{3}{8} & \frac{1^5}{1^5} \text{ by } \frac{3}{4} & \frac{1^5}{1^6} \text{ by } \frac{3}{4} & \frac{1^7}{1^7} \text{ by } \frac{3}{8} \\ 13. \quad 5\frac{3}{4} \text{ by } 4\frac{1}{2} & 5\frac{1}{2} \text{ by } 3\frac{1}{4} & 1\frac{3}{4} \text{ by } 2\frac{3}{4} & 2\frac{1}{2} \text{ by } 1\frac{1}{2} \\ 14. \quad 3\frac{1}{2} \text{ by } \frac{3}{4} & 2\frac{3}{4} \text{ by } \frac{3}{4} & 3\frac{3}{4} \text{ by } \frac{3}{4} & 5\frac{3}{4} \text{ by } \frac{3}{4} \end{array}$$



#### 4 MULTIPLICATION AND DIVISION OF FRACTIONS.

1. If  $\frac{1}{4}$  of an acre of land cost \$60, what will  $\frac{7}{15}$  of an acre cost?

SOLUTION.  $\$60 \times \frac{\overset{5}{\cancel{5}}}{\underset{3}{\cancel{4}}} \times \frac{7}{15} = \$35.$  If  $\frac{1}{4}$  of an acre cost \$60, one acre will cost as many dollars as  $\frac{1}{4}$  is contained times in \$60, or  $\$60 \times \frac{4}{1}$ . If one acre costs  $\$60 \times \frac{4}{1}$ ,  $\frac{7}{15}$  acres will cost  $\$60 \times \frac{4}{1} \times \frac{7}{15}$ .

2. If  $7\frac{1}{2}$  lb. of raisins cost 85 cents, what will  $4\frac{1}{2}$  lb. cost?

3. If 9 oranges cost  $22\frac{1}{2}$  cents, what will 16 oranges cost?

SOLUTION.  $\frac{\overset{5}{\cancel{45}}}{\underset{2}{\cancel{9}}} \times \frac{1}{9} \times \frac{\overset{8}{\cancel{16}}}{1} = 40$  cents.

4. What will  $4\frac{1}{2}$  tons of coal cost, if  $10\frac{1}{2}$  tons cost \$75.48?

5. What is the cost of a pile of wood 12 feet long, 4 feet wide, and 6 feet high, at \$5 $\frac{1}{2}$  a cord?

SOLUTION.  $\frac{\overset{3}{\cancel{12}} \times \overset{3}{\cancel{4}} \times \overset{3}{\cancel{6}}}{\underset{8}{\cancel{32}}} \times \frac{\$11}{\underset{2}{\cancel{2}}} = \frac{\$99}{8} = \$12\frac{3}{8}.$

The solution explains itself. The work is all indicated, and cancellation is then used to lessen the number of figures required.

6. Find the cost at \$3 $\frac{1}{2}$  a cord of wood which fills a shed 24 ft. long, 20 ft. wide, and 16 ft. high.

7. How many gallons of molasses at 37 $\frac{1}{2}$ ¢ a gallon are worth as much as 12 $\frac{1}{2}$  bu. of potatoes at 40¢ a bushel?

8. If 6 $\frac{1}{2}$  cd. of wood cost \$41 $\frac{1}{2}$ , what will 2 $\frac{1}{2}$  cd. cost?

9. The perimeter of a room is 56 ft. 8 in., and its height 8 ft. 9 in. Find the area of the 4 walls.

10. Find the volume of a rectangular solid whose dimensions are 2 $\frac{3}{4}$  ft., 2 $\frac{3}{4}$  ft., and 4 $\frac{1}{2}$  ft.

11. What must I pay for 37 $\frac{3}{4}$  tons of coal, if 12 $\frac{1}{2}$  tons cost \$74?

12. If 13 $\frac{1}{2}$  lb. of sugar cost 80 cents, what will 8 $\frac{1}{2}$  lb. cost?

13. How many times is 4 $\frac{1}{3}$  contained in 7 $\frac{2}{3}$ ?

1. In 96 oz. how many half-pounds?
2. When milk is \$.05 a quart, what will 5 gallons cost?
3. Find the cost of  $\frac{3}{4}$  of a pound of tea at \$1.00 a pound, and 1 pound of butter at 25¢ a pound.
4. If you can buy 6 cards for a cent, how many can you buy for \$1?
5. What will  $\frac{1}{2}$  of a ton of coal cost at \$6.30 a ton? At \$5.10?
6. Find the cost of 7 chairs at \$3 each, 12 at \$2 each, and a table at \$15.
7. A man spent \$24 out of \$36. What per cent of his money had he left?
8. If you should put 320 oz. of cloves into quarter pound packages, how many packages would you have?
9. How many pint bottles will be required to hold 5 gal. 2 qt. of syrup?
10. Find the cost of 4 lb. 8 oz. of cheese at 8 cents a half-pound.
11. How many quarts will it take to fill a bag holding  $2\frac{1}{2}$  pk.?
12. If \$18 is  $\frac{1}{3}$  of my money, how much money have I?
13. If you gather  $\frac{1}{2}$  bu. of walnuts, and sell 1 pk. for 50 cents and the rest at 10 cents a quart, how much will you receive?
14. Find the cost of  $6\frac{1}{2}$  lb. of sugar at 6¢ a pound.
15. Find the cost of  $6\frac{1}{2}$  lb. of meat at 9 cents a pound.
16. Paid 5 cents a pound for 2 packages of buckwheat, each package containing  $3\frac{1}{2}$  lb. Find the cost.
17. What is the cost of 4 pk. 4 qt. of nuts at 20¢ a peck?
18. What 4 equal numbers make 36?
19. Nellie had 36 peaches. After eating  $\frac{1}{4}$  of them, and giving away  $\frac{1}{2}$  of them, how many had she left?
20. What is meant by  $\frac{1}{2}$  of anything?
21. What part of 8 is 3? What per cent of it?
22. How many days are there in 7 wk. 3 da.?

1. Add  $\frac{3}{4}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ , and  $\frac{3}{10}$ . First reduce each to its lowest terms.

2.  $78\frac{1}{2} - 49\frac{1}{4}$ .  $4\frac{1}{2} + 5\frac{3}{4} + 7\frac{1}{2}$ .

3. Simplify  $\frac{\frac{2}{3} \times \frac{1}{2} \times 4\frac{1}{2}}{6\frac{3}{4}}$ .

4. What per cent of an acre is a rectangular piece of land 5 rd. long and 4 rd. wide?

5. Divide 73.8 by .0008.  $1\frac{1}{2}$  by  $\frac{1}{4}$ .

6. Find the prime factors that are common to 1,155 and 2,205.

7. At \$6.40 a ton, how much will 3,675 lb. of coal cost?

8. Add 7.46, 536, 73.76, 7.569, 165, and 46.78.

9. Divide  $\frac{1}{3}$  by  $1\frac{1}{2}$ .  $1\frac{1}{2}$  by  $5\frac{1}{2}$ .

10.  $246 \times 8 - 169 \div 13 - (225 \div 15) \times 16 + 207 \div 3$ .

11. Find the entire surface of a square pyramid whose slant height is 7 ft., and base a 3 ft. square.

12. What will it cost, at 65¢ a square yard, to paint a church spire whose base is an octagon 7 ft. on each side, and whose slant height is 95 ft.?

13. What is the area of a circle, if the distance around it is 314.16 ft.?

NOTE. — In finding the diameter or circumference, use 3.1416 in place of  $3\frac{1}{2}$ . It is more nearly correct.

14. What is the convex surface of a cone, the diameter of whose base is 6 feet, and whose slant height is 24 ft.?

15. Of another cone, the circumference of whose base is 16 ft., and whose slant height is 18 ft.?

16. Find the entire surface of a cylinder whose circumference is 16 ft., and whose altitude is 25 ft.

17. What will it cost, at \$1.25 a square yard, to polish the convex surface of a cylinder 3 ft. in diameter and 12 ft. long?

18. How many acres are there in a circular field whose diameter is 22 rods?

1. How many square feet in a floor 28 ft. long and  $19\frac{1}{2}$  ft. wide?

2. From the middle point of one side of an equilateral triangle to the vertex is 21 feet. The base is  $24\frac{1}{2}$  ft. Required the area.

3. If 45 sheep cost \$540, what number will \$228 purchase?

4. If 12 cd. of wood cost \$54, what will  $19\frac{1}{2}$  cords cost?

5. How much commission shall I pay an auctioneer for selling my house for \$5,600, at  $2\frac{1}{2}\%$ ?

6. From a cask of rice containing 760 lb., 90 lb. were sold at one time, 60 lb. at another, and 40 lb. at another. What per cent was sold?

7. Add 7 rd. 4 yd. 2 ft. 7 in.; 19 rd. 3 yd. 2 ft. 9 in.; 5 rd. 3 yd. 1 ft. 6 in.; 3 rd. 4 yd. 2 ft. 5 in.; 1 rd. 3 yd. 1 ft. 11 in.

8.  $646\frac{1}{2}$  pounds of sugar cost \$51.72. The merchant wishes to gain \$3.23 $\frac{1}{2}$ . How much must he ask a pound to do so?

9. A rectangular block of marble is 7 ft. long, 4 ft. wide, and  $3\frac{1}{2}$  ft. high. If it costs 60¢ a cubic foot, find its cost.

10. The perimeter of a rectangle is 124 rd., the width is 12 rd. Find the area.

11. A man built a barn 80 ft. long, 45 ft. wide, with 30 ft. posts. The gable is 15 ft. high, and the rafters are 28 ft. long. Find how many feet of inch boards will be needed for sides and ends, allowing for 2 double doors 14 ft. by 12 ft., and 2 single doors 9 ft. by 6 ft. Find the feet of lumber needed for floor, covered with 2-inch planks. Find the feet of roof-boards needed. Find the feet of lumber needed for doors, if  $1\frac{1}{2}$  in. boards are used. Find the cost of lumber at \$16 per M.

12. A merchant paid \$10 for an article, and sold it at an advance of 20%. Find the selling-price.

13. Find the value of a pile of wood 40 ft. long, 4 ft. wide, and 4 ft. 6 in. high, at \$5.50 a cord.

1. A man paid \$960 for a carriage and span of horses, paying three times as much for the horses as for the carriage. How much did he pay for each?

In examples like the above use the symbol  $x$ . This  $x$  stands for any unknown quantity.

Let  $x$  = the cost of the carriage.  
then  $3x$  = the cost of the horses.  
 $4x$  = the cost of all, which is \$960.  
 $4x = \$960.$        $x = \$240.$        $3x = \$720.$

2. A and B together had \$150, and B had four times as much money as A. How many dollars had each?

3. A man bought a harness and a robe for \$45. The harness cost 4 times as much as the robe. What was the price of each?

4. Three men, A, B, and C, form a company with a capital of \$6,000. C put in 2 times as much as A, and B 3 times as much as A. How many dollars did each put in?

5. 108 marbles are divided among 3 boys. A takes a certain sum, B takes 3 times as many as A, and C takes 5 times as many as A. What is each boy's share?

6. Divide \$81 among A, B, and C, so that B shall have 5 times as much as A, and C 3 times as much as A.

7. A horse and carriage are together worth \$450. The horse is worth twice as much as the carriage. What is each worth?

8. A man has 3 houses, which are together worth \$5,400. The second is worth twice as much as the first, and the third is worth as much as the other two. Find the cost of each.

9. Divide \$216 among A, B, and C, so that B may have 3 times as much as A, and C may have 5 times as much as A.

10. Divide the number 136 into three parts such that the second shall be 3 times the first and the third as much as the first and second.

1. The difference between 5 times a number and 3 times a number is 40. What is the number?

Let	$x$ = the number	The difference between $5x$ and $3x$
then	$5x = 5$ times the number	is $2x$ , hence $2x$ must equal 40.
and	$3x = 3$ times the number	
	$2x = 40$	
	$x = 20.$	

2. A has three times as much money as B. If the difference between their shares is \$200, how much money has each?

3. John had a certain number of papers, and George had 12 times as many. If George had 33 papers more than John, how many papers had each boy?

4.  $6x - 4x = 10$ . Make a problem and solve it.

Illustration: If from 6 times a certain number 4 times the number is taken, the remainder will be 10. Find the number.

Make problems for the following and solve them.

5.  $9x - 4x = 15.$

6.  $5x + 3x + x = 81.$

7.  $6x + 7x = 39.$

8.  $6x + 4x - 2x = 72.$

9.  $11x - 6x = 50.$

10.  $x + 2x + 3x = 48.$

11. Margaret is four times as old as Helen, and Ruth is half as old as Margaret. If the sum of their ages is 14 years, how old is each?

12. Robert has three times as many cents as Amy, and Charles has four times as many as Amy. If Charles has 12 cents more than Robert, how many cents has each?

13. Elizabeth had a certain sum of money when her father gave her 5 times as much more. She then had 72 cents. How much money had she at first?

14. Joseph had 7 times as many marbles as Henry, and Charles had 5 times as many as Henry. Joseph had 18 marbles more than Charles. How many marbles had each boy?

15. A, B, C and D have \$140. B has \$10 more than A, C has \$10 more than B, and D has \$10 more than C. How many dollars has each?

1. How many feet in two boards 14 ft. long, 12 in. wide?
2. How long is it from 15 minutes of 7 o'clock in the morning till 15 minutes past 6 o'clock in the evening?
3. How many cubic feet of earth must be removed to dig a cellar 30 ft. long, 20 ft. wide, and 4 ft. deep?
4. How many square yards in 27 sq. ft.? In 54 sq. ft.? 72 sq. ft.?
5. How many inches in  $4\frac{1}{2}$  ft.?
6. How many inches in  $2\frac{1}{2}$  yd.?
7. How high is a horse that is  $16\frac{1}{2}$  hands high?

NOTE. — A hand is 4 in., a term used only in speaking of the height of horses.

8. How many feet deep is water that is  $9\frac{1}{2}$  yd. deep?
9. If by selling a horse for \$160 I gain  $\frac{1}{4}$  of the cost, what did he cost?
10. Read the 9th example, using the equivalent per cent in place of the fraction.
11. If two-thirds of a dozen oranges cost 16 cents, how much will  $2\frac{1}{2}$  dozen cost?
12.  $\frac{2}{3}$  of 72 is  $\frac{2}{3}$  of how many?
13.  $\frac{3}{4}$  of 48 is  $\frac{3}{4}$  of how many?
14. A farmer sold  $\frac{1}{2}$  of his tobacco crop to one man, and  $\frac{1}{4}$  to another, and had 100 lb. remaining. How much had he at first?
15. If  $3\frac{1}{2}$  lb. of meat cost 50 cents, how much will  $2\frac{1}{2}$  lb. cost?
16. When  $\frac{2}{3}$  of my monthly salary is \$9, how much do I earn in  $\frac{2}{3}$  of a month?
17. What must I pay for  $9\frac{1}{2}$  bbl. of apples, if  $1\frac{1}{2}$  bbl. cost \$6?
18. If  $\frac{1}{4}$  of 8 cords of wood costs \$12, how much will  $\frac{1}{2}$  of 3 cords cost?
19. What will  $8\frac{1}{2}$  bbl. of apples cost, if  $2\frac{1}{2}$  bbl. cost \$9?
20.  $\frac{2}{3}$  of 100 is  $\frac{2}{3}$  of what number?
21. 8 is  $\frac{2}{3}$  of what number?  $1\frac{1}{2}$  is  $\frac{1}{3}$  of what number?
22. What will 1 gal. of cream cost at 30¢ a quart?

1. If I use another's house, what is the money I pay for the use of it called?

2. If I use a man's horse, what is the money I pay for the use of it called?

3. If I use another's money, what is the money I pay for the use of it called? Interest.

4. Learn : — Interest is money paid for the use of money. It is usually reckoned at a certain per cent for each year.

NOTE. — Interest differs from house-rent in that it is always a certain per cent of the money borrowed.

5. I asked a man to lend me some money. He said he would at 6%. What did he mean?

6. When one man borrows money of another, he usually gives him a paper called a promissory note, to prove that he has borrowed money.

\$100

Boston, Nov. 5, 1902.

On demand, I promise to pay to John Smith, or order, One Hundred Dollars, with interest at 6%.

Value received.

Geo. Whittaker.

7. In this note who promises to pay? To whom does he promise to pay? How much does he promise to pay?

8. The one who promises to pay is called the *promisor* or *maker*. The one to whom the promise is made is called the *promisee* or *payee*. The sum of money is called the *face*. Name the maker, payee, and face in this note.

9. When the words "on demand" are in the note, the payee can demand the payment of the money at any time. When the words "or order" are in the note, the payee can order the money paid to some one besides himself.

10. Write a demand note, using the following: date, to-day; maker, yourself; payee, your teacher; face, any sum.



1. Sometimes the words "on demand" are not in the note, but in their place a certain specified time. The following is a time note:

\$100

Boston, Nov. 5, 1902.

Six months after date I promise to pay John Smith, or order, One Hundred Dollars, with interest at 6%. Value received.

Geo. Whittaker.

2. If I must pay 6 cents for the use of \$1 for 1 year, what part of the year can I keep the dollar and pay only 1 cent interest? How many months is  $\frac{1}{6}$  of the year?

3. Then what is the interest of \$1 for 2 mo.?

4. If the interest of \$1 for 2 mo. is .01, what will be the interest of \$5? \$8? \$10? \$100?

5. If the interest of \$1 is .01 for 2 mo., the interest of \$100 will be 100 times \$.01, which is \$1.00. What short method can you see for multiplying .01 by any number?

6. Learn: Move the decimal point two places to the left to find the interest on any sum of money for two months at 6%.

7. Why does pointing off two places give the interest for 2 mo.?

Find the interest on the following sums of money for 2 mo. at 6%:

8.	\$60	\$ 88	\$305	\$548	\$5,678
9.	30	72	475	842	4,978
10.	48	46	267	648	9,876
11.	39	83	394	500	4,748
12.	62	450	412	402	8,649
13.	79	276	516	819	6,891
14.	12	118	618	549	4,206
15.	24	374	728	675	2,060
16.	31	333	175	715	1,008

**NOTE.** — In all examples in Interest on this page 6% is understood.

- 1. Find the interest of \$500 for 6 mo.**

\$5.00 = int. for 2 mo.      First find the interest for 2 mo. by pointing  
           3                               off two places; this is \$5. If \$5 is the interest  
 \$15.00 = int. for 6 mo.    for 2 mo., the interest for 6 mo. will be 3 times  
                                         as much.

2. How would you find the interest for 4 mo.? 8 mo.? 10 mo.? 1 year (12 mo.)? 1 yr. 4 mo. (16 mo.)?

3. Take the sums of money on page 12, and find the interest on each for 4 mo. 6 mo. 8 mo. 10 mo. 1 year.

- 4. Find the interest of \$600 for 10 mo.**

5. Find the interest for 6 mo. on \$1,200. On \$2,400.

- 6. Find the interest for 1 year on \$750. On \$300. On \$250.**

7. Find the interest for 1 yr. 2 mo. on \$405. On \$145.  
On \$69.

- 8. Find the interest for 1 yr. 8 mo. on \$1,216. On \$2,445.**

9. On \$1,218 for 1 yr. 10 mo.

10. On \$600 for 1 yr. 4 mo.

11. On \$436 for 8 mo.

- 12.** Find the interest of \$800 for 9 mo.

\$ 8.00 = 2 mo. First find for 8 mo. as usual. Then for the odd month, which must be  $\frac{1}{2}$  of the interest for 2 months.

$$\begin{array}{r} \$32.00 = 8 \text{ mo.} \\ 4.00 = 1 \text{ mo.} \\ \hline \$36.00 = 9 \text{ mo.} \end{array}$$

+ 1 mo.)

- 16.** On \$3,460 for 1 yr. 5 mo. (16 mo. + 1 mo.).

17. On \$489 for 1 yr. 2 mo. For 9 mo.

- 18.** On \$638 for 1 yr. 7 mo. (18 mo. + 1 mo.).

- 19.** On \$1,240 for 11 mo. For 1 yr. 5 mo.

- 20.** On \$9,876 for 5 mo. For 1 yr. 3 mo.

- 21.** On \$4,675 for 3 mo. For 1 yr. 2 mo.

- 22.** On \$4,106 for 7 mo. For 1 yr. 3 mo.

1. What is the interest on \$1 for 2 mo., or 60 days?
2. 6 days is what part of 60 days?
3. The interest on \$1 for 6 days is what part of the interest for 60 days?
4. What is  $\frac{1}{10}$  of .01?
5. What is the interest of \$1 for 6 days?
6. If the the interest on \$1 for 6 days is 1 mill, what is the interest on \$2? \$7?
7. At the same rate, the interest on \$100 is how many times .001?
8. Multiply .001 by 100; by 200; by 600; by 1000.
9. Tell a short way to multiply .001 by any number.
10. How many places to the left do you move the point?
11. Moving the point three places to the left is the same as multiplying by what?
12. Learn: Moving the point three places to the left gives the interest on any sum of money for six days.
13. Find the interest at 6% for 6 days on the sums of money given on page 12.
14. Tell a short way to find the interest for 12 days.
15. Find the interest on \$600 for 12 days.  

$$\begin{array}{r} \$600 \text{ interest for 6 days.} \\ \underline{\phantom{00}2} \\ \$1200 \text{ interest for 12 days.} \end{array}$$
16. How do you find the interest for 18 days? For 24 days?
17. Find the interest on \$1,240 for 18 days.
18. Find the interest on \$480 for 24 days.
19. Find the interest on \$697 for 12 days.
20. Find the interest on \$368 for 30 days.
21. Find the interest on \$267 for 36 days.
22. Find the interest on \$142 for 42 days.
23. Find the interest on \$612 for 18 days.
24. Find the interest on \$210 for 24 days.

The rate of interest in each of the following examples is 6%.

Find the interest of :

1. \$100 for 1 yr.                      \$100 for 1 yr. 6 mo.
2. \$100 for 1 yr. 6 mo. 6 da.      \$200 for 8mo. 12 da.
3. \$200 for 10 mo. 18 da.          \$200 for 1 yr. 5 mo.
4. \$200 for 1 yr. 7 mo. 24 da.      \$300 for 4 mo.
5. \$300 for 9 mo.                    \$300 for 3 mo. 18 da.
6. \$400 for 6 mo. 12 da.            \$400 for 1 yr. 3 mo. 18 da.
7. \$400 for 9 mo. 12 da.            \$400 for 1 yr. 8 mo. 6 da.
8. \$500 for 7 mo.                    \$500 for 1 yr. 2 mo.
9. \$500 for 1 yr. 4 mo. 12 da.      \$500 for 60 da.
10. \$500 for 36 da.                  \$600 for 1 yr. 1 mo. 6 da.
11. \$600 for 10 mo. 24 da.          \$600 for 48 da.
12. \$600 for 1 yr. 2 mo. 12 da.    \$1,000 for 9 mo.
13. Find the interest of \$200 for 2 mo.
14. Find the interest of \$300 for 3 mo.
15. Find the interest of \$400 for 4 mo.
16. Find the interest of \$500 for 5 mo.
17. Find the interest of \$600 for 6 mo.
18. Find the interest of \$700 for 7 mo.
19. Find the interest of \$800 for 8 mo.
20. Find the interest of \$900 for 9 mo.
21. Find the interest of \$1,000 for 10 mo.
22. Find the interest of \$400 for 6 days.
23. Find the interest of \$600 for 12 days.
24. Find the interest of \$800 for 18 days.
25. Find the interest of \$600 for 1 yr. 2 mo. 12 da.
26. Find the interest of \$700 for 24 days.
27. What is the interest of \$5 for 1 yr. 10 mo. 12 days?
28. What is the interest of \$200 for 1 yr.? For 2 yr.? For 6 mo.?
29. What is the interest of \$500 for 1 yr.? For 1 yr. 6mo.? For 2 yr. 6 mo.?

Find the interest at 6% :—

1. Of \$846 for 1 yr. 2 mo.
2. Of \$846 for 12 days.
3. Of \$846 for 1 yr. 2 mo. 12 days.
4. Of \$1,728 for 1 yr. 4 mo. 6 da. For 1 yr. 8 mo. 18 da.
5. Of \$466.56 for 8 mo. 12 da. For 1 yr. 6 mo. 12 da.
6. Of \$2,304 for 10 mo. 18 da. For 1 yr. 4 mo. 12 da.
7. Of \$450 for 1 yr. 3 mo. 6 da. For 1 yr. 7 mo. 18 da.
8. Of \$800 for 1 yr. 5 mo. 12 da. For 1 yr. 9 mo. 12 da.
9. Of \$375 for 24 days. For 1 yr. 5 mo. 18 da.
10. Of \$323.50 for 1 yr. 10 mo. 18 da. For 11 mo. 12 da.
11. Of \$960 for 1 yr. 3 mo. 18 da. For 5 mo. 6 da.
12. Of \$842 for 9 mo. 12 da. For 1 yr. 3 mo. 18 da.
13. Of \$700 for 6 mo. 18 da. For 1 yr. 9 mo. 24 da.
14. Of \$1,200 for 4 mo. 12 da. For 1 yr. 6 mo. 6 da.
15. Of \$900 for 1 yr. 7 mo. 18 da. For 1 yr. 5 mo. 24 da.
16. Of \$1,400 for 11 mo. 12 da. For 1 yr. 2 mo. 18 da.
17. Of \$976.25 for 4 mo. For 1 yr. 5 mo. 12 da.
18. Of \$846.78 for 1 yr. 6 mo. For 1 yr. 9 mo. 12 da.
19. Of \$180 for 1 yr. 3 mo. 24 da. For 1 yr. 5 mo. 18 da.
20. Of \$680.60 for 10 mo. For 1 yr. 11 mo. 12 da.
21. Of \$211.25 for 1 yr. 5 mo. For 1 yr. 6 mo. 18 da.
22. Of \$1,234.50 for 1 yr. 3 mo. 18 da. For 1 yr. 2 mo. 24 da.
23. Of \$666.60 for 1 yr. 2 mo. 12 da. For 1 yr. 3 mo. 18 da.
24. Of \$888.80 for 1 yr. 8 mo. 18 da. For 1 yr. 5 mo. 24 da.
25. Of \$555.50 for 1 yr. 4 mo. 6 da. For 1 yr. 7 mo. 30 da.
26. Of \$368.60 for 1 yr. 6 mo. 24 da. For 1 yr. 9 mo. 12 da.
27. Of \$169.50 for 1 yr. 10 mo. 12 da. For 1 yr. 11 mo. 18 da.
28. Of \$444.40 for 1 yr. 2 mo. 6 da. For 1 yr. 8 mo. 24 da.
29. Of \$333.30 for 1 yr. 4 mo. 12 da. For 1 yr. 5 mo. 18 da.
30. Of \$777.70 for 1 yr. 6 mo. 18 da. For 1 yr. 9 mo. 30 da.
31. Of \$999.90 for 1 yr. 8 mo. 24 da. For 1 yr. 9 mo. 24 da.
32. Of \$1,224.60 for 1 yr. 10 mo. 12 da. For 1 yr. 11 mo. 18 da.

1. What is the interest of \$600 for 60 days at 6%? How did you find it?
2. What is the interest of \$600 for 6 days at 6%? How did you find it?
3. Find the interest of \$600 for 7 days at 6%.

$$\$600 = \text{int. for 6 d.}$$

$$.10 = \text{int. for 1 d.}$$

$$\underline{.70} = \text{int. for 7 d.}$$

4. Find the interest of \$400 for 25 d.

$$\$4.00 = \text{int. for 60 d.}$$

$$1.333 = \text{int. for 20 d.}$$

$$.333 = \text{int. for 5 d.}$$

$$\underline{\$1.666} = \text{int. for 25 d.}$$

Point off 2 places to find the interest for 60 d.

To find the interest for 20 d., take  $\frac{1}{3}$  of the interest for 60 d. To find the interest for 5 d., take  $\frac{1}{4}$  of the interest for 20 d.

5. Find the interest on \$300 for 19 d.

$$\$3.00 = \text{int. for 60 d.}$$

$$.50 = \text{int. for 10 d.}$$

$$.30 = \text{int. for 6 d.}$$

$$.15 = \text{int. for 3 d.}$$

$$\underline{.95} = \text{int. for 19 d.}$$

To find the interest for 10 days take  $\frac{1}{6}$  of the interest for 60 days. To find the interest for 6 d. take  $\frac{1}{10}$  of the interest for 60 d. To find the interest for 3 d. take  $\frac{1}{2}$  of the interest for 6 d.

6. To find the interest on any sum of money for a certain number of days: Move the decimal point two places to the left to find the interest for 60 d., or three places for 6 d., and then take such parts of the interest thus found, as when united will give the interest for the specified time.

The following table will show the method:—

1 d. = $\frac{1}{6}$ of 6 days.	14 d. = 12 d. + 2 d.
2 d. = $\frac{1}{3}$ of 6 days.	15 d. = $\frac{1}{4}$ of 60 d.
3 d. = $\frac{1}{2}$ of 6 d.	16 d. = 15 d. + 1 d.
4 d. = 3 d. + 1 d.	17 d. = 15 d. + 2 d.
5 d. = $\frac{1}{2}$ of 60 d.	18 d. = 15 d. + 3 d. or $3 \times 6$ d.
7 d. = 6 d. + 1 d.	19 d. = 10 d. + 6 d. + 3 d.
8 d. = 6 d. + 2 d.	20 d. = $\frac{1}{3}$ of 60 d.
9 d. = 6 d. + 3 d.	21 d. = 20 d. + 1 d.
10 d. = $\frac{1}{6}$ of 60 d.	22 d. = 20 d. + 2 d.
11 d. = 10 d. + 1 d.	23 d. = 20 d. + 3 d.
12 d. = $\frac{1}{5}$ of 60 d.	24 d. = 20 d. + $\frac{1}{2}$ of 20 d.
13 d. = 12 d. + 1 d.	25 d. = 20 d. + $\frac{1}{4}$ of 20 d.

Find the interest on the following sums of money at 6% :

1. \$475 for 5 days. For 1 mo. 5 da.
2. 368 for 7 days. For 2 mo. 7 da.
3. 694 for 8 days. For 5 mo. 8 da.
4. 176.34 for 9 days. For 3 mo. 9 da.
5. 25.60 for 10 days. For 6 mo. 10 da.
6. 136.50 for 11 days. For 4 mo. 11 da.
7. 97.50 for 13 days. For 5 mo. 13 da.
8. 1,264 for 14 days. For 3 mo. 14 da.
9. 1,478 for 15 days. For 7 mo. 15 da.
10. 672 for 16 days. For 4 mo. 16 da.
11. 47.95 for 17 days. For 9 mo. 17 da.
12. 106.25 for 19 days. For 3 mo. 19 da.
13. 16.48 for 20 days. For 10 mo. 20 da.
14. 74.34 for 21 days. For 2 mo. 21 da.
15. 1,497 for 22 days. For 4 mo. 22 da.
16. 748.25 for 23 days. For 8 mo. 23 da.
17. 1,200 for 25 days. For 3 mo. 25 da.
18. 567 for 26 days. For 1 mo. 26 da.
19. 1,463 for 27 days. For 6 mo. 27 da.
20. 1,476.36 for 28 days. For 10 mo. 28 da.
21. 1,472.40 for 29 days. For 1 yr. 1 mo. 29 da.
22. 1,491.50 for 13 days. For 1 yr. 3 mo. 17 da.
23. 2,468 for 26 days. For 1 yr. 1 mo. 1 da.
24. 680.50 for 27 days. For 1 yr. 5 mo. 16 da.
25. 746.30 for 11 days. For 1 yr. 2 mo. 13 da.
26. 123.40 for 19 days. For 1 yr. 3 mo. 15 da.
27. 567.80 for 21 days. For 1 yr. 7 mo. 19 da.
28. 912.30 for 17 days. For 1 yr. 2 mo. 21 da.
29. 987.60 for 15 days. For 1 yr. 3 mo. 24 da.
30. 876.50 for 16 days. For 1 yr. 4 mo. 25 da.
31. 765.40 for 19 days. For 1 yr. 5 mo. 26 da.
32. 423.70 for 13 days. For 1 yr. 3 mo. 21 da.

1. Find the interest on \$1,200 for 1 yr. 2 mo. 18 da. at 5%.  
 $\$12.00 = 2 \text{ mo.}$   
 $\$84.00 = 1 \text{ yr. 2 mo.}$   
 $3.60 = 18 \text{ da.}$
- 6)  $\$87.60 = 6\%$ .  
 $14.60 = 1\%$ .  
 $\$73.00 = 5\%$ .
  - First find the interest at 6%, as usual. If \$87.60 is the interest at 6%, \$14.60 will be the interest at 1%, and \$73.00, the difference between 6% and 1%, will be the interest at 5%.
  2. How would you find the interest at 7%? What per cent would you add to 6%?
  3. What per cent would you subtract from 6% to find 4%?
  4. How do you find 2% when you know 6%?
  5. When you know 6%, what part of it must you find to know 3%?
  6. 8% is how many per cent greater than 6%? 2% is what part of 6%? What part of 6% then do you add to 6% to get 8%?
  7. 9% is how many per cent more than 6%? 3% is what part of 6%? What part of 6% then do you add to 6% to get 9%?
  8. How do you find interest at 3% the shortest way?
  9. How do you find interest at 4% the shortest way? At 5%? At 7%? At 8%? At 9%? At 10%? At  $4\frac{1}{2}\%$ ? At  $7\frac{1}{2}\%$ ?
  10. Find the interest on \$486.50 for 1 yr. 2 mo. 24 da. at 5%. 7%. 9%.
  11. Find the interest on \$247.65 for 1 yr. 3 mo. 15 da. at 4%. 8%.
  12. Find interest on \$1,264.30 for 1 yr. 5 mo. 13 da. at 5%. 7%.
- Find the interest of:
  13. \$798.81 for 1 yr. 1 mo. 1 d. at 8%.
  14. \$346.84 for 1 yr. 11 mo. 18 d. at 5%.
  15. \$816.24 for 1 yr. 7 mo. 6 da. at 7%.
  16. \$745.00 for 10 mo. 10 da. at 10%.



1. If 1 yd. of cloth cost  $\$5\frac{1}{2}$  what will  $\frac{3}{4}$  yd. cost?
2. If 5 bbl. of beef cost  $\$17\frac{1}{2}$ , what will  $1\frac{1}{2}$  bbl. cost?
3.  $\frac{3}{4}$  of 15 is  $\frac{1}{2}$  of how many times 2?
4.  $\frac{2}{3}$  of 18 is  $\frac{1}{3}$  of how many times 7?
5.  $\frac{3}{4}$  of 24 is  $1\frac{1}{2}$  times what number?
6. If  $\frac{1}{4}$  of an acre of land is worth  $\$15$  what are  $10\frac{1}{2}$  acres worth?
7. If you earn  $\$1\frac{1}{2}$  in a day and your brother  $\$3$  in a day, how much will you both earn in 8 days?
8. If Nellie has twice as many plums as Mary, and both have 18 plums, how many has each?
9. 84 is  $\frac{1}{2}$  of how many times  $\frac{1}{3}$  of 25?
10.  $7\frac{1}{2}$  are how many times 9?
11. Reduce to improper fractions:  $6\frac{1}{2}$ ,  $9\frac{1}{2}$ ,  $7\frac{1}{2}$ ,  $5\frac{1}{2}$ ,  $4\frac{1}{2}$ .
12. What will  $\frac{1}{3}$  of a dozen of oranges cost at  $\frac{1}{4}$  of a cent each?
13. What will 1 pt. of buckwheat cost if 3 pk. cost 48 cents?
14. Mr. C sold a farm for  $\$1,200$ , which was  $\frac{1}{4}$  of its cost. Find the loss.
15. How many cents will  $\frac{1}{2}$  of 100 oranges cost at  $\frac{1}{2}$  dime each?
16. How many times 8 are  $1\frac{1}{2}$  of 26?
17. If 7 bbl. of cider cost  $\$28$ , what will 6 bbl. cost?
18. If a boy can do a piece of work in 30 min., how many hours will it take him to do 12 times as much work?
19. If 8 yd. of cloth are worth  $\$40$ , and butter is  $\$3$  a box, how many boxes of butter will 9 yd. of cloth buy?
20. How much will 6 yd. of silk cost at  $\$2\frac{1}{2}$  a yard?
21. Mr. Brown is 40 years old, and his son is  $\frac{1}{4}$  as old. How old is the son?
22. A man sold  $2\frac{1}{2}$  yd. of velvet from a piece containing  $3\frac{1}{2}$  yd. How many yards had he left?
23. What is the ratio of 8¢ to 32¢?

1. What is the cost of carpeting a room  $16\frac{1}{2}$  ft. long, 12 ft. wide, with oil-cloth  $1\frac{1}{2}$  yd. wide, at 75¢ a yard?

NOTE. — Have the least waste.

2. What will it cost to carpet a room 18 ft. long, 13 ft. wide, with carpeting  $\frac{3}{4}$  yd. wide, at \$1.25 a yard, breadths to run lengthwise, and a waste on each breadth of 8 in. for matching the figures?

NOTE. — Sometimes, in order to have the figures in the carpet match, it is necessary to make each breadth longer than the room. This is called a waste, because it obliges the purchaser to buy more carpet than he needs for the room. Add the waste on each breadth to the length of the room when the carpet runs lengthwise, and to the width when it runs widthwise, to find the length of each breadth. Make a diagram to illustrate.

3. A church steeple is in the form of a pyramid. Its base is a square 15 ft. on each side, and its slant height is 70 ft. What is the cost of painting it at 30¢ a square yard?

4. Find the difference in cost of painting this spire had the base been a hexagon having the same dimensions.

5. What is the area of a semicircle whose radius is 12 ft.?

6. Draw a circle with a radius of 5 in. This is a drawing to represent a lot of land. If the scale is  $\frac{1}{4}$  in. to a rod, how many square rods in the field? How many acres?

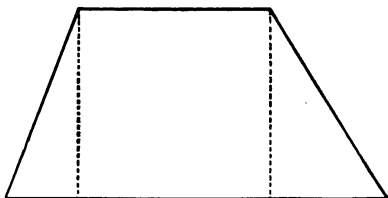
7. A rectangular cistern is 8 ft. long, 6 ft. 6 in. wide, and  $4\frac{1}{2}$  ft. deep. Find the expense of lining the sides and bottom with sheet lead weighing 9 lb. a square foot, at 5¢ a pound.

8. A pile of wood contains  $6\frac{1}{2}$  cd. If the pile is 32 ft. long and  $6\frac{1}{2}$  ft. high, how wide is it?

9. A room is 15 ft. 3 in. long and 12 ft. 6 in. wide. The carpet is  $\frac{3}{4}$  of a yard wide. There is a loss on each strip of 4 in. for matching. Which way should the breadths run to use the least number of yards of carpeting?

10. Find the area of a rectangle 15 yd. long, and 45 ft. wide.

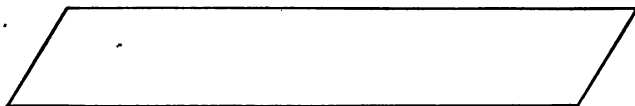
11. Find the area of a rectangle 150 ft. long, 4 rd. wide.



1. What figure is this?
2. Cut out of paper a trapezoid, twice as large as this figure.
3. Fold so that the two parallel edges will coincide. Crease. Cut on the crease.

Place the two pieces so that what were the two parallel lines shall form one continuous line.

4. What new form have you?



5. How do you find the area of a parallelogram?
6. The base of the parallelogram is the sum of what two lines in the trapezoid?
7. The height of the parallelogram is what part of the height of the trapezoid?
8. Formulate a rule for finding the area of a trapezoid.
9. Instead of multiplying the sum of the two parallel sides by one-half of the altitude, can you find another way for finding the area of a trapezoid?

NOTE. — Into what figures can you divide the trapezoid?

10. Find the area of a trapezoid whose parallel sides measure 11 ft. and 16 ft., and the perpendicular distance between them 60 ft.
11. Find the area of a trapezoid whose parallel sides are 60 ft. and 130 ft., and altitude 40 ft.
12. One parallel side of a field in the shape of a trapezoid is 150 yd. The other is 200 yd. How many square yards in the field, the perpendicular distance between the sides being 50 yards?

(See pages 271-275.)

1. Divide five thousand fifty and five tenths by five hundredths, and subtract twenty-five hundredths from the quotient.
2. Multiply seventy-three hundred-thousandths by one thousand.
3. 
$$\frac{.035 \times .0056}{.00007} = ?$$
4. Add fifteen thousandths, eighty-one ten-thousandths, fifty-six millionths, seventeen ten-millionths, two hundred five hundred-thousandths.
5. Change to decimals and add:  $1\frac{1}{2}$ ,  $4\frac{3}{4}$ ,  $5\frac{1}{8}$ ,  $2\frac{3}{8}$ ,  $1\frac{1}{16}$ .
6. Multiply .032 by .005, and then divide .0512 by your product.
7. Take 27 and 28 thousandths from 97 and 7 tenths.
8. Reduce to common fractions: .055, .008.
9. Reduce to decimals:  $\frac{1}{11}$ ,  $\frac{2}{5}$ ,  $\frac{108}{1000}$ ,  $13\frac{1}{3}$ .
10. Multiply 12 thousandths by 12 hundredths, and from the product take 12 millionths.
11. Multiply 160 by .016, and divide the product by .0025.
12. Add 8.3, 2.576, 3.42, 1.5, 6.279, .003, 1.417.
13. Multiply .084 by .0036.
14. Find the cost of 18,755 ft. of lumber at \$24.75 per M.
15. Divide .005,232 by .016.
16. Multiply the difference between 4.4 and .00027 by the product of 2.1 and .005.
17. Divide fifty and five thousandths by five millionths.
18. Multiply twelve thousandths by fifteen hundredths, and divide the product by five tenths.
19. Divide two tenths by five ten-millionths.
20. Add: 243 thousandths, 203 ten-thousandths, 546 millionths, 12 and 1,234 hundred-thousandths, 116 ten-millionths.
21. Divide 15.625 by  $31\frac{1}{4}$ .
22. Divide .08 by .0016. 670.08 by .016.

1. Name a number that is divisible by 4.
2. A number that is divisible by another number is a multiple of it.
3. Name some numbers that are divisible by both 4 and 6.
4. A common multiple of two or more numbers is a number that is divisible by all of the given numbers.
5. What is the least common multiple of 3, 4, and 6?
6. The least common multiple of two or more numbers is the least number that is divisible by the given numbers.
7. A multiple of a number contains all the prime factors of that number.
8. A common multiple of two or more numbers contains *all the factors* of each of the numbers.
9. Find the least common multiple of 8, 12, 20, and 30.

$$\begin{array}{r}
 2 \overline{) 8 \quad 12 \quad 20 \quad 30} \\
 2 \overline{) 4 \quad 6 \quad 10 \quad 15} \\
 3 \overline{) 2 \quad 3 \quad 5 \quad 15} \\
 5 \overline{) 2 \quad 1 \quad 5 \quad 5} \\
 \hline
 2 \quad 1 \quad 1 \quad 1
 \end{array}$$

$$2 \times 2 \times 2 \times 3 \times 5 = 120$$

Arranging the numbers in a line, we notice that 2 is a factor of all the numbers, hence (8) it must be a factor of their multiple. Remove the factor 2 by division. We notice again that 2 is a factor of 4, 6, and 10. It must then be a factor of any number that can be divided by 4, 6, and 10. As at first, remove the factor 2 by dividing, and since 2 is not a factor of 15, place the 15 in the next line with the quotients. In the same way continue to remove any factor of two or more of the numbers. The continued product of these divisors and the last quotients will be the least common multiple.

10. Find the least common multiple of 8, 16, 30, 48, 60, and 75.
11. Of 28, 30, 40, 56, and 60.
12. Of 42, 58, 84, 91, and 98.
13. Of 2, 3, 4, 5, 6, 7, 8, and 9.
14. Of 14, 18, 22, and 24.
15. Of 15, 27, 35, 42, and 70.
16. Of 27, 36, 45, 90.
17. Of 18, 24, 27, 45.

1. Find 125% of 12; 20; 80; 120; 240.
2. Find .66 $\frac{2}{3}$  of 27. Find 66 $\frac{2}{3}$ % of 27.
3. Express in common fractions in the lowest terms: 10%; 75%; 150%; 37 $\frac{1}{2}$ %; 116 $\frac{2}{3}$ %.
4. Find 50% of \$72; \$ $\frac{1}{2}$ ; \$12 $\frac{3}{4}$ ; 40 books; .4; 50%; \$500.
5. A man gained \$26 by buying flour at \$5 a barrel, and selling it at a gain of 20%. How many barrels did he sell?
6. What is 100% of \$10? 4 oranges? 79 m.?
7. A regiment of 400 men went into battle, where 25% of them were killed. How many men were not killed?
8. A merchant paid \$5.75 for an article, and sold it at a profit of 20%. How much did he gain? How much did he sell it for?
9. What part of 12 is 6? 16 is 4? 25 is 5? 12 is 2?
10. Substitute per cent for part in example 9, and give answer.
11. What part and what per cent of 10 is 5? 100 is 25? 500 is 50? 20 is 20? 120 is 40? 75 is 25? 40 is 32? 240 is 20?
12. A herd of 300 cattle was driven through a town. If the farmers bought 50 cows from the herd, what per cent of the herd did they buy?
13. I invested \$540 and lost \$90. What per cent did I lose?
14. I bought 5 dozen oranges, but threw away 6 because they were poor. What per cent did I throw away?
15. Of what number is 40, 20%? 12, 6%? 9, 100%?
16. 12 is  $\frac{1}{3}$  more than what number?
17. 12 is 133 $\frac{1}{3}$ % of what number?
18. 20 is  $\frac{1}{4}$  more than what number?
19. 20 is 125% of what number?
20. 20 is 25% more than what number?
21. How many inches in 50% of a yard?
22. A grocer paid 60¢ a pound for tea, and sold it so as to gain 25%. Find the gain.

(For Summary of Processes in Fractions, see pages 263-270.)

NOTE. — In addition and subtraction of fractions find the least common multiple of the denominators. This can often be found by inspection.

1. Add:  $20\frac{3}{4}$ ,  $128\frac{3}{4}$ ,  $4\frac{1}{2}$ ,  $81\frac{3}{4}$ .
2. Add:  $71\frac{1}{2}$ ,  $96\frac{3}{4}$ ,  $17\frac{1}{8}$ ,  $9\frac{3}{4}$ ,  $44\frac{3}{4}$ .
3. Add:  $136\frac{3}{4}$ ,  $7\frac{3}{4}$ ,  $35\frac{1}{4}$ ,  $71\frac{1}{2}$ .
4. From  $145\frac{3}{4}$  take  $76\frac{1}{4}$ .
5. From  $717\frac{3}{4}$  take  $196\frac{1}{8}$ .
6. From  $387\frac{3}{4}$  take  $214\frac{3}{4}$ .
7. Multiply:  $875\frac{3}{4}$  by  $234$ .
8. Multiply:  $645\frac{3}{4}$  by  $412$ .
9. Multiply:  $215$  by  $41\frac{3}{4}$ .
10. Multiply  $575$  by  $69\frac{3}{4}$ .
11. Find  $\frac{3}{4}$  of  $\frac{3}{4}$  of  $\frac{3}{4}$  of  $\frac{3}{4}$ .
12. Find  $\frac{3}{4}$  of  $\frac{1}{2}$  of  $4\frac{3}{4}$ .
13.  $(\frac{1}{2} + \frac{3}{4}) \times (\frac{1}{2} - \frac{1}{4})$ .
14. Multiply  $136\frac{3}{4}$  by  $41\frac{3}{4}$ .
15. Multiply  $612\frac{1}{4}$  by  $42\frac{3}{4}$ .
16. Divide  $6,345\frac{3}{4}$  by  $16$ .
17. Divide  $583\frac{1}{4}$  by  $43\frac{3}{4}$ .
18. Simplify:  $\frac{18\frac{1}{2} \div 12\frac{1}{2}}{16\frac{1}{2} - 15\frac{3}{4}}$ .  $\frac{\frac{3}{4} \text{ of } \frac{3}{4}}{\frac{1}{2} \text{ of } 2\frac{1}{4}}$ .
19. Simplify:  $\frac{\frac{1}{2} + 3\frac{1}{2}}{5\frac{3}{4} - 3\frac{1}{4}}$ .  $\frac{12\frac{1}{2} \times 11\frac{1}{2}}{68\frac{3}{4} + 1\frac{1}{4}}$ .
20. Simplify:  $\frac{\frac{7}{8} \times \frac{5}{8}}{\frac{3}{4} \times \frac{1}{8}}$ .  $\frac{\frac{3}{4} \times \frac{3}{4}}{\frac{3}{4} \times \frac{3}{4}}$ .
21. Divide:  $347\frac{3}{4}$  by  $15$ .  $692$  by  $19\frac{3}{4}$ .
22. Divide:  $19\frac{3}{4}$  by  $16\frac{3}{4}$ .  $786\frac{1}{4}$  by  $30\frac{1}{4}$ .
23. Add:  $2\frac{1}{8}$ ,  $8\frac{1}{8}$ ,  $27\frac{3}{4}$ ,  $9\frac{1}{8}$ .
24. Add:  $8\frac{1}{8}$ ,  $37\frac{3}{4}$ ,  $28\frac{1}{4}$ ,  $9\frac{3}{8}$ ,  $19\frac{1}{4}$ .
25. From  $27\frac{3}{8}$  take  $18\frac{3}{4}$ .  $12\frac{1}{2} - 9\frac{1}{4} = ?$
26. How many miles an hour does a man walk who walks  $21\frac{3}{4}$  miles in  $4\frac{3}{4}$  hours?

All kinds of lumber are measured by board feet. A board foot is 1 ft. long, 1 ft. wide, and 1 in. thick. Boards less than one inch in thickness are reckoned as one inch thick. In this book, when no thickness is mentioned, one inch is understood.

1. How many feet of lumber are there in 24 boards, each 12 ft. long, 10 in. wide, and 1 in. thick?

$$10 \text{ in.} = \frac{5}{8} \text{ ft.}$$

$$12 \text{ ft.} \times \frac{5}{8} = 10 \text{ ft.}$$

$$10 \text{ ft.} \times 24 = 240 \text{ ft.}$$

$$\text{or, } \frac{24 \times 12 \times 5}{6}$$

Cancel when possible.

If this board was 1 ft. wide, it would contain as many board feet as it has feet in length. Since it is only 10 in. ( $\frac{5}{8}$  ft.) wide, it will contain only  $\frac{5}{8}$  as many feet, or 10 ft. And 24 boards will contain 24 times 10 ft., or 240 ft.

2. How many feet of lumber in a plank 9 ft. long, 8 in. wide, and 2 in. thick?

$$\frac{9 \times 2 \times 2}{3}$$

Since a board is only 1 in. thick, a plank 2 in. thick will make 2 boards. Hence find the number of feet in one board as above, and then multiply by the thickness in inches.

Find the number of board feet in each of the following:

3. A board 16 ft. long, 12 in. wide, 1 in. thick.
4. A plank 16 ft. long, 9 in. wide, 2 in. thick.
5. 14 ft. long, 6 in. wide, 1 in. thick.
6. 14 ft. long, 6 in. wide, 3 in. thick.
7. 12 ft. long, 8 in. wide, 1 in. thick.
8. 18 ft. long, 9 in. wide, 3 in. thick.
9. 12 ft. long, 3 in. wide, 3 in. thick.
10. 14 ft. long, 12 in. wide, 2 in. thick.
11. 14 ft. long, 18 in. wide, 1 in. thick.
12. 15 ft. long, 8 in. wide, 1 in. thick.
13. 16 ft. long, 9 in. wide, 2 in. thick.
14. 18 ft. long, 18 in. wide, 3 in. thick.
15. 12 ft. long, 9 in. wide, 2 in. thick.
16. 9 ft. long, 10 in. wide, 2 in. thick.
17. 10 ft. long, 6 in. wide, 3 in. thick.



1. If I purchase 50 boards, each 12 ft. long and  $7\frac{1}{4}$  in. wide, for how many feet of lumber must I pay?

NOTE. — Fractions of an inch in the width of boards are never counted. Call it the nearest inch.

2. Find the number of feet of lumber required to floor a barn 36 ft. long, 17 ft. 6 in. wide, the planks being  $2\frac{1}{2}$  in. thick.

3. At \$18 per M., what will be the cost of the boards to build a fence 4 boards high round a field 160 yd. long, 120 yd. wide, if each board is 6 in. wide?

4. How many feet of 6-in. boards are required to build 20 rd. of fence 4 boards high?

5. How many feet of lumber are required for 140 ft. of tight board fence  $5\frac{1}{2}$  ft. high?

6. How many feet of boards will be necessary to cover both gables of a building 34 ft. wide, if the gable is  $8\frac{1}{2}$  ft. high?

7. How many feet of boards will be needed to cover both gables of a building 27 ft. wide, if the height of the gable is 9 ft.?

8. A floor is 12 ft. by 16 ft. How many feet of lumber will be needed for this floor, if each plank is 2 in. thick?

9. In the 7th example, which way should the planks run, if each plank is 16 ft. long?

10. Find how many feet of planks will be needed if 12-foot planks of the same thickness are used?

11. At \$16 per M., what is the cost of 42 16-ft. fence boards? Fence boards are 6 in. wide.

12. At \$28 per M., find the cost of 750 boards, each 14 ft. long, 12 in. wide, and 1 in. thick.

13. What is the cost of 46,250 ft. of pine lumber at \$28.40 per M.?

14. Find the number of board feet in 478 joists, each 24 ft. long, 10 in. wide, and 3 in. thick.

15. Bought 20 joists, each 18 ft. long, 5 in. wide, and 3 in. thick, at \$30 a M. What did they cost me?

(See Part I., page 135.)

1. The divisor is 675, the quotient 488, and the remainder 548.
2. A man owns a rectangular garden plot 320 ft. long, 210 ft. wide. Around the outside is a walk 6 ft. wide.
3. Two men purchased a lot of wood for \$31. In dividing the wood one man took  $6\frac{1}{2}$  cords, and the other 9 cords.
4. A man had \$12000. He lost in business the first year  $12\frac{1}{2}\%$  of it and 15% of the remainder the second year.
5. A square field is  $72\frac{1}{2}$  rd. on each side. The fence that incloses it cost \$1.85 a rod.
6. A grocer bought 1152 gal. of molasses. After 12% leaked out, the remainder was sold at 65¢ a gallon.
7. The largest circle possible was drawn on a sheet of paper 12 in. long and 10 in. wide.
8. The sum of two numbers is 528, and one of them is 11 times the other.
9. \$1,763.25 was on interest 1 yr. 3 mo. 14 da. at 5%.
10.  $\frac{3}{4}$  of a ton of hay cost \$12.30. I bought 4 T. 500 lb.
11. A man owned  $\frac{2}{3}$  of a ship, and sold  $\frac{1}{3}$  of his share for \$16,800.
12. A room measures  $18\frac{1}{2}$  ft. by 15 ft. The carpet is  $\frac{3}{4}$  of a yard wide. The breadths run lengthwise.
13. A man spent \$646 for board and expenses. This was  $34\%$  of his salary.
14. Having lost 28% of my money, I have \$17,640 left.
15. Goods that cost \$764 were sold at a loss of  $17\frac{1}{2}\%$ .
16. The quotient is 71, the divisor 42, and the remainder 15.
17. The radius of a circle is  $4\frac{1}{4}$  ft.
18. A piece of land is 120 ft. wide, and 150 ft. long.
19. A man bought a horse for \$72, and sold it for 25% more than it cost.

1. If one pipe will fill a cistern in 4 hours, and another in 6 hours, how long will it take to fill it when both pipes are running?

2. If water is running out of the second pipe while running in the first, how long then will it take to fill the cistern?

3. A man sold a watch for \$90, and gained 50%. What did it cost?

4. A man bought a hat for \$5, and sold it for \$6. What per cent did he gain?

5. John lost  $\frac{2}{3}$  of his money, and spent  $\frac{1}{3}$  of the remainder, and then had only 10 cents. How much money had he at first?

6. How old are you if  $\frac{3}{4}$  of 80 is 4 times your age?

7. How long will it take a man to save \$60, if he earns \$15 a week and spends \$9?

8. If  $\frac{1}{3}$  of a yard of cloth cost 63 cents, what will  $\frac{2}{3}$  of a yard cost?

9. A man sold a watch for \$120, which was  $\frac{4}{5}$  of what it cost him. How much did it cost?

10. If 4 men can do a piece of work in 12 days, how long will it take 6 men to do it?

11. If 4 pipes will fill a cistern in 40 min., how many pipes will fill it in 10 minutes?

12. A house was insured for \$3,200 at 1%. What was the premium?

13. An agent sold \$300 worth of property, and charged 5% for so doing. Find his commission.

14. Given the cost and loss per cent, what can be found?

15. A watch costing \$80 was sold at a loss of 10%. For how much was it sold?

16. What number increased by 12 equals 16?

17. 6 added to a number equals 14. Find the number.

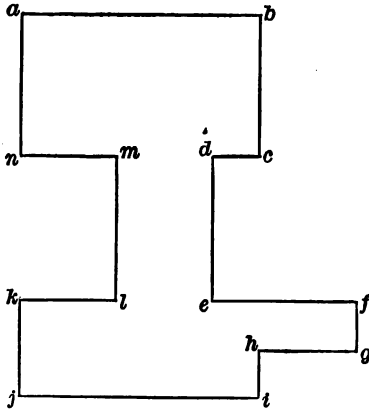
18. What number diminished by 5 equals 9?

19. What is the ratio of 108 to 12? Of 60 to 12?

(For definitions and review, see pages 285-289.)

1. A house which cost \$1,200 was sold at a gain of 25%. What was the selling-price?
2. Land which cost \$42 an acre was sold at a gain of  $8\frac{1}{2}\%$ . Find the selling-price.
3. A fruit-dealer bought apples at \$1.75 a barrel, and sold them at \$2.25 a barrel. What per cent did he gain?
4. A man bought a chair for \$2.34, and afterwards sold it for \$2.73. What per cent did he gain?
5. What does a bill for \$2,478 become after a reduction of 5%?
6. What is the cost of insuring 640 bbl. of flour, worth \$4 a barrel, if the cost of insurance is  $\frac{1}{2}\%$  of the value of the flour?
7. A horse was bought for \$175. At what price must he be sold to gain 12%?
8. A merchant bought apples at \$1.20 a barrel, and sold them at a gain of 25%. Find the selling-price a barrel. How many barrels did he sell if he received altogether \$187.50?
9. The population of a certain city was 48,000 this year. If it increases at the rate of  $2\frac{1}{2}\%$  each year, find its population 1 and 2 years hence.
10. A person gave \$1,500 for a piece of land, and sold it at a gain of 20%. Find the selling-price.
11. A man built a house for \$3,500, and rented it for \$400 a year. For what per cent of its value did he rent it?
12. Turn to your geographies, and find the area of the basins of the principal rivers, and then find what per cent of the area of North America is drained by each river.
13. A quantity of goods was sold at an advance of  $12\frac{1}{2}\%$ . If the gain was \$34, what was the cost?
14. When the cost is \$8,000 and the selling-price \$7,400, what is the rate of the loss?

1. Find the interest on \$800 for 2 yr. 6 mo. 15 da. at 6%.
2. Find the interest on \$346.50 for 1 yr. 8 mo. 2 da. at 6%.
3. Find the interest on \$750 for 3 yr. 3 mo. 3 da. at 6%.
4. A boy sold a sled for \$1.40, and by doing so lost  $12\frac{1}{2}\%$ .  
What did the sled cost?
5. A man paid \$270 for a horse, which was 10% less than his carriage cost. Find the cost of both.
6. A grocer bought 535 lb. of sugar at  $5\frac{1}{2}\text{¢}$  a pound, and sold it at a profit of 15%. What did he receive for it?
7. A merchant sold some cloth at \$3 a yard, and lost 25%.  
What did it cost him?
8. For what must a merchant sell a barrel of flour that cost him \$5.10 to gain  $9\frac{1}{4}\%$ ?
9. A miller bought a consignment of wheat, and ground it into flour. He sold 980 bbl. for \$6.50 a barrel, making 25%.  
How much did he pay for the wheat?
10. Mr. Longley sold his house for \$4,248, and by so doing gained 20%. Find what the house cost Mr. Longley.
11. How much will it cost to plaster and paint the walls of a room 28 ft. long, 20 ft. wide. and 12 ft. high, at  $33\frac{1}{3}\text{¢}$  a square yard?
12. At \$1.50 a yard, what will it cost to carpet a room 18 ft. long,  $15\frac{3}{4}$  ft. wide, the carpet being  $\frac{3}{4}$  of a yard wide?
13. What is the area of a circle whose diameter is 100 ft.?
14. How many square inches of gold leaf will be required to cover the convex surface of an equilateral triangular pyramid, each side of whose base is 10 in., and whose slant height is 4 ft.?
15. The greater of two numbers is five times the less. If their sum is 72, what are the numbers?
16. A father is five times as old as his son, and the sum of their ages is 36 years. How old is each?
17. If twice a number is added to four times the same number, the sum will be 66. What is the number?



This diagram represents a garden plot, drawn to a scale of  $\frac{1}{4}$  in. to 8 ft.

1. Find the distance round the garden.

2. Find the area of the garden.

3. If  $ml$  is extended to  $ji$ , it will cut off on the left Mary's flower-bed. Find the area and perimeter of her bed.

4. If she sets out plants six inches apart, how many plants

can she have in the garden?

5. Extend  $ih$  until it meets  $ef$ ; you have cut off on the right Bessie's flower-bed. The area of Bessie's bed is what per cent of the area of Mary's bed?

6. The perimeter of Bessie's bed is what per cent of the perimeter of Mary's bed?

7. If Bessie and Mary both start from  $a$ , and walk round the garden in opposite directions with equal speed, where will they meet?

8. If  $nm$  and  $cd$  are extended until they meet, the part cut off above is the vegetable garden. The area of Bessie's and Mary's flower gardens will be what per cent of the area of the vegetable garden?

9. The perimeter of Bessie's garden is what per cent of the perimeter of the vegetable garden?

10. Find the area of the garden not belonging to Mary, Bessie, or the vegetable garden.

11. How much will it cost, at \$18 per M., to build a fence 4 boards high round the entire garden, if the boards are 8 ft. long and 6 in. wide?

1. A man deposited in a bank \$475 $\frac{1}{2}$  Monday, \$370 $\frac{7}{10}$  Tuesday, \$400 $\frac{1}{8}$  Wednesday, and drew out \$563 $\frac{1}{2}$  Thursday, and \$145 $\frac{1}{10}$  Friday. How much had he in the bank Saturday?

2. What is the least common multiple of 20, 24, 36?

3. A man had 490 $\frac{1}{2}$  bu. of grain, and bought 784 $\frac{1}{4}$  bu. more, and then sold 900 $\frac{1}{2}$ . How many bushels had he left?

4. The area of an oblong is 24 square feet. What part of its area is that of a square whose side is 2 ft.?

5. Reduce 380 $\frac{1}{2}$  to 72ds.

6. If a car runs 48 $\frac{1}{2}$  miles in an hour, how far will it run in 16 days, running 14 $\frac{1}{2}$  hours a day?

7. Find the cost of 15 $\frac{1}{2}$  cd. of wood at \$6.37 $\frac{1}{2}$  a cord, and 7 $\frac{1}{2}$  cd. at \$5 $\frac{1}{2}$  a cord.

8.  $(\frac{1}{2} + \frac{3}{4} + \frac{1}{8}) \times (\frac{3}{4} \times \frac{7}{8} \times 16) = ?$

9. Find the cost of 19 $\frac{1}{2}$  cd. of wood @ \$6 $\frac{1}{2}$ .

10. A has \$540, which is  $\frac{3}{4}$  of 3 $\frac{1}{2}$  times as many as B has. How much money has B?

11. How many yards of cloth does a merchant buy if he spends \$1,200, of which \$680 was spent for cloth at \$5 $\frac{1}{2}$  a yard, and the remainder at \$4 $\frac{1}{2}$  a yard?

12. Divide 2 $\frac{1}{2}$  by 2 $\frac{1}{4}$ , and multiply the quotient by the quotient of 4 $\frac{1}{2}$  divided by 2 $\frac{1}{2}$ .

13. Find the cost of a house and lot when the house costs \$6,300, which is 1 $\frac{1}{2}$  times the cost of the lot.

14. From 20 $\frac{1}{2}$  take 16 $\frac{1}{4}$ .

15. The diameter of a circle is 15 ft. Find circumference.

16. The circumference is 33 rd. Find diameter.

17. The radius is 17 in. Find circumference.

18. The diameter of a circular pond is 15 rods; what is the area?

19. Find the convex surface of a triangular pyramid, each side of whose base measures 6 ft., and its slant height 24 ft.

20. Add 19 $\frac{2}{3}$ , 17 $\frac{1}{6}$ , 21 $\frac{3}{10}$ , and 18 $\frac{1}{5}$ .

1. If 3 apples are worth one peach, and 3 peaches are worth one orange, how many oranges can be bought for 45 apples?
2. Name two factors of each of the following numbers:  

10	12	14	15	16	18	20
22	24	25	26	27	28	30
32	33	35	36	40	42	44
45	48	49	50	54	55	56
60	63	64	66	70	72	77
81	88	96	108	121	132	144
3. What will 2 tons of iron cost if 1 lb. costs 10 cents?
4. How many inches in  $\frac{1}{2}$  yd.? In  $\frac{1}{4}$  yd.? In  $\frac{3}{4}$  yd.? In  $\frac{1}{8}$  yd.? In  $\frac{5}{8}$  yd.?
5. What will 7 quarts of wheat cost at 128¢ a bushel?
6. If a coat and vest cost \$20, and a pair of trousers  $\frac{1}{4}$  as much, how much will all cost?
7. If 1 pound of cheese costs 12 cents, what will  $2\frac{3}{4}$  lb. cost?
8. Nellie has 45 cents, and Grace has  $\frac{2}{3}$  as many. How many has Grace?
9. If I buy an article for \$60, and sell it for  $\frac{3}{4}$  of what it cost, how much shall I gain?
10. A man's salary is \$120 a month. If he spends  $\frac{1}{3}$  of it for a watch,  $\frac{1}{4}$  of it for a suit of clothes, and  $\frac{1}{6}$  of it for board, how much of his salary remains?
11. What will  $\frac{3}{4}$  of a gallon of molasses cost at 10¢ a pint?
12. Bought a third of a barrel of sugar for \$3. What will  $2\frac{3}{4}$  bbl. cost?
13. If  $\frac{1}{4}$  of a yard of cloth cost \$2, what will  $2\frac{1}{2}$  yd. cost?
14. If  $\frac{1}{4}$  of a bushel of corn cost 10 cents, what will 2 bu. cost?
15. If  $\frac{1}{5}$  of a pound of spice cost 15 cents, what will  $3\frac{1}{2}$  lb. cost?
16. Find the interest on \$600 for 19 da. at 6%.
17. In 672 eggs there are how many dozen eggs?



1. Find the interest of \$396.16 from July 15, 1901, to Feb. 6, 1903.

From July 15, 1901, to July 15, 1902, 1 yr.

From July 15, 1902, to Jan. 15, 1903, 6 mo.

From Jan. 15, 1903, to Feb. 6, 1903, 22 da.

\$ 3.9616 int. for 2 mo.		\$ 3.9616 int. for 2 mo.
<u>\$35.654</u> int. for 1 yr. 6 mo.		<u>\$35.654</u> int. for 1 yr. 6 mo.
1.188 int. for 18 da.		1.320 int. for 20 da.
.198 int. for 3 da.	or	.132 int. for 2 da.
.066 int. for 1 da.		
<u>\$37.106</u> int. for 1 yr. 6 mo. 22 da.		<u>\$37.106</u>

NOTE. — Use this method for finding the difference in dates. As soon as possible do the work mentally, and write only the results.

Find the interest at 6% of:

2. \$649.21 from June 8, 1900, to Aug. 15, 1903.
3. \$1460.78 from June 6, 1902, to April 23, 1904.
4. \$284.30 from Feb. 23, 1902, to Aug. 5, 1903.
5. \$366.44 from Jan. 5, 1903, to Jan. 27, 1905.
6. \$491.73 from Nov. 16, 1901, to Nov. 28, 1903.
7. \$91.36 from Aug. 12, 1900, to June 10, 1902.
8. \$436.74 from March 25, 1902, to July 29, 1904.
9. \$589.76 from May 11, 1901, to Jan. 7, 1903.
10. \$550 from May 8, 1900, to June 13, 1903.
11. \$125.40 from Sept. 25, 1901, to March 16, 1902.
12. \$679.08 from Feb. 10, 1902, to Dec. 7, 1903.
13. \$137.65 from Oct. 14, 1902, to Dec. 29, 1904.
14. \$146.35 from June 7, 1902, to Feb. 11, 1904.
15. \$154.25 from Apr. 18, 1903, to Jan. 25, 1906.
16. \$817.57 from Aug. 7, 1900, to Sept. 8, 1901.
17. \$132.25 from Nov. 13, 1901, to May 2, 1903.
18. \$446.50 from July 18, 1900, to Sept. 4, 1901.
19. \$8155.49 from Aug. 15, 1901, to May 1, 1905.

1. Add five, and three hundred eighty-two ten-thousandths; one thousand two hundred thirty-five hundred thousandths; eight hundred ninety-six, and fifty-one thousand three hundred twenty-seven millionths.

2. Add nineteen, and forty-nine ten-thousandths; seventy-three, and one hundred fifty-six millionths; thirty-four, and eight hundred-thousandths; five thousand eighty-two, and one thousand nineteen hundred-thousandths.

3. Multiply .0000915 by .0056.

4. Multiply .58273 by 1000.

5. Multiply 2.4675 by 100.

6. Multiply 4.3982 by 500.

7. Divide .0009 by .003.

8. Divide .0002784 by .032.

9. Divide 10 by .001.

10. Divide .31 by .0005.

11. Divide 18.45 by 10. By 100.

12. Divide 436.457 by 100. By 1,000.

13. Divide 1464.25 by 100. By 1,000.

14. Divide 1867.8 by 4,000.

15. Divide 375.82 by 500.

16. Find cost of 825 bu. @ \$1.66 $\frac{1}{2}$ .

17. Find cost of 72 gal. @ \$3.87 $\frac{1}{2}$ .

18. Add 8153 and 45 hundredths; 32 and 28 ten-thousandths; 237 and 483 thousandths; 5 and 165 hundred-thousandths; 6 hundredths.

19. Change to common fractions: .0419; .0048; .00625; 5.00125.

20. Change to decimals:  $\frac{1}{16}$ ,  $\frac{7}{81}$ ,  $\frac{5}{32}$ ,  $\frac{11}{14}$ ,  $\frac{1}{32}$ ,  $\frac{3}{4}$ ,  $\frac{17}{128}$ .

21. Divide 16 by 10,000.

22. Divide 1846 by 100. By 1,000.

23. Change to decimals: 6 $\frac{1}{2}$ , 32 $\frac{1}{2}$ , 46 $\frac{1}{16}$ , 24 $\frac{1}{128}$ .

24. Find the difference between .406 and .62.

1. A man owned 1,016 acres of land. He sold  $12\frac{1}{2}\%$  to one customer, and  $42\frac{1}{2}\%$  of the remainder to another customer. How many acres had he left?
2. A man's income is \$1,800 a year, of which he pays  $12\frac{1}{2}\%$  for house-rent. What rent does he pay each month?
3. What number increased by 40% of itself equals \$1,694?
4. A book-keeper spends \$600 a year, which is 24% of his salary. Required his salary.
5. What per cent of 675 is 135?
6. A merchant bought 275 bbl. of flour. After losing 20% of it, he sold 25% of the remainder. How many barrels remained? What per cent of the whole remained?
7. I bought \$820 worth of cloth, and sold it at a gain of 15%. What was the gain? The selling-price?
8. A dealer bought coal at \$4.25, and sold it at 5% advance. What was his selling-price?
9. Cost \$7.50, profit 18%. Find the selling-price.
10. Cost \$1,500, gain  $16\frac{2}{3}\%$ . Required the selling-price.
11. Cost \$80, gain \$35. Required the gain per cent.
12. Selling price \$125, loss 20%. Find the cost.
13. Gain 25%, cost \$5.50. Required the selling-price.
14. An agent sold 426 bales of cotton weighing 408 pounds each, at  $8\frac{1}{2}\%$  a pound. How much money did he receive? He kept  $2\frac{3}{4}\%$  of this as his commission. How much did he return to his employer?
15. A regiment went into battle with 960 men, and came out with 600 men. What per cent was lost?
16. A clerk's salary is \$800 a year. He spends 10% the first quarter, 15% the second, 16% the third, and 14% the fourth. How much does he save?
17. My agent sold goods for \$5,400; his commission was  $2\frac{1}{2}\%$ , and other charges \$17.50. What amount should he send me?
18. Find  $33\frac{1}{3}\%$  of 729.

1. \$625.80 was on interest at 6% from Nov. 28, 1900, to Sept. 16, 1904.
2. 18 men are at work on a piece of work that 24 men can do in 9 days.
3. I bought 42 yd. of cloth. 35 yd. cost \$12.25.
4. Goods that cost \$3,072 were sold for \$2,560.
5. A merchant withdrew \$2,058 from a bank. This was 28% of his deposit.
6. A farmer sold 525 bu. of wheat at \$1.12 a bushel, and 20% less of oats at 45¢ a bushel.
7. A farmer owned a flock of 580 sheep, but lost 20% of the flock in a snow-storm.
8. A plank is 16 ft. long, 1 ft. 3 in. wide, and 2 in. thick.
9. Wood cost me \$4.75 a cord. I paid \$28.50 for a pile. It was 24 ft. long and 4 ft. wide.
10. A farmer paid 48¢ a cubic yard for digging a ditch 22 ft. 9 in. long, 8 ft. 6 in. wide, and 8 ft. high.
11. Property worth \$7,500 was insured at  $1\frac{1}{2}\%$ .
12. A man's salary is \$2,500 a year. He spends 30% for board,  $12\frac{1}{2}\%$  for clothes, and 20% for other expenses.
13. A number diminished by 20% of itself is 936.
14. A merchant took a sum of money to market, and spent 24% of it. He brought back \$760.
15. A farmer raised 55 bu. of potatoes. He sold 20% to one man, and 25% of the remainder to another man.
16. A merchant bought 96 yds. of cloth at 40¢ a yard, and sold it at a gain of  $33\frac{1}{3}\%$ .
17. A man sold his library for \$840, which was 16% less than it cost.
18. A cylinder is 40 ft. long and 15 ft. in diameter.
19. A dealer sold 438 tons of coal at \$4.75 a ton, and a number of tons of another kind at \$5.20 a ton. He received for all \$4,254.10.

1. A man bought a horse for \$200, and sold him so as to gain 10% of his cost. Find the gain.

2. Mr. Jones invested \$2,000 in business, and gained 20% of his investment every year. How much did he gain in 1 year? 5 years?

3. An agent sold a house for \$1,200. His commission was 10%. How much did he keep as commission?

4. Plush cloth bought at \$5 a yard was sold at \$6 a yard. What part of the cost was gained? What per cent was gained?

5. A man receiving a salary of \$2,400 spends  $33\frac{1}{3}\%$  of it for expenses. How many dollars does he spend?

6. Ten bushels out of 100 bushels is what per cent?

7. \$1 out of every \$10 is what part? Is what per cent?

8. \$5 out of every \$20 is what part? Is how many hundredths? Is what per cent?

9.  $2\frac{1}{2}$  is  $\frac{1}{4}$  of what number?

10.  $2\frac{1}{2}$  is 25% of what number?

11. Yesterday I worked  $\frac{1}{3}$  of the day, and the day before  $\frac{1}{4}$  of a day. What part of a whole day did I work in all?

12. Change to a whole number:  $\frac{1}{8}$ ,  $\frac{1}{6}$ ,  $\frac{1}{4}$ ,  $\frac{1}{3}$ .

13. If I pay 8 cents for a pint of milk, what must I pay for a gallon at the same rate?

14. Make a problem to illustrate, Given two numbers to find their difference.

15. 45 is  $\frac{3}{5}$  of what number?  $\frac{2}{5}$  of what number?

16. 81 is  $\frac{3}{4}$  of what number?  $\frac{1}{4}$  of what number?

17. If 8 is  $\frac{2}{3}$  of a number, what is the number? What is  $\frac{1}{3}$  of it?

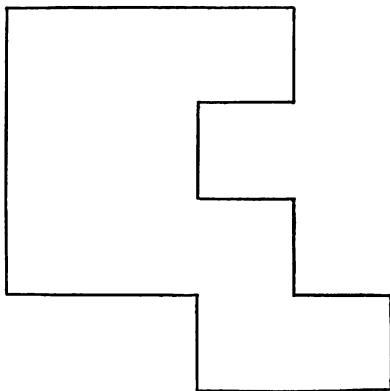
18. How many cubic feet in a box 4 ft. long, 3 ft. wide, 2 ft. high?

19. How many yards square is a floor that is 12 ft. square? How many square yards are there in the floor?

20. What is the ratio of a mile to a rod?

Starting at A, the boundary line of a garden runs east 5 rd., thence south 2 rd., thence west 2 rd., thence south 2 rd., thence east 2 rd., thence south 1 rd., thence east 1 rd., thence south 1 rd., thence west 6 rd., thence north to A.

1. Make a drawing, scale 1 in. to 1 rd.
2. Find the perimeter of the garden.
3. Find the area of the garden.
4. Find the number of posts, placed  $8\frac{1}{4}$  ft. apart, needed for a fence.
5. Find the feet of lumber in the two scantling (rails) running round the lot, each  $\frac{1}{4}$  in. by 3 in.
6. Find the feet of lumber in a tight board fence 5 ft. high all round it.
7. Find the cost of the posts at 25¢ each.
8. Find the cost of scantling and boards at \$16 per M.
9. Find the cost of painting the fence at  $33\frac{1}{3}$ ¢ a square yard.



10. This is a plan of a hall in a public building, drawn to a scale of  $\frac{1}{2}$  in. to 12 ft. Find the number of square feet in the floor.

11. If the walls are 12 ft. high, find the square feet in the walls and ceiling.

12. Beginning at a point called A, the northern boundary line of a park runs west 30 rd. to B; thence south 25 rd. to C; thence east 40 rd. to D; thence to place of beginning. Draw a diagram, scale  $\frac{1}{2}$  in. to 5 rd. Find the square rods in the park.

13. How many square yards of cloth will it take to cover a column that is 16 ft. high, and  $18\frac{1}{4}$  ft. in circumference?

1. Find the difference between 100,100,100 and 90,090,090.
2. If a man buys 395 acres at \$37 an acre, and sells his purchase for \$15,176, what will be his gain?
3. Multiply 183,600 by 427,000.
4. Multiply 630,000 by 3,800.
5. If the product of two numbers is 143,186,076, and one of the numbers is 32,871, what is the other number?
6. Divide 27,180,000 by 15,100.
7. A man had \$2,013.42. He bought 370 bu. wheat at \$1.12 a bushel; 980 bu. corn @ 54¢; 536 bu. rye @ 62¢; and invested the remainder of his money in flour at \$6.25 a barrel. How many barrels of flour did he buy?
8. A merchant sold 12 bbl. of pork, averaging 200 lb. a barrel, at 12¢ a pound, and took in payment 160 hams weighing 10 lb. each. Find the price of the ham a pound.
9. Reduce to improper fractions:  $27\frac{1}{8}$ ,  $42\frac{3}{8}$ .
10. Find the difference between  $438\frac{1}{8}$  and  $287\frac{3}{8}$ .
11. Multiply  $75\frac{1}{8}$  by  $64\frac{1}{8}$ .
12. Divide  $816\frac{1}{2}$  by  $16\frac{3}{8}$ .
13. A can finish a piece of work in 4 days, B in 6 days, and C in 8 days. In what time can the work be completed if all work together?
14. Find the cost of 619 lb. @  $27\frac{1}{2}$ ¢.
15. What is the cost of  $3\frac{1}{2}$  reams of paper at  $12\frac{1}{2}$ ¢ a quire?
16. A owns 15% of a business; B, 25%; C, 28%; and D, the remainder. What is the value of A's share if D's share is worth \$17,232?
17. The greater of two numbers is four times the less. If the difference is 36, what are the numbers?
18. Two trains leave New Haven at the same time, and move in the same direction. At the end of one hour they are 20 miles apart. If one has gone two times as far as the other, how far is each from New Haven?

1. What will it cost to floor a room  $17\frac{1}{2}$  ft. long and 16 ft. wide at \$1.10 a square yard?

2. A man had a capital of \$2,500. He put 25% of it into business,  $33\frac{1}{3}\%$  of it into a bank, and invested 28% of it in real estate. How much had he left?

3. A grocer bought 800 bags of coffee, each bag containing  $49\frac{1}{4}$  lb., at 18¢ a pound, and sold it at a profit of  $16\frac{2}{3}\%$ . How much did he receive for the whole lot?

4. I lost 10% by selling goods at 27¢ a yard. How much did I lose on 485 yd.?

5. What will be the cost of 35 3-in. planks, 24 ft. long, 16 in. wide, at \$16.75 per M?

6. A man sawed a pile of wood 40 ft. long, 4 ft. wide, and  $5\frac{1}{2}$  ft. high, for \$1.25 a cord. How much did he earn?

7. If  $\frac{3}{4}$  of my share of a farm is worth \$420, and I own  $\frac{3}{4}$  of the farm, what is the value of the farm?

8. Find the convex surface of a log whose circumference is 18 ft. and length 35 ft.

9. What is the area of a circle whose circumference is 160 yd.?

10. The parallel sides of a trapezoid are 25 yd. and 21 yd., and its altitude 16 yd. What is the area?

11. The length of a rhomboid is 17 ft., and the perpendicular height 16 ft. What is the area?

12. How many acres in a field 800 rd. long and 128 rd. wide?

13. Find the area of a triangle whose base is 49 yd., and altitude is  $\frac{1}{2}$  its base.

14. If  $4\frac{1}{2}$  tons of coal cost \$18, what will 18 tons cost?

15. If 5 bu. 3 pk. of potatoes cost \$4.60, what will 2 bu. 1 pk. cost?

16. Three boys, A, B, and C, together receive \$81. A receives twice as much as B, and C three times as much as B. How many dollars does each receive?



1. Find the convex surface of a triangular prism whose length is 12 ft. and each side of whose base is  $2\frac{1}{2}$  ft.

2. Required the number of square feet in the surface of a square pyramidal roof, the length of whose sides is 20 ft., and whose slant height is 18 ft.

3. What length of tire will it take to band a cart-wheel 5 ft. in diameter?

4. What is the difference between the area of a floor 40 ft. square, and that of two other floors, each 20 ft. square?

5. The diameter of a circular grass-plot is 17.5 ft. What is its circumference?

6. If the circumference of a tree is 50 in., what is its diameter?

7. How many board ft. in a plank whose length is 20 ft., breadth 16 in., and thickness 3 in.?

8. Required the area of a pasture in the form of a trapezoid whose parallel sides are 786 and 473 ft., and altitude 986 ft.

9. What is the cost of sawing a pile of wood 20 ft. long, 4 ft. wide, and 6 ft. high, at \$1.25 a cord?

10. A field contains  $199\frac{1}{2}$  sq. rd. It is  $18\frac{3}{4}$  rd. long. How wide is it?

11. Draw, name, and describe four kinds of quadrilaterals.

12. A certain flower-bed is in the form of a trapezoid. The two parallel sides are 10 ft. and 12 ft., and the perpendicular distance between them is 8 ft. Find the area.

13. What is the floor measurement of a house built in the form of an octagon, whose side is 12 ft., and the perpendicular distance from the center to the middle of each side is 8 ft. 6 in.?

14. A wheel whose diameter is 3 ft. turns how many times in going a half-mile?

15. A cistern is in the form of a rectangular prism, 12 ft. 8 in. long, 8 ft. 6 in. wide, and 12 ft. deep. How many square feet are there in the sides and bottom?

1. If I can do a piece of work in 4 days, what part can I do in 2 days?
2.  $\frac{1}{3}$  of 25 is  $\frac{1}{3}$  of how many?
3.  $\frac{1}{4}$  of 28 is  $\frac{1}{4}$  of how many?
4. By selling land at \$150 an acre I gained 25%. Find cost.
5. A merchant sold goods for \$500 at a loss of 20%. What was the cost?
6. I sold my horse for \$200, and by so doing lost 20%. What was the value of the horse?
7. Grace gave one-half of her oranges to her mother, and one-third of them to her father. What per cent had she left?
8. If you sell an article for  $12\frac{1}{2}$  cents that cost 10 cents, what will be your rate of gain?
9. If an article costs \$600, what will be the gain at 1%? At  $\frac{1}{3}$  of 1%?
10. A man paid \$500 for wheat, and sold the whole at a loss of 6%. Find the loss. Find the selling-price.
11. One-half of a lot of goods cost \$180. Find the loss at 10% on the whole lot.
12.  $\frac{1}{4}$  of 40 is what per cent of  $\frac{1}{4}$  of 20?
13.  $\frac{1}{4}$  of 30 is what per cent of  $\frac{1}{4}$  of 40?
14. How many barrels will it take to hold 25 bu., if 1 bbl. holds  $2\frac{1}{2}$  bushels?
15. Divide  $5\frac{3}{4}$  by  $\frac{3}{4}$ .  $6\frac{3}{4}$  by  $\frac{3}{4}$ .
16. A man paid \$24 for  $\frac{3}{4}$  of an acre of land. If he sold  $\frac{1}{4}$  of an acre for \$15, how much did he gain on the part sold?
17. Find the cost of 12 primers at  $12\frac{1}{2}$  each.
18. At the rate of \$7 $\frac{1}{2}$  a ton, what will 12 tons cost?
19. What is the difference between  $\frac{1}{3}$  of two and  $\frac{2}{3}$  of 1?
20. If 4 yd. of cloth cost 72 cents what will  $\frac{1}{2}$  yd. cost?
21.  $\frac{1}{3}$  of 30 is  $\frac{1}{3}$  of what number?
22. What is the area of a plot of ground 30 ft. by 50 ft.?

1. Subtract 8 bu. 3 pk. 7 qt. from 47 bu. 1 pk. 5 qt. 1 pt.

$$\begin{array}{r} 47 - 1 - 5 - 1 \\ 8 - 3 - 7 - 0 \\ \hline 38 - 1 - 6 - 1 \end{array}$$
 No pints from 1 pt. leaves 1 pt. 7 qt. from 5 qt. we cannot take, so we take 1 pk. from the column of pecks, which is equal to 8 qt. 8 qt. and 5 qt. are 13 qt. 7 qt. from 13 qt. leaves 6 qt. 3 pk. from 0 pk. we cannot take. 1 bu. equals 4 pk. 3 pk. from 4 pk. leaves 1 peck. 8 bu. from 46 bu. leaves 38 bu.

NOTE.—Numbers of the same denomination should be written in the same column.

2. From 46 gal. 1 qt. 1 pt. 2 gi. take 25 gal. 2 qt. 1 pt. 3 gi.

3. From 8 bu. 1 pk. 6 qt. take 3 bu. 2 pk. 4 qt.

4. Take 160 T. 1,800 lb. 6 oz. from 175 T. 298 lb.

5. From 7 mi. take 5 mi. 315 rd. 3 yd. 1 ft. 3 in.

6. From 471 cu. yd. 16 cu. ft. 972 cu. in. take 115 cu. yd. 17 cu. ft. 1,710 cu. in.

7. From 19 yr. 5 mo. 17 da. take 12 yr. 9 mo. 14 da.

8. From 20 gal.  $1\frac{1}{2}$  pt. take 3 qt.  $1\frac{1}{2}$  pt.

9. From 6 mi. 220 rd. 1 ft. 8 in. take 4 mi. 261 rd. 1 yd.

10. From 275 mi. take 50 mi. 130 rd. 3 yd. 1 ft. 3 in.

11. From 1,845 yr. 9 mo. 18 da. 20 hr. take 1,774 yr. 11 mo. 20 da.  $22\frac{1}{2}$  hr.

12. From 73 bu. 2 pk. 5 qt. take 59 bu. 3 pk. 7 qt.

13. From 17 mi. 311 rd. 1 yd. 1 ft. 3 in. take 3 mi. 79 rd. 1 yd. 2 ft. 7 in.

14. From 6 mi. take 4 mi. 64 rd.

15. From 116 cd. 4 cd. ft. take 105 cd. 5 cd. ft.

16. Subtract 5 mi. 215 rd. 5 yd. from 8 mi. 216 rd. 3 yd.

17. From 17 bu. 2 pk. 6 qt. take 8 bu. 3 pk. 4 qt.

18. From 14 cu. yd. 6 cu. ft. 1,011 cu. in. take 9 cu. yd. 17 cu. ft. 1,108 cu. in.

19. From 17 mi. 58 rd. take 10 mi. 117 rd. 2 ft.

1. A merchant by selling cloth at \$2.50 a yard gained 25%. Find the cost.
2. An agent bought 50 carriages for \$140 each, and charged me  $3\frac{1}{2}\%$  commission. He also paid \$75 for freight, and \$35 for cartage. What did the carriages cost me?
3. Find the interest of \$475.05 for 6 yr. 10 mo. 10 da. at  $6\frac{1}{2}\%$ .
4. A man sold a cow for \$37.50, and lost 25%. Find the cost.
5. A man sold a wagon for \$41.25, and gained 25%. Find the gain.
6. I sold a horse for \$240, and lost 20%. Find the cost.
7. A man owned 156 A. of land, and sold 75% of it for \$5,265. What was the price an acre?
8. A man bought a farm 180 rd. long and 160 rd. wide, for \$6,750, and sold it at a gain of 20%. How much did he receive an acre?
9. I purchased 417 bbl. of flour at \$5.25 a barrel. For how much must I sell the whole to gain 35%?
10. If it costs a man who earns \$90 a month \$72 for expenses, what per cent of his money can he save?
11. A manufacturer made 5,280 barrels of flour. He sold  $12\frac{1}{2}\%$  to one man,  $33\frac{1}{3}\%$  of the remainder to another, and 50% of what was then left to another. If he received \$4,666.20 for what was then left, how much was that a barrel?
12. Find the interest of \$875 for 80 days at 7%.
13. \$596.50 for 33 days at 4%.
14. \$1,375 for 5 mo. 3 days at 6%.
15. \$7,000 for 2 yr. 2 mo. 16 da. at 10%.
16. If I lost 10% by selling goods at 18¢ a yard, what did they cost?
17. A merchant sold tea at 60¢ a pound, gaining 20%. Find the cost.

1. How far will light travel in 12 min. 45 sec., if it travels 186,000 miles in one second?

2. Divide 567,891 by 729. 900,972 by 843.

3. Divide 120,000,000 by 12,000,000.

4. Add: Twenty-six and fifteen thousandths; eighty-one and one thousand nine hundred ten-thousandths; eleven and twenty thousand seven hundred four hundred-millionths; twelve hundred and twelve hundred-thousandths.

5. Multiply 36.03 by .06006.

6. Divide 5.958 by .0009.

7. Divide 16.27704 by 14.664.

8. If a man can build .425 of a rod of fence in an hour, how many rods can 12 men build in 6.5 days, working 8.25 hours a day?

9. Thirty-five hundredths of a cargo of 8,000 bushels of wheat were destroyed by fire: What was the value of the part left at \$.875 a bushel?

10. A farmer exchanged wood for coal. If he bought 9.5 tons of coal at \$4.25 a ton, how many cords of wood at \$3.75 a cord did he give in exchange?

11. What is .15 of \$47.65?

12. Divide 10.201 by 101.

13. Divide 1.125 by 937.5.

14. Find the cost of 8,724 roofing-slate at \$5.75 per hundred.

15. The bricks in a schoolhouse cost \$10,875. If the price was \$7.25 a thousand, how many bricks were used?

16. Find the cost of 17,250 lb. of hay at \$15.75 a ton.

17. A ton of coal costs \$2.75 to mine it, \$.85 for freight, and \$.25 for delivery. A dealer sold 425,600 lb. at \$5.50 a ton. How much did he make?

In buying coal at wholesale, a ton is 2240 pounds. This is called a *long* ton.

18. Divide 16 ten-millionths by 25 thousandths.

1. A can do a piece of work in 6 days, B in 7, and C in 8. In what time can they do it working together?

2. How many casks (40 gal.) of water will a cylindrical cistern hold, whose diameter is  $9\frac{1}{2}$  ft., and depth 10 ft.?

3. If the diameter of my carriage wheel is  $4\frac{1}{2}$  ft., how many revolutions will it make in going 2 miles and back again?

4. The height of a cylinder is 6 ft., and the diameter of the base is  $2\frac{1}{2}$  ft. Find the entire surface and volume.

5. A field in the form of a trapezoid contains  $23\frac{1}{2}$  acres. One of its parallel sides is 95 rd., and the other 65 rd. What is its altitude?

6. Write a promissory demand note. Find the interest on it for 1 yr. 3 mo. 17 da. at 7%.

7. Find the interest at  $7\frac{1}{2}$ % on \$256.34 from Nov. 17, 1901, to Aug. 24, 1903.

8. Find the commission on \$46,912.60 at  $1\frac{3}{4}$ %.

9. At \$1.85 a yard, find the cost of carpet 1 yd. wide to cover the floor of a room 22 ft. long, 19 ft. wide, strips to run lengthwise.

10. How many 4-oz. bottles ( $\frac{1}{4}$  pt.) can be filled from 4 gal. 2 qt. 1 pt. 3 gi. of alcohol?

11. Find the cost of 86 pieces of maple flooring, 3 in. wide, 16 ft. long, at \$38.50 per M.; same number of pieces 4 in. wide, 15 ft. long, at \$35 per M.; 48 boards 10 in. wide, 18 ft. long, @ \$28.75 per M.

Find the interest of:

12. \$248 for 90 days at 7%.

13. \$636 for 1 yr. 5 mo. 10 da. at 5%.

14. \$1,478 for 1 yr. 2 mo. 13 da. at 6%.

15. Divide 200 into three parts so that the second part shall be three times the first, and the third part two times the second.

16. Divide \$800 between A and B, giving B \$3 as often as you give A \$5.

Find the cost of one pound or one yard in the following:

1. 9 lb. figs cost \$1.08.      3 lb. steak cost \$.75.
2. 15 yd. ribbon cost \$1.05.    14 lb. rice cost \$1.12.
3. A newsboy bought 75 papers at 2¢ each, and sold them at 3¢ each. How much less than \$1.00 did he gain?
4. I have a number in my mind. If I take 15 from it, 35 will remain. What is the number?
5. When is the selling-price equal to the cost and something added?
6. When is the selling-price equal to the cost and something subtracted?
7. When is the selling-price more than the cost?
8. When is the selling-price less than the cost?
9. When is the selling-price equal to the cost?
10. The cost is always considered what per cent?
11. When the gain is 20% what is the selling per cent?
12. When the loss is 10% what is the selling per cent?
13. When there is neither gain nor loss what is the selling per cent?
14. Eggs are 38¢ a dozen at one store, and 45¢ a dozen at another. I bought 9 doz. at the second store. How much would I have saved had I bought at the first store?
15. Make change from three quarters for a 69¢ purchase.
16. Make change from a half-dollar for a 31¢ purchase.
17. Find the change due from three dimes for a 27¢ purchase.
18. A man worked for a farmer at 20¢ an hour, and received 4 bu. of potatoes at 50¢ a bushel. How many hours did the man work?
19. If a man receives \$6.60 for 22 hours' work, how much does he receive an hour?
20. What part of a pound is 12 oz.? 4 oz.?
21. Find the cost of 5 T. 400 lb. of coal at \$5 a ton.
22. What is the volume of a prism 10 ft. by 5 ft. by 4 ft.?



Fig. 1.



Fig. 2.

NOTE. — These drawings were made from a cut-up cylinder. One should be in the hands of the teacher when this lesson is given.

1. Of what is Fig. 1 a drawing?
2. Of what is Fig. 2 nearly a drawing?

NOTE. — The more parts into which the cylinder is cut, the more nearly will it approach a prism, when arranged as in Fig. 2.

3. How do you find the contents of the prism?
4. How does the length of the prism compare with the circumference of the cylinder?
5. How does the width of the prism compare with the diameter of the cylinder?
6. How does the height of the cylinder and prism compare?
7. Every cylinder can be changed into a rectangular prism, with  $\frac{1}{2}$  the circumference as the length, the radius as the width, and the same height. Learn: To find the contents of a cylinder, multiply the area of the base by the height.
8. What shape is one end of the cylinder?
9. What does this circle become when the cylinder is changed into a prism?
10. Prove that the way to find the contents of cylinders is identical with that of finding the contents of prisms. Find the number of cubic units that can be placed in one layer on the base, and multiply by the number of layers.
11. Find the volume of a cylinder whose altitude is 7 ft. 4 in. and the diameter of the base 5 ft.



1. Find the volume of a cylinder whose altitude is 8 ft. 6 in. and the diameter of the base 3 ft.

2. Find the solid contents of a cylinder whose altitude is 15 ft., and the radius of the base 1 ft. 3 in.

3. What is the entire surface of a cylinder 6 ft. 6 in. long, and the radius of its base 4 ft.?

4. What are the contents of a cylinder whose length is 5 ft. and diameter of the base 15 in.?

5. How many gallons will a circular cistern hold that is 6 ft. in diameter and 10 ft. deep?

NOTE. — There are 231 cu. in. in a gallon. For practical purposes it is sufficient to say, there are  $7\frac{1}{4}$  gal. in a cubic foot. Use the latter method unless otherwise directed.

6. Find the entire surface of a cylinder 30 ft. long, and 30 in. in diameter.

7. How many gallons of water will a cylindrical vessel hold that is 9 ft. deep and 3 ft. in diameter?

8. A cylindrical vessel 8 ft. high and 5 ft. in diameter is filled with potatoes. What is the value of the potatoes at 75¢ a bushel? Approximate measurement.

9. How many gallons of water are in a well 5 ft. in diameter, if the water is 7 ft. deep?

10. Find the entire surface and volume of a cylindrical column  $3\frac{1}{2}$  ft. in diameter, and 28 ft. high.

11. At 32¢ a cubic foot what is the value of a log 45 ft. long and 2 ft. in diameter?

12. A cylindrical water-tank is 25 ft. high and its diameter is 30 ft. How many gallons of water will it hold?

13. A circular reservoir is 80 ft. in diameter, and 20 ft. deep. How many gallons of water will it hold when full?

14. How much more gilding will it take to cover a 9-inch cube, than to cover a cylinder whose height and diameter are each 9 inches?

1. A boy sold 2 bu. 8 pk. of pears at the rate of 3 for 5 cents. Each peck averaged  $3\frac{1}{2}$  doz. pears. How much did he receive for all?

2. If a number diminished by  $\frac{1}{3}$  of itself is 7,296, what is the number?

3. A horse and carriage are worth \$763. The carriage is worth  $\frac{2}{3}$  as much as the horse. What is the value of each?

4. A house and lot cost \$13,600. The cost of the lot was  $\frac{1}{4}$  of the cost of the house. Find the cost of each.

5. A man divided his estate, giving his oldest son  $\frac{1}{4}$  of it and his youngest son  $\frac{1}{3}$  of it. If \$375 was the difference between the sons' shares, what was the value of the estate?

6. A and B can do a piece of work in 6 days. A can do it alone in 10 days. In what time can B do it?

7. I called for bids in a piece of work. A agreed to do it in  $1\frac{1}{2}$  mo. at \$2.75 a day; B agreed to do it in  $2\frac{1}{2}$  mo., at \$2.25 a day; and C in  $3\frac{1}{2}$  mo., at \$1.50 a day. Each counted only 24 working days to a month. Which bid should I accept?

8. If  $\frac{1}{6}$  of A's money is equal to  $\frac{1}{8}$  of B's money, and B has \$8,000, how many dollars has A?

9. A drover bought cows at \$27.40; if he had paid \$28 $\frac{1}{4}$ , they would have cost him \$120.70 more. How many cows did he buy?

10. When 9 hours is a day's work, and \$1.50 is a day's pay, find each man's pay in the following time sheet:

	MON.	TUES.	WED.	THURS.	FRI.	SAT.
A	9	8 $\frac{1}{2}$	9 $\frac{1}{2}$	7 $\frac{1}{2}$	8	6 $\frac{1}{2}$
B	8	10	7	6 $\frac{1}{2}$	8	8 $\frac{1}{2}$
C	6 $\frac{1}{2}$	8 $\frac{1}{2}$	9	8	5	7
D	6 $\frac{1}{4}$	5 $\frac{1}{2}$	9	7 $\frac{1}{4}$	6	5
E	8	9	7	8	9	8

11.  $53\frac{1}{3}$  is what part of  $83\frac{1}{3}$ ?

1. 2, 2, 5, and 7 are four of the five factors of 1,680.
2. A owns  $\frac{1}{3}$  of a ship, and sells  $\frac{1}{3}$  of his share for \$3,600.
3. A pole stands  $\frac{1}{3}$  in the ground,  $\frac{1}{4}$  in the water, and 33 ft. above water.
4. A room is 40 ft. long,  $31\frac{1}{2}$  ft. wide, and 12 ft. high.
5. A man's salary is \$1,500 a year. He spends 35% of it.
6. A merchant paid \$10,050 for stock, and sold it an advance of  $33\frac{1}{3}\%$ . His expenses were \$1,500.
7. A rectangular tank of water is 25 ft. long, 18 ft. wide, and 16 ft. 9 in. deep.
8. A merchant sold a lot of goods for \$129 at a loss of  $33\frac{1}{3}\%$ .
9. The same merchant sold another lot for \$73.85 at a gain of  $16\frac{2}{3}\%$ .
10. A stove-pipe is 16 ft. long and 7 in. in diameter.
11. A pile of wood contains  $4\frac{1}{2}$  cords. The pile is 7 ft. 6 in. high and 4 ft. wide.
12. The circumference of a circular pond is 150 rd.
13. Bought a horse and carriage for \$650, and sold them for \$806.
14. An article cost \$90. It was sold at a gain of  $12\frac{1}{2}\%$ .
15. A farmer bought a twelve-acre field of wheat for \$225. He paid \$1.45 an acre for cutting, and 5¢ a bushel for thrashing, and \$1.75 a load (42 bu.) for teaming. The wheat yielded 28 bu. to the acre, and was sold for \$1.12 $\frac{1}{2}$  a bushel.
16. A train leaves New Haven at 8.45 A.M., and goes 27 $\frac{1}{2}$  miles an hour. Another train follows at 9 A.M., and goes 41 miles an hour.
17. The inside dimensions of a rectangular fort are 240 ft. by 190 ft. The wall surrounding this fort is 6 ft. thick and 15 ft. high.
18. Two men had each \$420. One of them spent 15% and the other  $18\frac{1}{2}\%$  of this sum.

1. A merchant bought 5 boxes of butter for \$50, and sold them so as to gain \$10. What did he receive for each?
2. What number added to twice itself gives 18?
3. If one man can do a piece of work in 44 days, how many men can do the same work in 4 days?
4. At 80¢ a bushel, what is a peck and a half of corn worth?
5. Five boys bought a ball for 85 cents, and sold it for 70 cents. How much did each boy lose if they divided the loss equally?
6. A man bought a calf for \$12, and sold it to the butcher so as to gain 5%. How much did he gain?
7. What is the amount of \$60 for 60 days?
8. What is the interest for \$60 for 1 yr. 6 mo. at 5%?
9. What is the interest of \$100 for 1 yr.? For 2 yr.? 1 yr. 6 mo.? 2 yr. 3 mo.?
10. \$20 is  $16\frac{2}{3}\%$  of what sum?
11. A grocer sold tea at 50¢ a pound, and thereby gained 25%. What was the cost a pound?
12. What is  $8\frac{1}{3}\%$  of 48 books?
13. If  $3\frac{3}{4}$  bbl. of flour cost \$20 $\frac{3}{4}$ , what will  $6\frac{1}{2}$  bbl. cost?
14. If a family consume  $\frac{3}{4}$  barrels of flour in a month, how long will  $3\frac{1}{2}$  barrels last them?
15. At  $\frac{1}{2}$  of a dollar a rod, what will it cost to build  $\frac{3}{4}$  of a rod of fence?
16. A boy had 36 hens, and sold  $\frac{2}{3}$  of them at 50¢ each.
17. How many are  $\frac{1}{2}$  of  $\frac{2}{3}$  of 36?
18. In one room there are 32 pupils.  $\frac{2}{3}$  of these are  $\frac{3}{4}$  of the number in the other. How many pupils are there in the second room?
19. In a pile of wood there are 24 cd. Three-eighths of the pile are worth \$54. What is 1 cd. worth?
20. 54 are how many times  $\frac{3}{4}$  of 24?
21. What is the ratio of a day to 3 hours?

1. A man had \$60,000. He put 37% of it in the bank, 17% in a store, and the remainder in a railroad. How many dollars did he invest in the railroad?

2. A man owes \$8,496, but can pay only \$6,372. What per cent can he pay?

3. I have invested \$4,896 in business, which is 16% of all my money. How much money have I?

4. What number increased by  $62\frac{1}{2}\%$  of itself equals 3,942 $\frac{1}{4}$ ?

5. The Mississippi River is 4,200 miles long, which is 5% longer than the Nile, and that is  $6\frac{1}{3}\%$  longer than the Amazon. Find the length of the Nile and of the Amazon.

6. A drover sold 34% of his cattle, and had 990 left. How many did he sell?

7. A man bought 800 tons of coal at \$3.50 a ton, and sold it so as to gain 45%. What did he receive for it?

8. A grocer sold flour at \$3.50 a barrel, and lost 30%. Find the cost.

9. A man sold an engine for \$1,650, and gained 25%. Find the gain.

10. The same man sold another engine for the same price, and lost 25%. Find the loss. How does the loss compare with the gain in example 9?

11. An agent collected \$1,680 for me, but only sent me \$1,600. How much did he keep as commission? What per cent of what he collected did he keep?

12. Find the interest of \$2,763 from Sept. 5, 1901, to Jan. 13, 1903, at  $4\frac{1}{2}\%$ .

13. Find the interest of \$106.45 from Nov. 28, 1900, to June 6, 1903, at  $5\frac{1}{2}\%$ .

14. Find the amount of \$1,047.50 for 1 yr. 9 mo. 10 da., at 7%.

15. Find the interest of \$750 from Sept. 8, 1901, to Aug. 8, 1904, at  $6\frac{1}{2}\%$ .

1. A and B can do a piece of work in 8 days, A and C in 9 days, and A alone in 12 days. In how many days can B and C do it?

2. How many board feet are there in 56 joists, each 20 ft. long, and 9 in. by 3 in.?

3. Add  $\frac{1}{2}$ ,  $\frac{3}{4}$ ,  $\frac{1}{8}$ , 9.647, 8.93 $\frac{1}{2}$ , 59.7 $\frac{1}{2}$ , 4.07 $\frac{1}{2}$ .

4. At 75¢ a square yard it cost \$99 to pave a walk. If the walk is 6 ft. wide, how long is it?

5. How much will it cost at 36¢ a square yard, to plaster the walls and ceiling of a room 18 ft.  $\times$  14 ft.  $\times$  9 ft., allowing 180 sq. ft. for openings.

6. A room is 24 ft. long and 19 ft. 8 in. wide. How many yards of carpet 1 yd. wide will it take for this room if the breadths run lengthwise, and there is a waste of 10 in. on each breadth for matching? Would it take any less if the breadths ran the other way.

7. Find the cost of building a tight board fence 4 ft. 6 in. high, on two sides of a lot 32 rd. long, and  $\frac{1}{2}$  as wide. The boards are nailed to 2 scantlings each 2 by 4 in. The posts are 8 ft. apart, and cost \$27 a hundred. The scantlings cost \$16 per M., and the boards \$18 per M.

8. Find the interest of \$2,679.13 from June 6, 1901, to July 6, 1903, at 7%.

9. Find the area of a triangle whose altitude is 16 ft. and base 18 ft.

10. Find the area of a trapezoid when the parallel sides are 84 rd. and 66 rd., and the altitude 38 rd.

11. What must be the height of a pile of wood 32 ft. long and 6 ft. wide, to contain 9 cords?

12. A man bought a horse, cow, and pig for \$160. If he paid three times as much for the cow as for the pig, and four times as much for the horse as for the cow, what was the price of each?

(See Part II., pages 64, 65.)

## 1. What is the ratio of:

36 to 12?	12 to 4?	2 ft. to 4 in.?
49 to 7?	4 to 40?	1½ yd. to 6 in.?
72 to 8?	16 to 7?	1 lb. to 8 oz.?
42 to 6?	8 to 56?	\$4 to 50 cents?

## 2. What is the ratio of:

3 to 6?	4 to 20?	8 to 24?
2 to 4?	6 to 8?	12 to 60?
2 to 10?	3 to 12?	18 to 54?
6 to 30?	16 to 48?	36 to 48?

## 3. Find the ratio of:

1 da. to 2 hr.	4 mo. to 1 yr.	2 wk. to 2 da.
1 ft. to 1 yd.	2 min. to 30 sec.	1 pt. to 1 qt.
1 gal. to 2 qt.	4 oz. to 1 lb.	2 pk. to 1 bu.
¾ to ⅓.	⅔ to ⅓.	⅓ to ⅓.

## 4. Find the ratio of:

8.7 to 2.9.	.64 to .08.	3½ to 4¾.
6 mi. 15 rd. to 2 mi. 5 rd.	3 ft. 9 in. to 8 ft. 4 in.	
5 lb. 4 oz. to 6 lb. 10 oz.	2½ to 3½.	

## 5. Find the ratio of:

1.25 : 37.5	2½ : 3½	2 mi. : 640 ft.
6.25 : 2.5	5¼ : 6½	5 wk. : 4½ da.
16.8 : 8.4	8½ : 1½	5 tons : 500 lb.

## 6. Find the ratio of:

65 : 15	225 : 75	93 : 31.
25 : 625	275 : 550	48 : 72.
342 : 228	75 : 125	144 : 48.

## 7. Find the ratio of:

900 : 300	96 : 72	144 : 60
200 : 600	63 : 108	33 : 88
150 : 450	56 : 84	36 : 72
600 : 200	54 : 18	24 : 16

1. In every ratio the first term is called the antecedent, and the second term the consequent.

2. What is the ratio of 8 : 4? What is the ratio of 6 : 3?

These two ratios being equal may be written  $8 : 4 = 6 : 3$ . This is a proportion. Proportion is an equality of ratios. A proportion may be read in two ways: The ratio of 8 to 4 equals the ratio of 6 to 3, or 8 is to 4 as 6 is to 3.

3. In the proportion above, how many ratios are there? Name the antecedent of each ratio. Name the consequent of each ratio.

4. In every proportion the first and fourth terms are called extremes, the second and third means.

5. Name the extremes and means in the above proportion.

6. Multiply the extremes together; multiply the means together. What is true of your products?

7. Since this is true, if any three terms of a proportion are given, the other can be found.

8. In the proportion  $x : 4 = 6 : 3$ , we know that  $3 \times x = 6 \times 4$ . Hence,  $3x = 24$ .  $x = 8$ .

9. In the proportion  $8 : x = 6 : 3$ , we know that  $8 \times 3 = x \times 6$ . Hence  $6x = 24$ .  $x = 4$ .

10. In the proportion  $8 : 4 = x : 3$ , we know that  $8 \times 3 = 4 \times x$ . Hence  $4x = 24$ .  $x = 6$ .

11. In the proportion  $8 : 4 = 6 : x$ , we know that  $8 \times x = 6 \times 4$ . Hence  $8x = 24$ .  $x = 3$ .

Find the missing terms in the following proportions:

12.  $4 : 6 = 8 : x$ .  $x : 8 = 10 : 40$ .  $9 : x = 7 : 21$ .

13.  $6 : 3 = x : 11$ .  $8 : 4 = 12 : x$ .  $x : 5 = 8 : 20$ .

14.  $9 : 5 = x : 15$ .  $8 : x = 6 : 12$ .  $10 : 6 = 6 : x$ .

15.  $140 : 8 = 70 : x$ .  $8 : x = 5 : 120$ .  $12 : x = 42 : 63$ .

16.  $x : 8 = 7 : 3$ .  $54 : 9 = x : 10$ .  $63 : 7 = x : 5$ .

17.  $35 : x = 21 : 3$ .  $48 : x = 12 : 2$ .  $36 : 6 = 24 : x$ .

18.  $\frac{2}{3} : x = \frac{1}{2} : \frac{1}{3}$ .  $x : \frac{3}{4} = \frac{1}{2} : \frac{1}{3}$ .  $\frac{7}{8} : \frac{3}{4} = x : \frac{1}{4}$ .

19.  $2 : 5 = x : 10$ .  $3 : 4 = 9 : x$ .  $x : 3 = 12 : 36$ .



1. What is the ratio of 35 to 5? Of 6 to 18?
2. What is the ratio of 60 to 12? Of 8 to 16?
3. What is the ratio of 1 to  $\frac{1}{2}$ ? Of  $\frac{1}{2}$  to 1?
4. What is the ratio of  $\frac{1}{2}$  to  $\frac{1}{4}$ ? Of  $\frac{1}{4}$  to  $\frac{1}{2}$ ?

Find the value of  $x$ :—

$x:4 = 6:3$	$2:x = 6:12$	$3:5 = x:10$
$4:3 = 8:x$	$x:6 = 2:4$	$8:x = 4:2$
$8:6 = x:4$	$7:14 = 3:x$	$x:6 = 4:8$
$2:x = 4:8$	$10:5 = x:3$	$6:2 = 9:x$

(For solving the following problems see Part II, page 59, note.)

6. If 4 oranges cost 12 cents, what will 8 oranges cost?
7. If 5 yd. of cloth cost \$15, what will 6 yd. cost?
8. How many oranges can be bought for 50¢, at the rate of 4 for 10 cents?
9. What will 5 tons of coal cost, if 3 tons cost \$18?
10. If 4 men can mow 8 acres in a certain time, how many acres can 8 men mow in the same time?
11. If 7 men can build a wall 14 ft. long, how long a wall will 4 men build?
12. If 4 men can build a fence 12 rd. long, how long a fence will 7 men build in the same time?
13. If 5 men can dig a ditch 10 rods long in a day, how long a ditch can 8 men dig?
14. What will 20 yd. of cloth cost, if 5 yds. cost \$15?
15. If \$100 gains \$6 in a year, what will \$200 gain?
16. If 5 oranges cost 15¢, what will 8 oranges cost at the same rate?
17. If 3 tons of hay cost \$40, what will  $1\frac{1}{2}$  tons cost at the same rate?
18. Required the cost of 21 bbl. of apples, at the rate of \$6 for 3 barrels.
19. The ratio of the value of a purse to its contents is  $\frac{1}{3}$ . If the purse is worth \$2, how much money does it contain?

1. If 6 tons of coal cost \$28, how many tons can be bought for \$84?

(a) In examples like the above we have two ratios. \$28 is to be compared with \$84. 6 tons is to be compared with  $x$  tons. These two ratios will be equal, for they bear the same relation to each other as cause to effect. The sums of money are directly proportional to the tons of coal bought with the money.  $\$28 : \$84 = 6 \text{ tons} : x \text{ tons}$ .  $28x = 504$ .  $x = 18$ .

(b) These problems may be solved without forming a proportion if desired. The ratio of \$84 to \$28 is 3. Hence we can buy 3 times 6 tons of coal, or 18 tons.

2. If 6 men can build a wall in 24 days, how long will it take 18 men to build the same wall?

The first example is an illustration of direct proportion, where *more* dollars will buy *more* tons. This example is an illustration of an inverse ratio, where *more* men need *less* days; i.e., the days are not directly proportional to the men.

$$18 \text{ men} : 6 \text{ men} = 24 \text{ days} : x \text{ days} ?$$

$$18x = 144. \quad x = 8 \text{ days.} \quad \text{Ans.}$$

The following explanations may seem clearer to some, and are therefore inserted.

1.  $\$28 : \$84 = 6 \text{ tons} : x \text{ tons}$ .

$$\frac{\begin{array}{c} 3 \\ 84 \times 6 \\ \hline 28 \end{array}}{28} = 18 \text{ tons.}$$

Since we are seeking for tons, we place 6 tons in the third place, because the third and fourth terms form a ratio and must be of the same kind.

We know that \$28 will buy 6 tons, then \$84 will buy *more* tons. This shows us that the consequent of our second ratio is more than its antecedent; therefore to have an equality of ratios, the consequent of our first ratio must be larger than its antecedent. Hence our ratio must be \$28 : \$84.

Cancelling, we have 18 tons.

2.  $18 \text{ men} : 6 \text{ men} = 24 \text{ days} : x \text{ days}$ .

$$\frac{\begin{array}{c} 8 \\ 6 \times 24 \\ \hline 18 \end{array}}{18} = 8. \quad \text{Ans.}$$

Since we are seeking for days, we place 24 days as our third term. Why? We know that 6 men can build a wall in 24 days; then 18 men can build it in less days.

This shows us that our second consequent is less than its antecedent; hence to have an equality of ratios we must make our first consequent less than its antecedent, and our ratio must be 18 men to 6 men. Cancelling, we have 8 days.

NOTE. — Before comparing the quantities in the following examples, first determine whether the terms are *directly* or *inversely* proportional.

1. If 45 sheep cost \$225, what will 165 sheep cost?
2. If 35 acres of land cost \$937.50, what will 175 acres cost?
3. If 240 acres of land cost \$1,500, how many acres can be bought for \$4,500?
4. If 15 yd. of cloth cost \$24, what will 65 yd. cost?
5. If 24 bbl. of flour will last 160 men for 5 weeks, how many barrels will last 180 men the same time?
6. If 8 men can dig a ditch in 4 days, how long will it take 7 men to do it?
7. If 9 men can build a wall in 15 days, how long will it take 5 men to build it?
8. If 14 men can mow 25 acres of grass in a day, how many acres can 35 men mow?
9. If a staff 3 ft. high casts a shadow 5 ft. long, how long a shadow will be cast by a pole 120 ft. high at the same time?
10. If 12 lb. of sugar cost \$2, what will 30 lb. cost at the same rate?
11. A family of 6 persons pays \$21 a week for board; at this same rate what must a family of 8 persons pay?
12. A tree 18 ft. high casts a shadow 45 ft. long; how high must a steeple be to cast a shadow 135 ft.?
13. If 16 men can build a house in 24 days, how long will it take 12 men to build it?
14. If 24 men can build a house in 15 days, how many men can build it in 24 days?
15. If 12 tons of hay cost \$264, what will 19 tons cost?
16. If a train travels 220 mi. in 8 hr., how long will it be in traveling 330 miles?
17. If 16 horses eat a ton of hay in 12 days, how many horses will eat a ton in 32 days?

1. If  $\frac{2}{3}$  of a store is worth \$5,200, how much is  $\frac{1}{3}$  of it worth?

$$\frac{\$5,200 \times \frac{1}{2}}{\frac{2}{3}} = \frac{5,200 \times 7 \times 9}{8 \times 5} = \$8,190$$

Arrange the ratios as in whole numbers. Arrange for cancellation as usual. \$5,200 multiplied by  $\frac{1}{2}$  and divided by  $\frac{2}{3}$  equals \$5,200 multiplied by  $\frac{3}{4}$  and multiplied by  $\frac{1}{2}$ . This clears the work of fractions.

NOTE. — To arrange for cancellation, place fractions in the second or third term as for multiplication. Since we are to divide by the fraction in the first term, it will be the same to multiply by the same fraction inverted.

2. If it requires 42 yd. of carpeting  $\frac{3}{4}$  yd. wide to cover a floor, how many yards  $\frac{1}{2}$  yd. wide will cover the same floor?

3. If  $6\frac{1}{2}$  cd. of wood cost \$19 $\frac{1}{2}$ , how many cords can be bought for \$78?

4. If  $3\frac{1}{2}$  yd. of velvet cost \$5 $\frac{1}{2}$ , how much will  $7\frac{1}{2}$  yd. cost?

5. If  $4\frac{1}{2}$  lb. of butter cost \$1.35, what will  $17\frac{1}{2}$  lb. cost?

6. If  $\frac{1}{3}$  of a yard of silk cost \$2.10, what will  $33\frac{1}{2}$  yd. cost?

7. If 9 weeks' board cost \$94 $\frac{1}{2}$ , what will 12 weeks' board cost?

8. If 7 men can lay 2 miles of water-pipe in 15 days, how many days will 48 men require?

9. If  $\frac{1}{3}$  of a yard cost \$1 $\frac{1}{2}$ , what will  $\frac{2}{3}$  of a yard cost?

10. If a loaf of bread weighs 8 oz. when flour is worth \$5 a barrel, what should it weigh when flour is worth \$6 a barrel?

11. If  $2\frac{1}{2}$  yd. of cloth can be bought for \$3.30, what will  $15\frac{1}{2}$  yd. cost?

12. If 25 men can do a piece of work in 12 days, in how many days can 10 men do the same work?

13. If a steeple 126 ft. high casts a shadow 93 ft. long, how long a shadow will a steeple 168 ft. high cast at the same time?

14. If a steeple 216 ft. high casts a shadow 162 ft. long, how long a shadow will be cast by a steeple 124 ft. high at the same time?

15.  $83\frac{1}{2} : 55\frac{1}{2} = x : 93\frac{1}{2}$ . Find  $x$ .

1. Find the feet of lumber in the following boards: 2 boards 18 ft. long, 8 in. wide, and 1 in. thick; 3 boards 16 ft. long, 9 in. wide,  $\frac{3}{4}$  in. thick; 1 board 12 ft. 8 in. long,  $8\frac{1}{4}$  in. wide, 1 in. thick; 2 planks 16 ft. long, 10 in. wide at one end, 8 in. wide at the other, and  $1\frac{1}{8}$  in. thick.

NOTE. — Boards less than 1 in. in thickness are called 1 inch. Fractions of an inch in width are omitted, and the nearest integer taken. When the fraction is one-half the next integer is taken. When boards are not of uniform width, the average width is taken.

2. How many square feet in the surface of a spire which is in the form of an hexagonal pyramid, whose slant height is 80 ft. and each side of its base 12 ft.?

3. How many bushels of corn will a box contain which is 8 ft. long, 3 ft. wide, and 20 in. deep? (Allow  $1\frac{1}{4}$  cu. ft. to 1 bu.)

4. What will it cost, at 13¢ a square yard, to plaster the walls and ceiling of a room 16 ft. long,  $15\frac{1}{2}$  ft. wide, and 9 ft. high, deducting 95 sq. ft. for openings?

5. A rectangular monument of granite is 3 ft. square at the base, and 8 ft. high. How many cubic feet does it contain?

6. At 19¢ a square yard, what will it cost to paint the outside and the inside of a cylindrical tank 9 ft. long, and 6 ft. in diameter, no attention being paid to thickness of the material?

7. Find the cost of 1,860 ft. of lumber at \$24.50 a thousand.

8. A house is 32 ft. 8 in. long. The rafters are 22 ft. long. How many shingles laid 4 inches to the weather will cover the roof? (Allow 1000 shingles to 100 sq. ft.)

9. A cylinder is 3 ft. 8 in. long, and has a diameter of  $1\frac{1}{2}$  ft. How many gallons will it hold? (Allow  $7\frac{1}{2}$  gal. to 1 cu. ft.)

10. A rectangular lawn measures 15 yd. 2 ft. in length, and 33 ft. 8 in. in width. At 18¢ a square yard, find the cost of sodding the lawn. Outside the lawn is a walk 3 ft. 8 in. wide. Find the square feet in it. At 16¢ a running foot, find the cost of a fence just outside the walk.

Give the names of the following figures:

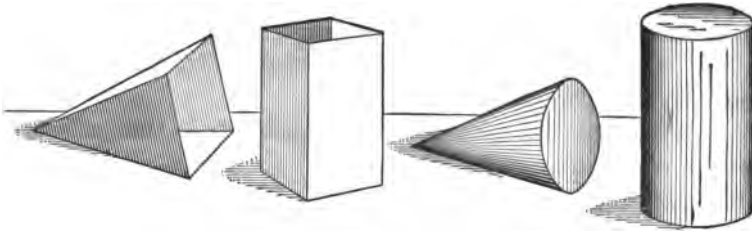
1. A figure having three sides.
2. A figure having three equal sides.
3. A figure having three sides, two of which are equal.
4. A figure having four equal sides and four right angles.
5. A four-sided figure, having opposite sides parallel, but containing no right angle.
6. A figure having four equal sides and opposite sides parallel, but containing no right angle.
7. A figure having four sides, with opposite sides parallel and equal.
8. A figure having four sides with only two parallel sides.
9. Any figure having four sides.
10. A figure having four sides with no parallel sides.
11. A rose garden is 12 ft. long and 9 ft. wide. How many bouquets can I gather, if 3 sq. ft. will furnish 1 bouquet?
12.  $9 + 7 - 6 + 5 - 8 + 4 - 9 + 7 - 6 - 3 + 2 = ?$
13. How many cubic feet in a wall 30 ft. long, 4 ft. high, 2 ft. thick?
14. How many one-foot cubes can be placed in a cubical box one yard long, one yard wide, and one yard high?
15. A farmer sold his wheat for \$267, and his oats for \$234. How much did he receive for both?
16. At 45¢ a pound, how many pounds can be bought for \$1.35?
17. Divide 18 lb. 12 oz. by 3.
18. Divide 30 da. 15 hr. 45 min. by 3.
19. What part of 2 gal. is 2 qt. 1 pt.?
20. 15 is  $\frac{3}{4}$  of  $\frac{1}{5}$  of what number?
21. If  $\frac{1}{3}$  of a stack of hay is worth \$42, what will 2 stacks be worth?
22. What is the least common multiple of 8, 12, and 24?
23. Divide 15,000,000 by 30,000.

1. A merchant sold goods which cost him \$4,768.75 at a profit of 18%. Find the gain.
2. A man raised 1,640 bu. of grain, and sold 246 bu. What per cent did he sell?
3. A teacher's salary, having been decreased 83%, is now \$1,200. What was it at first?
4. A grocer bought \$400 worth of oranges. 25% spoiled before he sold them. The remainder he sold  $16\frac{2}{3}\%$  above cost. Did he gain or lose? and how much?
5. A farm which cost \$3,400 was sold at a gain of 22%. Find the selling-price.
6. By selling hay at \$15 a ton a dealer loses 10%. Find the cost.
7. What is the commission, at  $3\frac{1}{2}\%$ , on the sale of \$4,769 worth of goods?
8. An agent sells 276 bbl. of flour at \$6.50 a barrel. His commission was  $2\frac{1}{2}\%$ . How much money should he return to me?
9. A ship was valued at \$72,000, and insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ . Find the premium.
10. A man bought 6,200 bu. of grain at \$1.50 a bushel. He sold 20% of it at a 5% loss, 40% of it at a 10% gain, and the remainder at cost. What was gained on the whole?
11. Find the cost when \$17.25 is the loss from selling an article at 15% below cost.
12. Find the interest on \$1,500 for 1 yr. 3 mo. 27 da. at 9%.
13. Find the amount of \$960 for 3 yr. 7 mo. 9 da. at 5%.
14. A farmer raised 2,480 bu. of grain. 36% of it was rye, 24% of it oats, and the rest corn. How many bushels of each kind of grain did he raise?
15. A man bought 120 acres of land at \$60 an acre, and paid 25% of the cost of the land for repairs and building. For how much must he sell to gain \$2,000?

1. What are the prime factors of 5,075?
2. Reduce  $1\frac{2}{3}\frac{3}{4}$  to lowest terms.
3. When  $17\frac{1}{2}$  lb. cost \$11 $\frac{1}{2}$ , how many pounds can be bought for \$5 $\frac{1}{2}$ ?
4. What will  $94\frac{1}{2}$  yd. of cloth cost, if  $165\frac{1}{2}$  yd. cost \$94?
5. If a boat sails 254 miles in  $19\frac{1}{2}$  hours, what is the rate an hour?
6. What will  $117\frac{1}{2}$  yd. of cloth cost, if  $378\frac{1}{2}$  yd. cost \$1,515?
7. What will  $143\frac{1}{2}$  bu. of apples cost, if  $584\frac{1}{2}$  bu. cost \$1,022 $\frac{1}{2}$ ?
8. A owns  $\frac{1}{4}$  of a store, and B the remainder. If A owns \$465 more than B, what is the value of the store and of each one's share?
9. How many marbles have two boys, when one owns  $\frac{1}{2}$  of all, and has 60 marbles more than the other?
10. Eight-elevenths of 2,728 is  $3\frac{1}{2}$  of what number?
11. A man gave 180 acres to his son, and had  $\frac{1}{3}$  of his farm left. What is the value of the remainder at \$39 $\frac{1}{2}$  an acre?
12. An estate was divided between two persons, so that A received  $\frac{1}{4}$  of the whole, and B the remainder, or \$6,400. Find the value of the whole estate.
13. An estate was divided among 3 children, so that one received  $\frac{1}{3}$  of it, the second  $\frac{1}{3}$  of it, and the third \$4,840. Find the value of the estate.
14. How much more is  $4\frac{1}{2}$  times  $9\frac{1}{2}$  than  $2\frac{1}{2}$  times  $5\frac{1}{2}$ ?
15. A owns  $\frac{1}{4}$  of a store, B  $\frac{1}{3}$  of it, and C the remainder, or \$31,000. What is the value of the store?
16. A room is 15 ft. by 20 ft., with walls 12 ft. high. Find how many square feet there are in the walls and ceiling. If there are 3 windows  $2\frac{1}{2}$  ft. by 6 ft., and 2 doors  $3\frac{1}{2}$  ft. by 8 ft., find how many square feet there are in the doors and windows. Find how many square feet there are in the walls and ceiling, after taking out the doors and windows.
17. From two take two ten-millionths.



1. What is the area of a circular garden whose circumference is 180 rd.?
2. Find the area of a trapezoid when the parallel sides are 120 in. and 96 in., and the altitude 86 in.
3. A and B together have 2,588 acres of land, and B has 5 times as much as A. How many acres has each.
4. A drover has 427 sheep and cows. If he has 125 more sheep than cows, how many has he of each?
5. If  $13\frac{1}{2}$  bu. of corn cost \$6 $\frac{1}{4}$ , what will  $16\frac{1}{2}$  bu. cost?
6. If 25 oxen eat the grass from 36 acres in a month, for how many oxen would 468 acres furnish feed for the same time?
7. Find the interest of \$23.75 for 6 yr. 7 mo. 21 da. at 5%.
8. How long must 3 piles of wood be to contain 27 cd., if each is  $5\frac{1}{2}$  ft. high, and 4 ft. wide?
9. A can do a piece of work in 6 days, B in 8 days, and C in 12 days. In how many days can all do it together?
10. Divide .00017 by .034.
11. Add  $9\frac{4}{5}$ , .845,  $7\frac{3}{4}$ ,  $56\frac{1}{2}$ , .65 $\frac{1}{2}$ , 59.84,  $37\frac{3}{4}$ .
12. Reduce 1,345,165 seconds to higher denominations.
13. Find the cost of 2,400 qt. of onions at 18¢ a pk.; 3,200 qt. of milk @ 24¢ a gallon; and 4,200 qt. sirup @ 12¢ a pint.
14. Find the gain in buying 2,480 gal. of vinegar at 30¢ a gallon, and selling it at 9¢ a quart.
15. A man divided a field containing 16 acres into lots containing 40 square rods each. He sold the lots at \$175 each. How much did he receive?
16. Find the cost of 17 bu. 5 qt. of oats at 2¢ a quart, and 7 lb. 2 oz. of spice at 15¢ an ounce.
17. A mill is worth \$9,900, a house \$3,000, and a farm  $\frac{7}{8}$  of the difference between the value of the house and mill. Find the value of all.
18. In a school there are 495 pupils, and twice as many girls as boys. How many girls are there?



To illustrate this principle there should be in the room a hollow cone and cylinder, each having the same base and altitude, also a hollow pyramid and prism, each having the same base and altitude. These can easily be made from cardboard. Using sand or sawdust, let each pupil determine, by measuring, the ratio or relation of the contents of the cone to the contents of the cylinder, and the contents of the pyramid to the contents of the prism. This they will find to be one-third.

NOTE. — To find surface, the slant height must be known; but to find volume or contents, the altitude must be known.

LEARN: To find the contents of cones and pyramids, multiply the area of the base by  $\frac{1}{3}$  of the altitude.

1. Find the volume of a square pyramid each side of whose base is 4 ft. and altitude 18 ft.

2. Find the volume of a cone the circumference of whose base is 9 ft., and whose altitude is 14 ft.

3. What is the volume of a square pyramid, the perimeter of whose base is 8 ft., and whose altitude is 10 ft.?

4. What is the volume of a pyramid whose base is 6 ft. square, and whose altitude is 21 ft.?

5. Find the volume of a cone whose diameter is 6 ft., and altitude 18 ft.

6. Find the volume of a cone whose circumference is 31.416 feet and altitude 20 feet.

7. Find the volume of a square pyramid with a base 10 ft. square, and an altitude 25 ft.

8. Find the volume of a cone, when the circumference of its base is 48 ft., and its altitude 50 ft.

1. If a boy buys oranges at 40¢ a dozen, how must he sell them apiece so as to make 20% on each orange?
2. If a yard of silk costs a merchant 80¢, for how much must he sell it to gain  $12\frac{1}{2}\%$ ?
3. What is the gain per cent on an article that is bought for 40 cents and sold for 60 cents?
4. I went out into the country and bought eggs at 25¢ a dozen, and brought them to the city and sold them for 30¢ a dozen. What was my gain per cent?
5. At the same time I bought potatoes at 60¢ a bushel, and sold them at 25¢ a peck. What was my gain per cent?
6. A lawyer agreed to collect for me at 5% commission. He collected  $\frac{1}{2}$  of a debt of \$1,200. How much commission did he keep?
7. At 8%, find the interest on \$50 for one year.
8. At 7%, find the interest on \$40 for a year.
9. At 10%, find the interest on \$60 for 60 days.
10. At 6%, find the interest on \$200 for 33 days.
11. A watch cost \$70 more than a chain, and together they cost \$160. How much did each cost?
12. A can do a piece of work in 3 days, B in 4 days, and C in 5 days. How much can each do in a day? How much can all do in a day? How many days will it take, all working together, to do the whole work?
13. Two men can do a piece of work in 3 days. If one man can do it in 5 days, in how many days can the second man do it?
14. A is 40 yd. ahead of B. If B runs 5 yd. while A runs 4, how many yards must B run to overtake A?
15. A is 30 ft. ahead of B, but B runs 5 ft. while A runs 2 ft. How many feet will B run to overtake A?
16. Divide 64, 76, 96, 104, 112, 216 by 4.
17. If  $\frac{1}{2}$  a yard of cloth costs \$4, what will  $1\frac{1}{2}$  yd. cost?
18.  $\frac{2}{3}$  of 24 are how many eighths of 12?

1. The difference between two numbers is 2,001,005; the larger number is 89,009,089. Find the smaller.

2. At \$5.75 a barrel, how many barrels of flour can be bought for \$1,161.50?

3. The quotient is 4,769. What will the quotient be if the dividend is multiplied by 9?

4. The quotient is 4,664. What will the quotient be if the divisor is multiplied by 8?

5. The quotient is 805. What will the quotient be if the divisor is  $\frac{1}{2}$  of what it is now?

6. The quotient is 909. What will the quotient be if both dividend and divisor are multiplied by 15?

7. The quotient is 478. What will the quotient be if both divisor and dividend are divided by 12?

8. If the divisor were 8 times as large as it is, the quotient would be 489. What is the quotient?

9. What number must be taken from 7,684 that it may be exactly divisible by 33?

10. A book agent bought 112 books at \$3.20 each. He sold them at \$4.90 each. If his expenses were \$75, and he could not collect the money for 4 books, how much did he gain or lose?

11. If 37 is added 99 times to itself, the result will be how much less than 3,750?

12. If 37 be added to a certain number, 85 can be subtracted from it 113 times. Find the number.

13. Of what number is 463 both divisor and quotient?

14. If 593 is subtracted 347 times from a certain number, the remainder is 287. Find the number.

15. A house and lot cost \$9,600. The house cost 5 times as much as the lot. Find the cost of each.

16. How much will a dealer gain by buying 2,464 bu. of potatoes at  $87\frac{1}{2}$ ¢ a bushel, and selling them at \$1.12 $\frac{1}{2}$  a bushel?

1. The base of a cone is 32 in. in diameter, and its altitude is 3 ft.
2. A rectangular prism has an altitude of 12 ft. Its base measures 18 in. by 14 in.
3. The area of a rhombus is 720 sq. in. Its altitude is 24 in.
4. The area of a triangle is 7 sq. yd., and its altitude is 21 ft.
5. A man traveled 65 days at the rate of 768 miles in 4 days.
6. An article that cost \$75 was sold for \$95.
7. \$953 was on interest from Aug. 9, 1901, to April 1, 1902, at 5%.
8. A merchant insured his store for \$7200, at  $2\frac{1}{2}\%$  a year for three years.
9. A collector received \$121.40 for collecting a debt at 5%.
10. By selling flour at \$5.85 a barrel, a merchant lost 22%.
11. When the selling-price was \$162.50, there was a profit of 30%.
12. A boy earned \$8.40, which is 15% of what he had before.
13. A man lost \$1770 out of a business of \$2950.
14.  $83\frac{1}{4}\%$  of a debt of \$8400 has been paid.
15. A room measures 21 ft. by 16 ft. The carpet is  $\frac{3}{4}$  yd. wide, and costs \$1.40 a yd. The breadths run crosswise, and there is a loss on each breadth of 1 ft. for matching.
16. From an acre of land there was sold a piece in the form of a trapezoid 10 rd. long, 7 rd. wide at one end, and 5 rd. wide at the other.
17. It is 75 ft. in a straight line across a circular pond.
18. The area of a parallelogram is 300 sq. yd. The distance between its parallel sides is 50 ft.
19. Some 4-foot wood is piled 6 ft. high. There are 24 cords in the pile.

1. If 16 cd. of wood cost \$96, what will 25 cd. cost?

In solving problems like the above, it is a good plan to make statements like the following :

1. I am asked to find the cost of 25 cd. of wood.
2. I know the cost of 16 cords.
3. I need to know the cost of 1 cord.

4. Process  $\frac{96 \times 25}{16} = \$150$ .

5. Is the result reasonable? 25 cords are a little more than  $1\frac{1}{2}$  times 16 cords. My result should be a little more than  $1\frac{1}{2}$  times \$96.

NOTE. — If pupils were always obliged to take the 5th step we should have fewer nonsensical answers.

Solve the following problems by this method :

2. If a young man earns \$36 a month, in how many years will he earn \$5,616?

3. If 7 tons of coal cost \$35, what will 4 tons cost?

4. My sister Alice hires a bicycle for 30¢ an hour. What does she pay for its use if she uses it from 10 A.M. till 3.30 P.M.?

5. A has  $\frac{1}{3}$  of a sum of money, B  $\frac{1}{2}$  of it. B has \$30 more than A. How much has each?

6. If a bushel of beans weighs 60 lb., and a barrel holds 3 bu., how many barrels will it take to hold 5 tons?

7. If an ocean steamer uses 300 tons of coal in a day, how many pounds will it use in a month of 30 days?

8. Four men cut a pile of wood. If the first man cut  $\frac{1}{4}$  of it, the second  $\frac{1}{3}$ , and the third  $\frac{1}{5}$ , how much did the fourth cut?

9. I hired 4 men to work at \$3 a day each. The first worked  $\frac{1}{2}$  a day, the second  $\frac{1}{3}$ , the third  $2\frac{1}{4}$  days, and the fourth  $1\frac{1}{2}$  days. How much must I pay them?

10. If 3 bbl. of oil cost \$15.75, what will  $18\frac{1}{2}$  bbl. cost?

11. If I buy a 42-gal. barrel of oil for \$3.50, and retail it at  $12\frac{1}{2}$ ¢ a gallon, how much do I make?

12. My house cost me \$5,000, and my tax is \$16 on a thousand. If I rent my house for \$30 a month, what is my annual income?

1. Reduce to whole or mixed numbers:  $2\frac{1}{2}$ ,  $3\frac{2}{3}$ ,  $5\frac{2}{3}$ ,  $1\frac{2}{3}$ ,  $2\frac{1}{2}$ ,  $3\frac{2}{3}$ ,  $2\frac{2}{3}$ ,  $1\frac{2}{3}$ ,  $3\frac{2}{3}$ ,  $3\frac{2}{3}$ .

2. Reduce to improper fractions:  $516\frac{1}{2}$ ,  $84\frac{1}{2}$ ,  $134\frac{1}{2}$ ,  $125\frac{1}{2}$ ,  $160\frac{1}{2}$ ,  $105\frac{1}{2}$ ,  $113\frac{1}{2}$ ,  $63\frac{1}{2}$ .

3. Reduce  $\frac{3}{4}$  and  $\frac{1}{2}$  to 12ths.  $\frac{3}{4}$  and  $\frac{1}{2}$  to 8ths.  $\frac{3}{4}$  and  $\frac{1}{2}$  to 24ths.  $\frac{3}{4}$  and  $\frac{1}{2}$  to 30ths.

4. Reduce to equivalent fractions having a common denominator:  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$ ,  $\frac{1}{8}$ ,  $\frac{1}{9}$ .

5. Add:  $16\frac{1}{2}$ ,  $22\frac{1}{2}$ ,  $45\frac{1}{2}$ ,  $50\frac{1}{2}$ ,  $8\frac{1}{2}$ ,  $10\frac{1}{2}$ ,  $14\frac{1}{2}$ .

6. Add:  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ;  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ .

7. From  $20\frac{1}{2}$  take  $8\frac{1}{2}$ . From  $47\frac{1}{2}$  take  $19\frac{1}{2}$ .

8. What is  $\frac{1}{2}$  of 3,218?  $\frac{1}{3}$  of 3,002?

9.  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$ ?  $\frac{1}{3}$  of  $\frac{1}{2}$  of  $\frac{1}{4}$ ?  $\frac{1}{4}$  of  $\frac{1}{2}$  of  $\frac{1}{3}$ ?

10. Multiply  $40\frac{1}{2}$  by 20. 122 by  $6\frac{1}{2}$ .

11. Divide 231 by  $\frac{1}{2}$ .  $65\frac{1}{2}$  by  $18\frac{1}{2}$ .

12. A man lost  $\frac{1}{2}$  of his money and had \$411 remaining. How much had he at first?

13. If  $\frac{1}{2}$  of a farm is valued at \$2,253 $\frac{1}{2}$ , what is the value of  $\frac{1}{3}$  of it?

14.  $96\frac{1}{2} - \frac{770\frac{1}{2}}{16\frac{1}{2}}$ ?

15. If  $8\frac{1}{2}$  tons of coal cost \$30 $\frac{1}{2}$ , how many tons can be bought for \$127 $\frac{1}{2}$ ?

16. A farmer raised 2,146 bbl. of apples. He sold  $\frac{1}{2}$  of them at \$1 $\frac{1}{2}$  a barrel, and the remainder at \$2 $\frac{1}{2}$  a barrel. How much did he receive?

17. A boy bought 120 oranges at the rate of 5 for 2 cents. He sold  $\frac{1}{2}$  of them at the rate of 3 for 1 cent and the remainder at 3 for 2 cents. How much did he gain or lose?

18. If  $1\frac{1}{2}$  lb. of beef and  $1\frac{1}{2}$  lb. of flour are allowed to each man as a ration, how much will rations for 618 men cost if beef is  $11\frac{1}{2}$ ¢ a pound and flour  $3\frac{1}{2}$ ¢ a pound?

19. From  $126\frac{1}{2}$  take  $13\frac{1}{2}$  times  $1\frac{1}{2}$ .

1. If 4 men can do a piece of work in 6 days, how long will it take 12 men to do the same work?
2. If 8 men can do some work in 5 days, how long will it take 4 men to do it?
3. If  $\frac{3}{4}$  of a man's age is 18 years, how old is he?
4. If  $\frac{1}{2}$  of a ton of coal is worth \$3 $\frac{1}{2}$ , how much are 3 $\frac{1}{2}$  tons worth?
5. A boy spent  $\frac{1}{2}$  of his money, then earned  $\frac{1}{2}$  as much as he spent, and had \$20. How much had he at first?
6. Five-sixths of 60 is  $\frac{2}{3}$  of how many times 5?
7. A farmer sold a cow worth \$45 at a loss of 10%. What did he receive for her?
8. Three-fifths of 80 is what per cent of  $\frac{1}{2}$  of 120?
9. A butcher buys pork at 6¢ a pound, and sells it at 10¢ a pound. What is his gain per cent?
10. One-fourth of 80 is how many times 5?
11. Seventy is  $\frac{2}{3}$  of how many times 5?
12. A farm was sold for \$1,800, which was  $\frac{3}{4}$  of its cost. What was the loss?
13. Goods bought for \$120 must be sold for what price to gain 33 $\frac{1}{3}$ %?
14. From what number must  $\frac{1}{2}$  of 20 be taken 3 times to leave 3?
15. Twenty-four is 60% of what number?
16. A can do a piece of work in 4 days; B can do it in 5 days. In what time can A and B do it if they work together?
17. A rectangular field contains 1 acre, and is 40 rd. long. What is its perimeter?
18. A man spent 16 $\frac{2}{3}$ % of his month's salary. If he spent \$15, what was his month's salary?
19. The selling-price of a cow is \$60, the gain is 25%. What two things can be found? Find them.
20. What part of 100 is 12 $\frac{1}{2}$ ? 37 $\frac{1}{2}$ ? 87 $\frac{1}{2}$ ?



1. A man bought a house for \$4,200, and sold it at a gain of 25%. Find the cost.

2. A man spent 30% of his income for family expenses, and 25% of it for books and clothing, and saved the rest. If he saved \$900, what was his income?

3. Change  $\frac{1}{4}\%$  to a decimal. Change .005 to a per cent.

4. Of a lot of goods a man sold \$528 worth, which was 5% of the cost. What was the cost?

5. 75 is 15 per cent of what number? 24 is 40 per cent of what number?

6. 26 is 13 per cent of what number? 105 is 35 per cent of what number?

7. If an agent's salary was increased 30%, making it \$2,600, what was it before it was increased?

8. A man bought a farm for \$1,250, and sold it at a gain of \$250. Find the gain per cent.

9. What per cent of 800 is 48? Of 700 is 35? Of 450 is 45? Of 75 is 37.5?

10. If the ore is 35% pure, how much iron will 6,893 lb. of ore produce?

11. A man owned a part of a mill for which he paid \$12,250. He was obliged to sell at a loss of 15%. Find his loss.

12. Given the amount gained and the rate of gain. What can you find? Illustrate by an example.

13. Given the cost of an article and the rate of gain. What can you find? Illustrate by an example.

14. Given the cost of an article and the gain. What can you find? Illustrate by an example.

15. Given the cost and the selling-price. What can you find? Illustrate.

16. A dealer lost 20% by selling a carriage for \$120. What was the cost?

17. Find 65% of \$684. Find  $6\frac{1}{4}\%$  of  $8\frac{1}{2}$ .

(See general summary, pages 276-282.)

1. Find the contents of a rectangular prism  $12\frac{1}{2}$  yd. by 10 ft. by 16 in.
2. A rectangular prism is  $20\frac{1}{2}$  ft. long,  $17\frac{1}{2}$  ft. wide, and 6 in. deep. Find its contents.
3. At \$3.00 a cord, find the cost of a pile of wood 24 ft. long, 4 ft. wide, and  $7\frac{1}{2}$  ft. high.
4. I bought 6 boards, each 16 ft. long and  $1\frac{1}{2}$  in. thick. Their width was as follows: 8 in., 10 in., 12 in., 13 in., 14 in., 9 in. How many feet of lumber did I buy?
5. The entire surface of a cube is 294 sq. in. How long is the cube?
6. Find the entire surface of a 9-in. cube.
7. Find the entire surface and the contents of a prism 20 ft. long, 14 ft. wide, and 10 ft. high.
8. What is the difference in volume between a square prism 4 in. wide and 25 in. long and the largest cylinder that can be cut from the prism?
9. A well is 32 ft. deep and 5 ft. in diameter. How many cubic feet of water in it if it is  $\frac{3}{4}$  full?
10. The area of a rectangular field is 12 acres. If its length is 20 rd., what is its altitude?
11. In digging a trench 3 ft. wide and  $4\frac{1}{2}$  ft. deep, 330 cu. yd. of earth were removed. How long was the trench?
12. From a lot of land 40 rd. square I sold 40 sq. rd. What is the remainder worth at \$120 an acre?
13. A railroad company fenced 8 miles of its road at  $67\frac{1}{2}$ ¢ a rod. Find the cost of the fence.
14. On a pond covering 1 acre the ice when removed was 15 in. thick. If a cubic foot weighs  $57\frac{1}{2}$  lb., how many tons were cut?
15. What will it cost to polish the sides and top of a granite shaft 6 ft. by 2 ft. by 22 in. at  $1\frac{1}{4}$ ¢ a square inch?

1. At \$50 an acre, find the cost of a quadrangular piece of land whose parallel sides are 25 rd. and 35 rd., and 50 rd. apart.
2. What will it cost to sod a yard  $28\frac{3}{4}$  ft. long, and  $24\frac{1}{4}$  ft. wide, at 75¢ a square yard?
3. A and B can build a wall in 10 days; A can build it in 18 days. How long will it take B to build it alone?
4. What will 5 cd. ft. of wood cost at \$3.76 per cord?
5. What will it cost at 30¢ a square yard, to plaster a room 27 ft. long, 18 ft. wide, and 9 ft. high, allowing for 2 windows and 1 door, each 6 ft. by  $2\frac{1}{3}$  ft.?
6. 25% of 560 is  $33\frac{1}{3}\%$  of what number?
7. If 6 acres of land cost \$72, what will  $\frac{3}{8}$  of 80 acres cost?
8. One-eighth of a certain number is 16 more than one-ninth of it. What is the number?
9. I sold an article to a man for  $\frac{1}{3}$  more than it cost me. He sold it for \$12, which was  $\frac{2}{3}$  less than it cost him. What did it cost me?
10. A pile of wood contains 200 cords. It is 8 ft. wide and 8 ft. high. How long is it?
11. What will it cost to dig a ditch 2 ft. deep and  $1\frac{1}{2}$  ft. wide around a lot 4 rd. long and  $3\frac{1}{2}$  rd. wide, at 54¢ a cubic yard?
12. How many rods of fence will inclose a rectangular field of 20 acres, whose width is 40 rd.?
13. If 27 bu. of apples cost \$60 $\frac{1}{2}$ , how many bushels can be bought for \$461 $\frac{1}{2}$ ?
14. If a stick of timber 20 ft. long, 12 in. wide, 10 in. thick, is sawed into boards 1 in. thick, how many board feet will there be?
15. Four times a certain number added to three times the same number gives 112. Find the number.
16. The sum of two numbers is 344, and the greater is 7 times the less. What are the two numbers?

1. Find the cost of carpet at \$1.25 a yard, for 30 in. wide carpet, for a room 18 ft. by 14 ft., if the strips run lengthwise.
2. Find the cost of papering a room 32 ft. long, 22 ft. wide, 13 ft. high, with paper 18 in. wide, 8 yd. in a roll, at 65¢ a roll, if 50 sq. yd. are allowed for openings.
3. How many board feet in twelve 4-in. planks 16 ft. long and 10 in. wide?
4. What will it cost to shingle a roof, each side of which is 30 ft. long, and 25 ft. wide, at \$4.50 a square?
5. What will it cost to build a wall 90 ft. long,  $7\frac{1}{2}$  ft. high, 2 ft. thick, at \$6 a cubic yard?
6. Find the number of bushels of grain in a bin that is 6 ft. long, 5 ft. wide, 4 ft. deep. ( $1\frac{1}{4}$  cu. ft. in 1 bu.)
7. What is the breadth of a rectangular field containing  $7\frac{1}{2}$  acres, if the length is 242 yards?
8. Make out a bill that shall contain five debit and one credit items.
9. A man sold 40 horses at \$200 each. On one-half of them he gained 25%, and on the rest he lost 20%. Find the entire gain or loss.
10. Find the interest on \$1250 from Nov. 15, 1902, to Mar. 1, 1904, at 5%.
11. If 2.45 tons of straw cost \$29.40, how many tons can be bought for \$9?
12. Find the surface and volume of a prism measuring 4' 8"  $\times$  3' 10"  $\times$  3' 6".
13. Three men, A, B, and C, earned \$330. A earned four times as much as B, and C as much as both A and B. How much did each earn?
14. In an orchard there are three times as many pear trees as apple trees, and four times as many peach trees as pear trees. In all there are 224 trees in the orchard. How many are there of each kind?

1. For how much must silk that cost \$1.20 a yard be sold to gain 20%?
2. A dealer bought cloth at \$4 a yard, and sold it at \$6 a yard. What per cent of profit did he make?
3. A grocer bought flour at a profit of \$1.20 a barrel, which was a gain of 25%. What was the cost a barrel?
4. What per cent was lost on a horse which cost \$90 and was sold for \$75?
5. When a hat was sold for \$2 there was a gain of  $33\frac{1}{3}\%$ . What was the cost?
6. A mowing-machine was sold for \$36 at a loss of 25%.
7. A house was sold for \$1,500 at a gain of 25%.
8. A house was sold for \$1,800 at a loss of 25%.
9. A cow that cost \$45 was sold for \$40.
10. When goods are sold for  $\frac{3}{4}$  of their cost what per cent is lost?
11. When  $\frac{3}{4}$  of an article is sold for what the whole article cost, what per cent is gained?
12. A grocer sold a pound of butter for 24 cents, by which he gained 20% on its cost. What would have been his gain had he sold it at 30 cents a pound?
13. Mr. Smith sold a city lot for \$700, losing \$100. What per cent did he lose?
14. A gain of \$5 on goods that sold for \$25 is a gain of what per cent?
15. How shall I mark cloth that cost 12¢ a yard to make 25%?
16. What per cent of an acre is a rectangular lot 4 rd. by 5 rd.?
17. If one-fourth yard of cotton cloth cost one and one-half cents, what will 12 yards cost?
18. At 20¢ a yard what will 42 ft. of ribbon cost?
19. What is the interest on \$400 at 6% for 10 months?

1. Name a divisor of 8.
2. A *divisor* is a number that will exactly divide another number.
3. Name a common divisor of 8 and 12.
4. A *common divisor* is a number that will exactly divide two or more numbers.
5. Name the greatest common divisor or factor of 8 and 12.
6. The *greatest common divisor* or factor of two or more numbers is the greatest number that will exactly divide them.
7. What is the greatest common divisor of 18, 30, and 48?

2) 18 30 48  
 3) 9 15 24  
 ) 3 5 8

By the definition we see that any factor or divisor must be a divisor of all the numbers. We arrange the numbers in a horizontal line, and by division remove all factors that are common to all. The product of these factors must be the greatest common factor. Removing 2, we find our quotients to be 9, 15, and 24. Any number, to be a part of our greatest common divisor, must be a part or factor of these quotients. We remove three as common to them all. Our quotients are 3, 5, and 8. Since these numbers are prime to each other, there can be no more factors common to all.

$2 \times 3$  or 6 then must be the greatest common divisor.

8. What is the greatest common divisor of 60, 72, 48, 84?
9. What is the greatest common divisor of 45, 75, 90, 135, 150, 180?
10. What is the greatest common divisor of 75, 300, 450?
11. Find the greatest common divisor of 108, 270, 432.
12. Find the greatest common divisor of 16, 20, 24.
13. Find the greatest common divisor of 44, 110, 154.

In the following examples find the greatest common measure of the numerator and denominator of each fraction, and divide each term by it to reduce the fraction to its lowest terms.

14. Reduce to lowest terms:

$$\frac{18}{31}, \frac{11}{44}, \frac{14}{63}, \frac{24}{156}, \frac{32}{78}, \frac{18}{96}, \frac{32}{14}, \frac{45}{80}$$

15. Reduce to lowest terms:

$$\frac{31}{71}, \frac{13}{32}, \frac{15}{28}, \frac{16}{25}, \frac{33}{47}, \frac{13}{23}, \frac{12}{15}, \frac{72}{84}$$

NOTE. — These topics have little value, and may be omitted without loss.

1. Multiply 7. bu. 3 pk. 5 qt. by 6.

7 bu.	3 pk.	5 qt.	Multiply as in whole numbers. 6 times 5 qt.
		6	are 30 qt. 6 times 3 pk. are 18 pk. 6 times 7
<u>42 bu.</u>	<u>18 pk.</u>	<u>30 qt.</u>	bu. are 42 bu. Draw a line under this answer,
47 bu.	1 pk.	6 qt.	and change to the next higher denomination if
			possible. 30 qt. equal 3 pk. and 6 qt. Write
			the 6 qt. 18 pk. and 3 pk. are 21 pk., which equal 5 bu. and 1 pk. Write
			the 1 pk. 42 bu. and 5 bu. are 47 bu.

2. What is the weight of 9 loads of hay, each weighing 1 T. 545 lb. 12 oz.?

3. Multiply 217 rd. 4 yd. 2 ft. 7 in. by 32.

4. Multiply 13 gal. 2 qt. 1 pt. 7 gi. by 17.

5. If it takes 4 d. 5 hr. 15 min. to build a machine, how long will it take to build 24 machines?

6. Multiply 46 T. 439 lb. 8 oz. by 7.

7. Multiply 14 m. 85 rd. 9 ft. 11 in. by 11.

8. Divide 50 bu. 3 pk. 2 qt. by 6.

8      1      7

- 6)50 bu. 3 pk. 2 qt.

48 bu.	One-sixth of 50 bu. is 8 bu. (write it above bushels) and 2 bu.
<u>2 bu.</u>	remaining. 2 bu. equals 8 pk., and 3 pk. make 11 pk. One-
4	sixth of 11 pk. equals 1 pk. and 5 pk. remaining. 5 pk. equal
8 pk.	40 qt., and 2 qt. make 42 qt., one-sixth of 42 qt. equals 7 qt.

3 pk.	Divide :
<u>11 pk.</u>	9. 17 hr. 32 min. 24 sec. by 4. 23 sq. yd. 9
6 pk.	sq. ft. 117 sq. in. by 9.

<u>5 pk.</u>	10. 35 wk. 5 d. 15 hr. 12 min. 18 sec. by 6.
8	11. 21 cu. yd. 20 cu. ft. 17 cu. in. by 7. 89 bu.

<u>40 qt.</u>	1 pk. 7 qt. by 7.
2 qt.	12. 23 cu. yd. 12 cu. in. by 4. 53 T. 176 lb.

<u>42 qt.</u>	by 28.
42 qt.	13. 112 A. 8 sq. rd. by 9. 332 lb. 8 oz. by 19.

14. 125 cd. 7 cd. ft. 7 cu. ft. by 9.

NOTE. — See note on page 82.

1. Change  $\frac{1}{4}$  gal. to integers of lower denominations.

$\frac{1}{4} \times 4 \text{ qt.} = \frac{1}{4} \text{ qt.} = 2\frac{3}{4} \text{ qt.}$       Since there are 4 qt. in one gal., in  $\frac{1}{4}$  gal. there are  $\frac{1}{4}$  times 4 qt., or  $2\frac{3}{4}$  qt. Since there are 2 pt. in 1 qt., in  $\frac{3}{4}$  qt. there are  $\frac{3}{4}$  times 2 pt. or  $1\frac{1}{2}$  pt. Since there are 4 gi. in 1 pt., in  $\frac{1}{2}$  pt. there are  $\frac{1}{2}$  times 4 gi., or 2 gi.

Ans. 2 qt. 1 pt. 2 gills.

2. Change  $\frac{1}{2}$  gal. to lower denominations.

3. Change  $\frac{7}{8}$  of a ton to lower denominations.

4. Change  $\frac{1}{2}$  bu. to quarts and pints.

5. Change .6875 of a gallon to integers of lower denominations.

.6875 gal.      The explanation is the same as for common fractions. See above.

$$\begin{array}{r} 4 \\ 2.7500 \end{array} \text{ qt.}$$

6. Change .85 lb. to integers of lower denominations.

$$\begin{array}{r} 2 \\ 1.50 \end{array} \text{ pt.}$$

7. Change .325 T. to integers of lower denominations.

$$\begin{array}{r} 4 \\ 2.0 \end{array} \text{ gi.}$$

8. Change .0135 cd. to cubic feet.

9. Change .08 $\frac{1}{2}$  yd. to feet, etc.

Ans.

2 qt. 1 pt. 2 gi.

10. Express .09375 A. in square rods.

11. Change .015625 bu. to pecks, etc.

12. Change 2 pk. 6 qt. to the decimal of a bushel.

8)6 qt.

4)2.75 pk.

.6875 bu.

Since 8 qt. make a peck, 6 qt. are equal to .75 pk., which, united with the 2 pk., make 2.75 pk. Since there are 4 pk. in a bushel, 2.75 pk. are equal to .6875 bu.

13. Change 2 pk. 6 qt. to the fraction of a bushel.

6 qt.  $\div$  8 =  $\frac{3}{4}$  pk. =  $\frac{3}{4}$  pk.

$2\frac{3}{4}$  pk. =  $\frac{11}{4}$  pk.

$\frac{11}{4}$  pk.  $\div$  4 =  $\frac{11}{16}$  bu.

The explanation is the same as for changing to a decimal.

14. Change 4 yd. 2 ft. 5.25. in. to the fraction of a rod.

15. Add  $\frac{3}{4}$  of a gallon and .375 gal.

16. Add 103.75 ft. and .845 miles.



21. Multiply 45 bu. 3 pk. 6 qt. 1 pt. by 15.
22. Divide 212 m. 26 rd.  $1\frac{1}{2}$  yd. by 7.
23. Add 43 A. 32 sq. rd. 127 sq. ft.; 240 A. 20 sq. rd. 200 sq. ft.; 95 A. 25 sq. rd. 75 sq. ft.; 12 A. 100 sq. ft.; 137 sq. rd. 30 sq. ft.
24. How many acres in a farm 225 rd. long and 175 rd. wide?
25. How many cubic feet in a room 18 ft. long, 17 ft. wide, and 15 ft. high?
26. In 3,538,944 cu. in. how many cubic yards?
27. Reduce 4 mi. 213 rd. 15 ft. to inches.
28. How much rice at 8¢ a pound will pay for 5 bu. 3 pk. of cherries at 9¢ a quart?
29. Find the least common multiple of 13, 39, 56, 63.
30. Find the greatest common divisor of 315, 945, and 63.
31. If  $\frac{3}{4}$  of an acre of land cost \$66 $\frac{3}{4}$ , how much will 8 $\frac{3}{4}$  acres cost?
32. What is the value of a pile of wood 48 ft. long, 8 $\frac{3}{4}$  ft. high, and 4 ft. wide, at \$6.50 a cord?
33. Multiply 27 millionths by 12 hundredths, and divide the product by 324 thousandths.
34. A farmer had  $\frac{1}{4}$  of his sheep in one pasture,  $\frac{1}{3}$  in another,  $\frac{1}{5}$  in another, and the remainder, 26, in a fourth. How many sheep had he?
35. Take four hundred and twenty-five ten-thousandths from ten thousand, and multiply the remainder by ten hundredths.
36. The sum of two numbers is 56. The larger is seven times the smaller. Find the numbers.
37. A boy has a certain number of pears and four times as many peaches. His pears and peaches together number 25. How many of each has he?
38. The difference of two numbers is 24. The larger is five times the smaller. Find the numbers.

1. Find the cost of 1,000 cords of wood at \$7.37½ a cord.
2. A man earned \$100 a month. If he spent \$4 out of every \$10, how many dollars did he save every month?
3. How much did a grocer pay for a quart of milk, if he sold a gallon for \$.32, at a gain of 33½%?
4. A rectangular field contains 1 acre. If it is 80 rd. long, how wide is it? What is its perimeter?
5. I sold a dozen oranges costing 24 cents for 18 cents. What per cent did I lose?
6. I sold a dozen oranges costing 18 cents for 24 cents. What per cent did I gain?
7. What will 27 lb. of coffee cost at 33⅓¢ a pound?
8. What will 40 pt. of milk cost at 2½¢ a pint?
9. What is the interest of \$2,468 for 30 days at 6%?
10. If 2½ qt. of beans cost 25 cents, what will 10 qt. cost?
11. Add all the prime numbers between 1 and 10.
12. 5 qt. is what decimal of a peck? What per cent of it?
13. A can do a piece of work in 4 days, B can do it in 5 days. In what time can A and B do it working together?
14. I bought a table for \$4. At what price must I sell it to gain 12½%?
15. How many 2-in. cubes in a 10-in. cube?
16. If a dealer sells goods for double what they cost him, what per cent does he make?
17. I sold my watch for \$18, which was 12½% more than it cost. Find the cost.
18. What part of a dollar did I pay for 6¼ lb. of candy at 12¢ a pound?
19. How many hours is it from 2.45 P.M. to 5.15 P.M.?
20. A man bought a gallon of milk for 28 cents, and sold it at 4¢ a pint. How much did he gain on a quart?
21. How many cows at \$25 a head can be bought for \$250?
22. What part of \$100 is 2 times \$16⅔?

(For general summary, see pages 293, 294.)

What is the interest of:

1. \$845 for 6 mo. 24 da. at 4%?
2. \$46.50 for 123 days at 6%?
3. \$74.60 for 1 yr. 5 mo. at 5%?
4. \$2,463.75 for 11 mo. 23 da. at  $4\frac{1}{2}$ %?
5. \$5,900 for 3 yr. 6 mo. 17 da. at 7%?
6. \$400.50 for 2 yr. 11 mo. 3 da. at 4%?
7. \$10,000 for 63 days at 5%?
8. \$640.80 for 4 yr. 7 mo. 11 da. at 7%?
9. \$16,420 for 9 mo. 24 da. at  $6\frac{1}{2}$ %?
10. \$734.75 for 3 yr. 9 mo. at 4%?
11. \$459.28 from Dec. 14, 1901, to May 5, 1902, at  $4\frac{1}{2}$ %?
12. \$658.48 from Aug. 17, 1900, to Apr. 4, 1902, at 8%?
13. \$2,184 from Jan. 24, 1901, to Mar. 30, 1904, at 6%?
14. \$609.50 from Mar. 5, 1899, to Sept. 14, 1901, at  $4\frac{1}{2}$ %?
15. \$489.25 from May 5, 1901, to Aug. 11, 1902, at 5%?
16. \$625.57 from Aug. 15, 1900, to Dec. 29, 1905, at  $3\frac{1}{2}$ %?
17. \$647.48 from Sept. 30, 1900, to May 5, 1902, at  $7\frac{1}{2}$ %?
18. \$492 from Aug. 31, 1901, to Dec. 30, 1902, at  $3\frac{1}{2}$ %?
19. \$1,827 from Jan. 16, 1901, to Oct. 11, 1902, at 4%?
20. \$945.96 from June 4, 1901, to Sept. 10, 1903, at  $4\frac{1}{2}$ %?
21. \$2,846 for 8 yr. 4 mo. 12 da. at  $6\frac{1}{2}$ %?
22. \$862 for 4 yr. 7 mo. 22 da. at 8%?
23. \$8,624 for 1 yr. 2 mo. 17 da. at 5%?
24. \$946.25 for 89 days at  $4\frac{1}{2}$ %?
25. \$3,010 for 2 yr. 7 mo. 7 da. at 8%?
26. \$480 for 3 yr. 1 mo. 24 da. at 7%?
27. \$847.25 for 1 yr. 8 mo. 7 da. at 4%?
28. \$756.75 for 2 yr. 2 mo. 5 da. at  $7\frac{1}{2}$ %?
29. \$1,050 for 3 yr. 5 mo. 3 da. at 6%?
30. \$2,500 for 4 yr. 11 mo. 9 da. at 6%?
31. \$800 for 115 days at  $3\frac{1}{2}$ %?

NOTE. — This page may be omitted without loss.

1. What principal at 6% will yield \$225 int. in 2 yr. 6 mo.?

$$\begin{aligned} & \$15 = \text{int. of } \$1 \text{ for 2 yr. 6 mo.} \\ & \$225 : \$15 = \$x : \$1. \\ & .15x = \$225. \\ & x = \$1500. \end{aligned}$$

What is the interest of \$1 for 2 yr. 6 mo.? What is the interest of \$x as stated in the example? What is the ratio of \$225 to \$.15? What is the ratio of \$x to \$1? Since these two ratios

are equal, write them as a proportion. Find the value of x.

2. In what time will \$940 at 6% gain \$432.40?

$$\begin{aligned} & \$56.40 = \text{int. of } \$940 \text{ for 1 yr. at 6\%}. \\ & \$432.40 : \$56.40 = x \text{ yr.} : 1 \text{ yr.} \\ & 56.40x = 432.40. \\ & x = 7\frac{2}{3} \text{ yr.} \\ & x = 7 \text{ yr. 8 mo.} \end{aligned}$$

What is the interest of \$940 for 1 yr. at 6%? What is the interest stated in the example for x years at 6%?

Form the time and interest into a proportion, and find the value of x.

3. At what rate will \$900 gain \$231 in 3 yr. 8 mo.?

$$\begin{aligned} & \$198 = \text{int. on } \$900 \text{ for 3 yr. 8 mo. at 6\%}. \\ & \$33 = \text{int. on } \$900 \text{ for 3 yr. 8 mo. at 1\%} \\ & \$231 : \$33 = x\% : 1\%. \\ & 33x = 231. \quad x = 7. \end{aligned}$$

What is the interest on \$900 at 1% for 3 yr. 8 mo.? What is the interest of \$900 at x% as stated in the ex-

ample? How does the interest given at x% compare with the interest you get at 1%? Form the two ratios into a proportion, and find the value of x.

4. What principal on interest at 6% will gain \$15 in 2 yr.?
5. At 5% what principal will gain \$20 in 4 yr.?
6. What principal at 7 % will yield \$350 in 1 yr.?
7. In what time will \$142.64 gain \$13.105 at 4%?
8. In what time will \$500 gain \$5.00 at 4%?
9. In what time will \$900 gain \$13.50 at 6%?
10. At what rate will \$426 gain \$63.90 in 2 yr. 6 mo.?
11. At what rate will \$62.75 gain \$8.785 in 2 yr. 4 mo.?

1. *Profit* is the excess of the selling-price over the cost.
2. *Loss* is the excess of the cost over the selling-price.
3. *Selling-price* is always the cost plus the profit, or cost less the loss.
4. The gain or loss is always reckoned as a percentage of the cost.
5. I bought \$640 worth of goods, and sold them at a gain of 12%. Find the selling-price.
6. An agent buys \$650 worth of goods at 40% off, and sells at a gain of 25% on the cost. For what does he sell?
7. George bought a building-lot for \$850, and sold it to Henry at a gain of 25%. Henry sold it at a loss of 20%; what did Henry receive for it?
8. A grocer lost 15% by selling eggs at 17 cents a dozen. Find the cost.
9. A dealer sold a horse at  $12\frac{1}{2}\%$  loss, and lost \$25. Find the cost.
10. If  $\frac{3}{4}$  of an article is sold for  $\frac{1}{2}$  of its cost, what per cent is lost?
11. A dealer lost 16% by selling goods for \$4200. Find the cost.
12. If I sell goods that cost me \$.84 a yard for \$.63, what is my loss per cent?
13. If I sell a horse for \$175, and gain 5%, what per cent should I have gained if I had sold him for \$200?
14. Sold some goods for \$200, and thereby gained 25%. What per cent should I have gained had I sold them for \$220?
15. A lady spent \$64.50 for jewelry and dress goods, paying 15% more for dress goods than jewelry. How much did she pay for each?

NOTE. — 100% = jewelry. 115% = dress goods. 215% = both.

16. A and B together have \$1,680, and A has 25% less money than B. How much has each?

1. I sold a bicycle for \$45 at a loss of 10% ; for what should it be sold to gain 10% ?

1. S. P. at 1st loss, = \$45.

The first three statements need no explanation.

2. S. % at 1st loss, = 90.

What is the ratio or comparison of 110% (which we wish to find) to 90% (which we know) ? It is as 110 : 90 or

3. S. % at  $x$  gain, = 110.

$110 : 90 = \frac{110}{90} = \frac{11}{9}$ .

$\frac{110}{90}$  or  $\frac{11}{9}$ . If it is  $\frac{11}{9}$  of the second statement, it must be  $\frac{11}{9}$  of the first

$\frac{11}{9}$  of \$45 = \$55.

statement, because the first and second are equal, both representing the Selling-Price at first loss.

2. If by selling a farm for \$1,425 a farmer gains 14%, what per cent would he have gained by selling it for \$1,600 ?

1. S. P. at 1st gain, \$1425.  $1600 : 1425 = \frac{1600}{1425} = \frac{320}{285} = \frac{64}{57}$ .

2. S. % at 1st gain, 114.  $\frac{64}{57}$  of 114% = 128%.

3. S. P. at  $x$  gain, 1600.  $128\% - 100\% = 28\%$ .

(See explanation above.)

3. Sold a farm for \$8,128, and made 27 per cent on the cost. What per cent should I have gained had I sold it for \$9,600 ?

4. A pair of horses were sold for \$297, at a gain of 35%. What would have been the gain per cent if sold for \$253 ?

5. A horse was sold for \$144, at a gain of 20%. At what price would it have been sold if there had been a loss of 20% ?

6. A piano was sold for \$252, which was at a gain of 12%. At what price would it have been sold to gain 25% ?

7. A boy sold a pair of skates for 92 cents, and gained 15%. At what should he have sold them to gain 18 $\frac{1}{4}$ % ?

8. Sold a lot of hay for \$644, at a gain of 15%. At what price should it have been sold to gain 20% ?

9. A merchant's income is \$5760. This is a gain of 18 $\frac{1}{4}$ % on the capital invested. His income last year was 25% of the capital. Find his income of last year.

10. I sold a farm for \$5000, and made 25%. What per cent should I have gained or lost if I had sold it for \$3,500 ?

1. What is the loss per cent on goods costing 15¢ a yard, if sold at 12¢?
2. What per cent of a score is a dozen?
3. What per cent of an acre is a lot that is 5 rd. by 16 rd.?
4. If 3.5 acres of land cost \$35, what will a farm of 40 acres cost?
5. A farmer sold a calf for \$4, and lost 30%. What did it cost?
6. A boy earned \$20, which was 20% of what he had before. How much has he now?
7. A man sold a wagon for \$15 less than it cost him, and lost 20%. What did the wagon cost?
8. An article that cost \$4.50 was sold for \$6. What was the gain per cent?
9. An article sold for \$32 at a loss of 20%. What was the first cost?
10. A man gained  $12\frac{1}{2}\%$  by selling a wagon for \$8 more than it cost him. What did it cost?
11. Find the cost of a hat that was sold for \$1.60 at a loss of 20%.
12. By selling a watch for \$60 I gained 20%. Find its cost.
13. Flour sold at \$6 a barrel yields a profit of 20%. Find the cost of 10 barrels.
14. Sold a cow for \$5 less than cost, and lost  $12\frac{1}{2}\%$ . Find the cost and selling-price.
15. I sold a harness for \$48, which was 20% below cost. What did it cost?
16. An article that cost \$40 was sold for \$50. What was the gain per cent?
17. If a merchant buys shoes at \$5 a pair, at what price must he sell them to gain 25%?
18. How much will a 40 qt. can of milk cost at 24¢ a gallon?
19. If  $\frac{3}{4}$  yd. of cloth cost 24¢, what will  $1\frac{1}{4}$  yd. cost?

1. My furniture, worth \$1,800, is insured for  $\frac{3}{4}$  of its value at  $\frac{3}{4}\%$ .
2. A grocer bought flour at \$4.35 a barrel, paying for it \$1,148.40, and sold it at \$5.15 a barrel.
3. In a school 174 pupils are present, and 6 are absent.
4. Bought 175 bu. of wheat for \$315, and sold it for \$2 a bushel.
5. On a commission of  $2\frac{1}{4}\%$  I sold 400 bales of cotton, each weighing 480 pounds, at 32¢ a pound.
6. Selling-price, \$125.45 ; profit,  $8\frac{1}{3}\%$ .
7. Cost \$88.60, loss  $7\frac{1}{2}\%$ .
8. A house which cost \$4,800 rents for \$24 a month. The expenses on it are \$48 annually.
9. Land which cost \$5,600 was sold at a profit of 25%.
10. A line of wire 1,560 ft. long is supported by 13 posts, placed the same distance apart.
11. 17 gal. 3 qt. 1 pt. 2 gi. were sold from a cask containing 25 gal. 2 qt. 1 gi.
12. I lost 12% by selling some goods for \$215.60.
13. I bought 320 lb. of sugar at  $5\frac{3}{4}\%$  a pound. I lost  $3\frac{1}{8}\%$  by drying, and sold the rest at 6¢ a pound.
14. A man lost  $\frac{1}{3}$  of his money, and then  $\frac{1}{3}$  of what was left. He then had \$12.60.
15. In a school the girls are 52% of the whole number, and there are 240 boys.
16. James spent  $\frac{1}{2}$  of his money, then  $\frac{3}{4}$  of what remained, then  $\frac{3}{4}$  of what still remained. He then had \$15.
17. A lawyer collected a note of \$2375. His commission was 5%.
18. Twelve pairs of shoes were bought at \$3.25 a pair, with a discount of 5%.
19.  $22\frac{1}{2}$  yd. of ribbon, costing \$6.75, were sold at a gain of 20%.



1. Divide \$336 between two persons so that one may have  $\frac{2}{3}$  as much as the other.

2. A tank 18 ft. long and 15 ft. wide requires 96 sq. yd. of lead to line its sides and bottom. How deep is it?

3. A room 20 ft. long, 17 ft. 6 in. wide, will require how many yards of carpet 2 ft. 6 in. wide to cover it, if no allowance be made for waste?

4. Find the area of a gravel walk 6 ft. wide just inside a fence surrounding a lot 320 ft. long, 210 ft. wide.

NOTE. — Find area of one rectangle 320 by 210 ft., and of another 308 by 198 ft. Find their difference.

5. What is the difference between 65 divided by .65, and .65 divided by 65?

6. By selling a carriage for \$117 a dealer lost 10%. For how much should he have sold it to gain 10%?

7. A man gave  $\frac{2}{3}$  of his money to his wife,  $\frac{1}{3}$  of the remainder to his oldest daughter, and the remainder, \$5,000, he divided equally between his two younger daughters. How much was the man worth?

8. Add seven hundredths, thirty-six ten-thousandths, and seventy-two millionths.

9. An agent charged \$25.50 as commission at  $2\frac{1}{2}\%$  for selling 200 bbl. of flour. At what price a barrel was the flour sold?

10. I paid \$25 for an insurance policy on my house. If the rate is  $\frac{1}{2}\%$ , for how much is my house insured?

11. I collected 80% of a debt of \$5,600, and charged  $4\frac{1}{2}\%$  commission. How much ought I to return to my employer?

12. A man spent  $\frac{2}{3}$  of his money, and invested  $\frac{1}{3}$  in business. He had the rest, \$1,850, in a bank. How much was he worth at first?

13. Find the interest on \$769.74 for 3 yr. 9 mo. 17 da. at 4%.

14. \$2,300 is 15% more than what?

1. *Insurance* is security against financial loss on account of the destruction of property, or by the injury or death of a person.

2. The three common forms of insurance are Fire, Accident, and Life.

3. *Premium* is the sum paid for insurance. In Fire insurance it is estimated at a certain per cent of the amount insured. In life insurance it is estimated at a certain amount a year for each thousand dollars of insurance, and varies with the age of the person insured.

4. A man insures his life for \$2,500 at the rate of \$22.50 for every \$1,000. What is his annual premium?

5. A man insures his life for \$3,000, paying \$14.24 semi-annually for every \$1,000. If he dies in twelve years, how much more than his premiums will his heirs receive?

6. A mill was insured for  $\frac{3}{4}$  of its value. The premium paid was \$350. The rate of insurance 5%. Find the value of the mill.

7. A factory valued at \$50,000 is insured for  $\frac{3}{4}$  of its value. The premium is \$500. What is the rate of the insurance?

8. How large an insurance can I place on my house by paying a premium of \$122.50, if the rate of insurance is  $1\frac{1}{4}\%$ ?

9. A ship worth \$52,000 was insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ . The cargo worth \$8,640, was insured for  $\frac{3}{4}$  of its value at 3%. Find the premium.

10. A house worth \$12,000 is insured for  $\frac{3}{4}$  of its value. Find the premium at  $\frac{3}{4}\%$ .

11. I insure my house, worth \$6,000, at  $\frac{1}{2}\%$  on  $\frac{3}{4}$  of its value, and \$1,800 worth of furniture on  $\frac{3}{4}$  of its value at  $\frac{3}{4}\%$ . How much premium must I pay?

12. Mr. J. paid \$250 for insurance on his stock of goods. The face of the policy is \$10,000. Find the rate.

13. Find the amount insured, when \$68.24 is the premium at  $\frac{3}{4}\%$ .

1. *Commission* is compensation paid by one person to another for transacting some business.

2. These business transactions are usually buying or selling property or collecting bills.

3. The person doing business for another is called Agent, Factor, Broker, Commission Merchant.

4. The person for whom the business is done is called Principal, Employer.

5. The services performed by agents are of two kinds: *a*, Where they receive money for selling or collecting, to be remitted to their principal; *b*, Where money is sent them to be expended for their principal.

6. The agent's commission is usually some per cent of the amount collected or expended.

7. The Net Proceeds is the sum of money due the Principal after the commission and other charges have been deducted.

8. The Entire Cost of a purchase is the price paid, plus the commission and all other expenses.

9. My agent bought 70 bbl. of flour at \$5.25 a barrel. His commission was 3%. Find the agent's commission.

10. A commission merchant sold a lot of goods for \$1,480, charging  $2\frac{1}{2}\%$  commission. What should he send his employer?

11. A broker bought for me some goods for which he paid \$2,146. His commission for buying was  $\frac{1}{2}\%$ . What did the goods cost me?

12. My collector charges me 3% for collecting bills. In one month I paid him \$345. How much did he collect?

13. My agent sold 23 wagons at \$85 each. What were the net proceeds if his commission was 6%?

14. I collected 95% of a debt of \$2,148. My commission was  $3\frac{1}{2}\%$ . What sum must I send my employer?

15. The commission for selling some property was \$985. If the rate was 5%, find the value of the property sold.

1. What will it cost to get a house insured for \$4,000 for 10 years, at  $\frac{1}{4}\%$  a year?
2. A house valued at \$3,500 was insured at 1%. What was the premium?
3. If a book-agent received \$50 for selling \$150 worth of books, what was his rate of commission?
4. An auctioneer sold \$500 worth of goods at a commission of 4%. What was his commission?
5. Can you lose more than 100% in selling an article?
6. Give a case where you lose 100%.
7. Can a gain be greater than 100%? If so, make an example to illustrate.
8. If not greater than 100%, tell why not. Make an example where the gain is just 100%.
9. A man sold a horse for \$90, and gained 20%. Find the cost of the horse.
10. Read the 9th example, using a common fraction instead of per cent.
11. When cloth that cost \$6 a yard is sold for \$4 a yard, what is the loss per cent?
12. If you buy an article for \$80, and sell it at 25% profit, what will be your selling-price?
13. A man paid \$160 for a horse, which was 20% less than the cost of a carriage. Find the cost of both.
14. What per cent of a number is  $\frac{3}{4}$  of it?
15. What per cent of a number is  $\frac{1}{4}$  of it?
16. What per cent of a number is  $\frac{3}{8}$  of it?
17. A horse that cost \$84 was sold at a gain of 25%. Find the selling-price.
18. If 25% of the cost of a bicycle is \$9, what is the cost of the bicycle?
19. Cloth which cost \$.75 a yard was sold at a loss of  $33\frac{1}{3}\%$ . Find the selling-price.

1. Charles and Henry together had 72 cents. If Charles had 3 times as many as Henry, how many did each have?

2. Three men, A, B, and C, formed a company with a capital of \$8,000. B put in 4 times as much as A, and C three times as much as A. How many dollars did each put in?

3. The sum of two numbers is 99, and the greater is twice the less. What are the numbers?

4. Three times a certain number added to two times the same number gives 75. Find the number?

5. The sum of two numbers is 366, and the greater is 5 times the less. What are the two numbers?

6. A man bequeathed \$48,000 to his wife, son, and daughter. The will provided that the son should receive twice as much as the daughter, and the wife 3 times as much as the daughter. What was the share of each?

7. A man divided 80 cents between two children. If he gave the second three times as much as the first, what did he give to each?

8. Divide the number 88 into three parts, so that the second part shall be three times the first, and the third four times the first.

9. Divide the number 120 into three parts so that the second part shall be two times the first, and the third part as much as the sum of the first and second.

10. Four men have together \$480. B has 4 times as much as A, C has 6 times as much as B, and D has  $\frac{1}{2}$  as much as C. How many dollars has each?

11. A boy, being asked how many marbles he had, said that if he had 5 times as many more he should have 240. How many had he?

12. In a school there are 495 pupils, and twice as many boys as girls. How many boys are there?

13.  $4x - x = 7 + 5$ . Find  $x$ .

Make accurate drawings to represent the examples in this lesson. Find the answers by measuring.

1. Two men, Brooks and Scott, start from A. Brooks walks directly north 36 miles, and Scott directly east 48 miles; how far apart are they in the shortest line? Scale,  $\frac{1}{12}$  in. to a mile.

2. A house is 18 ft. wide. From the attic-floor to the ridge-pole it is 12 ft. How long rafters must be used to project 6 in. over the wall of the house? Scale,  $\frac{1}{4}$  in. to 3 ft.

3. If the length of this roof is 32 ft., how many boards 16 ft. long and 6 in. wide will it take to cover it?

4. At a point, A, draw a line east 6 in. to B; south 2 in. to C; west 2 in. to D; south 1 in. to E; west 1 in. to H; south 4 in. to I; west to a point directly south of A to J. Connect A and J. How far is it round this field? Scale,  $\frac{1}{4}$  in. to a rod.

5. From B to H is what part of the whole distance?

6.  $\frac{4}{5}$  of the distance round this figure is how many yards?

7. A house is 40 ft. long, 26 ft. wide, and has 18 ft. posts. The gable is 13 ft. high, and there is no allowance for doors and windows. What will it cost at 22¢ a square yard to paint the outside?

8. Three boys were standing by a tree. William remained there; George and Henry walked east 24 ft. to another tree. George stopped there, and Henry walked north 32 ft. to another tree and stopped. How far is Henry from William? Scale, 1 in. to 8 ft.

9. Beginning at the north-east corner, the boundary-line of my lot runs as follows: west 40 rd. to a tree, called B; south 16 rd. to a point, C; east 24 rd. to another tree, called D; then north-east to the point of beginning. How many square rods in the lot? How many rods round the field? Scale, 1 in. to 8 rd.

10. Find the area of a quadrilateral, 55 yd. by 16 ft.

1. Sold 96 yd. of carpeting at  $\$1.87\frac{1}{2}$  a yard, and thereby gained  $\$38.40$ . How much did it cost me a yard?
2. What would you gain by selling at  $6\frac{1}{4}\%$  a pound, 1,522 lb. of rice costing  $\$4.25$  a hundred, and 636 lb. of barley costing  $\$5.60$  a hundred?
3. Multiply seventy-eight ten-thousandths by five hundredths; divide the product by thirteen thousandths, and reduce the quotient to a common fraction.
4. A commission merchant sold 500 pieces of cloth for  $\$130$  a piece, and paid his employer  $\$55,250$ . What was the rate of his commission?
5. What will be the net proceeds of a sale of 525 bbl. of beef @  $\$18.25$ , allowing 3% commission, and  $5\%$  a barrel for storing?
6. How much is  $5\frac{1}{2}$  tons of coal worth if  $17\frac{1}{2}$  tons are worth  $\$100$ ?
7. If the numerator of a common fraction is divided by 3, what is the effect upon the value of the fraction? Illustrate it.
8. Two men start from two towns 105 miles apart, and walk toward each other. They meet at the end of 15 hours. If the first traveled 3 miles an hour, how many miles did the second travel an hour?
9. What is the area in acres of a triangle whose base is 56 rd. and altitude 63 rd.?
10. How many farms containing 90 acres each can be formed in the town of Granby, if it is in the form of a rectangle 6 miles long and  $4\frac{1}{2}$  miles wide?
11. What is a fraction? Explain addition of fractions, and give the reason for every step.
12. What is interest? Explain your method of finding the interest on any sum for any time at any rate.
13. From four billion take two billion, one hundred five million, two hundred fifty thousand, forty-seven.

1. If  $\frac{3}{4}$  yd. of cloth cost  $\$1\frac{2}{3}$ , what will  $\frac{1}{2}$  of a yard cost?
2. Make out the following bill, supplying names and dates:  
 32 lb. soap, @  $15\frac{1}{2}\%$ ; 35 lb. starch, @  $5\frac{1}{2}\%$ ; 85 lb. sugar, @  $8\frac{1}{2}\%$ ; 62 $\frac{1}{2}$  gal. of vinegar, @  $25\%$ ; 28 lb. coffee, @  $23\%$ ; 112 lb. butter, @  $33\frac{1}{2}\%$ . Discounts, 10% and 5%.
3. A merchant bought 76.75 yd. of cloth for  $\$115\frac{1}{2}$ , and sold  $\frac{1}{2}$  of it at an advance of  $\$1\frac{1}{2}$  a yard. How much did he receive for the part sold?
4. What will 76 lemons cost if three dozen cost  $\$1\frac{1}{2}$ ?
5. What will 8 lb. 12 oz. of butter cost, if  $1\frac{1}{2}$  lb. cost 30 cents?
6. When  $2\frac{1}{2}$  tons of hay cost  $\$33$ , what will  $18\frac{3}{4}$  tons cost?
7. If  $3\frac{1}{2}$  yd. of cloth cost  $\$10$ , what will  $2\frac{3}{4}$  yd. cost?
8. A man sold a horse for  $\$125\frac{1}{2}$  and gained  $\$26\frac{1}{2}$ . How much did the horse cost?
9. A man divided 6.3 bu. of potatoes among his workmen, giving each  $\frac{1}{6}$  of a bushel.
10. What is a complex fraction? Write one. Change it to a simple fraction.
11. What is a mixed number? Write one. To what other form can you change it? Do so. Have you changed its value?
12. What are the terms of a fraction called? Why are they so called?
13. Show what effect it has upon the value of a fraction to (1) multiply its numerator by 2; (2) to divide its denominator by 2; (3) to multiply both numerator and denominator by 2; (4) to multiply its numerator by 2, and divide its denominator by 2.
14. At  $7\frac{1}{2}\%$  a pound, how many pounds of sugar can you buy for  $\$4.80$ ?
15. How many feet in  $5\frac{3}{4}$  rd. and  $4\frac{1}{2}$  yd.?
16. What fraction of  $4\frac{1}{2}$  is  $6\frac{3}{4}$ ?
17. What part of  $5\frac{1}{2}$  is  $2\frac{3}{4}$ ? Of  $14\frac{3}{4}$  is  $4\frac{3}{4}$ .



1. If a certain number diminished by  $\frac{1}{2}$  of itself is 15, what is the number?
2. Nellie has 18 buttons, and  $\frac{3}{4}$  of her buttons equals  $\frac{1}{2}$  of Mary's buttons. How many buttons has Mary?
3. What is  $\frac{1}{2}$  of  $\frac{3}{4}$ ?  $\frac{3}{4}$  of  $\frac{1}{2}$ ?  $\frac{1}{4}$  of  $\frac{3}{4}$ ?
4. Having lost  $\frac{1}{2}$  of his money, Harry found  $\frac{1}{3}$  of what he lost, and then had 70 cents. How much had he at first?
5. If a man can walk  $3\frac{1}{2}$  miles in 2 hours, how far can he walk in 8 hours?
6. How far apart are two places, when 7 times  $3\frac{1}{2}$  miles is  $5\frac{1}{2}$  times the distance between them?
7. If cloth costs \$5 a yard, how much can you buy for \$2 $\frac{1}{2}$ ?
8. If your brother can earn \$6 a week, how long will it take him to earn \$4 $\frac{1}{2}$ ?
9. If 5 pints of milk cost 12 cents, what will 25 pints cost?
10. If 4 men can do a piece of work in  $4\frac{1}{2}$  days, how long will it take 12 men to do the same work?
11. A girl gave  $3\frac{1}{2}$  apples to each of her girl friends. If she gave away 14 apples, to how many friends did she give her apples?
12. A watch cost \$40, and a chain cost \$12. What per cent of the cost of the watch is the cost of the chain?
13. A cow cost \$24, and  $\frac{1}{3}$  of the cost of the cow is  $\frac{2}{3}$  of the cost of the horse. Find the cost of the horse.
14. Five-sixths of 72 is how many times  $\frac{1}{2}$  of 16?
15. A horse cost \$150, and  $\frac{1}{3}$  of this is three times the cost of a sleigh, and the sleigh cost twice as much as a harness. Find the cost of the sleigh and the harness.
16. A vessel was sunk in 9 fathoms of water. How many feet deep was the water? A fathom is 6 ft.
17. The distance round the school-yard is 160 paces. How many feet is it? How many yards is it?
18. 8 is one factor of 24. What is the other?

(For summary, see page 291.)

Commercial discount is a reduction from the nominal price of anything.

1. Find the net amount of a bill of \$1,440, with 25%, 10%, and 5% off.

$$25\% \text{ of } \$1,440 = \$ 360.00$$

$$\$1,440 - \$360 = 1080.00$$

$$10\% \text{ of } \$1080 = 108.00$$

$$\$1080 - \$108 = 972.00$$

$$5\% \text{ of } \$972 = 48.60$$

$$\$972 - \$48.60 = 923.40. \text{ Ans.}$$

\$1,440 is the gross amount of the bill.

Find 25% of this amount and deduct it. The result is \$1,080.

Find 10% of this amount and deduct it. The result is \$972. Find

5% of this amount and deduct it. The result is \$923.40. This is the net amount of the bill.

2. Find the net amount of a bill of \$1,920 with 25% and 7½% off.

Find the net amount of a bill of:

3. \$1,275 with 20% and 15% off.
4. \$562 with 35% and 15% off.
5. \$1,088 with 50% and 10% off, and an additional 5% off for cash.
6. 8 doz. bolts, at \$3.00 with 40%, 5%, and 25% off.
7. 11 gross screws, at \$2.25, with ¾, and 30% off.
8. 6 doz. handles, at \$1.50, with 40%, 5%, 25%, and 17½% off.
9. 480 lb. tea, at 62½¢, with 37½%, and 15% off.
10. 560 articles at 87½¢, with 25%, 16⅓%, and 10% off.
11. 25 lb. crushed sugar, at 10¢; 40 lb. maple sugar, at 12¢; 6 lb. cheese, at 13¢; 8 lb. butter, at 28¢; 4 lb. raisins, at 13¢; 2 lb. cream tartar, at 35¢; with 15%, 20%, and 5% off on the whole.
12. 20 oxen, at \$53.50, with 45%, 15%, and 5% off.
13. 15 cows, at \$23.25, with 25%, 15%, and 10% off.
14. 15 reams note paper, at \$1.25, and 25 reams letter paper, at \$1.75 with 30%, 22½%, and 12½% off.

1. I bought books listed at \$12 a dozen for  $\frac{1}{3}$  and 10% off. I sold them at \$1.00 each. What per cent did I gain?

2. A farmer sold 1,000 bu. of corn at \$1 a bushel, and estimated his loss at 5%. What per cent would he have gained had he sold at \$1.20 a bushel?

3. A dealer sold skates at \$3.60 a pair, and made 20%. What per cent would he have made if he had sold them at \$4.00 a pair?

4. Find the net gain on two houses sold for \$2,100 each, if on one there is a gain of  $16\frac{2}{3}\%$  and on the other a loss of  $12\frac{1}{2}\%$ .

5. A house was insured for  $\frac{3}{4}$  of its value at  $\frac{1}{4}\%$ . The premium was \$13.50. What was the value of the house?

6. A real-estate dealer sold for me 75 lots of land at \$275 a lot. If he charged me 2% commission for selling, and \$5 a lot for recording the deed, how much will I receive for all my land?

7. A merchant sold 73,680 ft. of lumber at \$20 per M., and gained \$294.72. What was his per cent of gain?

8. By selling a piece of land for \$160, I lost 25%. At what price should I have sold it to have made 20%?

9. I bought a house for \$2,250, and sold it for \$2,700. What per cent did I gain?

10. A dealer bought 56 house-lots for \$256.00 each, and sold them at an advance of  $9\frac{1}{4}\%$ . How much did he receive for all the lots?

11. A merchant paid  $18\frac{3}{4}\%$  a yard for cloth, and exchanged  $12\frac{1}{2}$  yd. for 16 doz. eggs at 25¢ a dozen. What per cent of profit did he make?

12. By selling a piece of land for  $16\frac{2}{3}\%$  profit I cleared \$150. What did it cost?

13. If I pay \$150 for insuring \$8,000 worth of goods, what is the rate?

14.  $32\frac{1}{3}$  is  $6\frac{1}{4}\%$  of what number?

1. What will it cost to dig a cellar 70 ft. long, 35 ft. wide, 5 ft. deep, at  $62\frac{1}{4}$ ¢ a cubic yard?

2. At \$4.25 a cord, what is the value of a pile of wood 72 ft. long, 4 ft. wide, and 12 ft. high?

3. At \$18 per M. find the cost of planks for flooring a barn 40 ft. by 32 ft., if each plank is 16 ft. long, 15 in. wide, 2 in. thick?

4. The width of a gable is 30 ft. and its perpendicular height 20 ft. What will be the cost of boarding two gables at \$16 per M.?

5. A bin is 8.5 ft. long, 4.25 ft. wide, and 3.75 ft. deep. How many bushels of oats will it hold?  $1\frac{1}{4}$  cu. ft. in 1 bu.

6. How many gallons of water will a tank hold 4 ft. by  $3\frac{1}{2}$  ft. by 2 ft. 4 in.?  $7\frac{1}{2}$  gal. in 1 cu. ft.

7. A rectangular garden is 200 ft. long and 150 ft. wide. There is a walk 4 ft. wide running all round it, and also through the center in both directions. What part of the whole area of the garden is taken up with the walk?

8. A section of land is one mile square. At 60¢ a rod for fencing, and \$1 $\frac{1}{4}$  an acre for plowing, find the sum paid out for a quarter-section of land.

9. A room is 18 ft. by 24 ft. A carpet is  $\frac{3}{4}$  of a yard wide, and runs lengthwise. There is a waste of 9 in. on each breadth for matching. At \$1.25 a yard find the cost of the carpet.

10. What will it cost to build a fence round a square 660 ft. on a side, if the posts are placed 6 ft. apart, and cost 18¢ each? The pickets are 2 in. wide, and placed 2 in. apart, and cost \$3.25 a hundred. The two rails are 4 in. by 4 in., costing \$12 per M. board feet. The labor is \$125.

11. At \$72 an acre, a farm is worth \$12,240. What will it cost to fence it at \$1.25 a rod if it is in the form of a rectangle 160 rd. wide?

12. Find area of a rectangle 33 rd. by 34 rd. 2 yd.

1. A man paid \$87 for insuring his house, worth \$7,250. What was the rate of insurance?

2. If I pay \$27 for insuring property at  $\frac{3}{4}\%$  premium, what is the value of the property?

3. Find the net amount of a bill of \$1,875, the discounts being 20% and 5%.

4. A merchant imported 175 chests of tea. Each chest contained 45 lb., valued at 48¢ a pound. He sold it at a gain of 25%. Find the selling-price.

5. An elevator in Minneapolis is valued at \$24,000, and the grain in it at \$25,000. The elevator is insured for  $\frac{3}{4}$  of its value at  $\frac{3}{4}\%$ , and the grain is insured for  $\frac{3}{4}$  of its value at  $\frac{1}{2}\%$ . Find the entire premium.

6. The perimeter of a rectangle is 42 inches. The horizontal sides are twice as long as the vertical sides. How long is each side?

7. A earns 18% more in a week than B, and the sum of their wages is \$76.30. How much does each earn?

8. A merchant paid \$1.50 for a book, and marked it to be sold for \$2.00. He discounted  $12\frac{1}{2}\%$  from his price. How much did he gain?

9. An agent purchased  $4\frac{1}{2}$  tons of raw sugar at  $3\frac{1}{2}\%$  a pound. What was his commission at  $2\frac{1}{2}\%$ ?

10. The net proceeds of a sale were \$1,368. The commission was \$57. What was the rate of commission?

11. Find the interest of \$2,862 for 93 days @ 6%.

12. Divide 96,496 by 592; 76,368 by 516.

13. Add .96, 7.3004, 8010, .00093, 1.24650.

14. A flag-pole 140 feet high was broken off, so that the part broken off was 6 times as long as the part left standing. How long was the part broken off?

15. The sum of two numbers is 49. The larger is 6 times the smaller. Find the numbers.

1. If 6 bbl. of flour cost \$33, what will 11 bbl. cost?
  2. If 9 tons of coal cost \$54, how many cords of wood at \$4 a cord will cost as much as 5 tons of coal?
  3.  $27 + 15 + 18 + 25 + 9 = ?$
  4. If 4 lb. of cheese cost 36 cents, how much cheese can be bought for 3 cents?
  5. If 2 bu. of cider apples cost 40 cents, what will 3 pk. cost?
  6. 25% of \$24 is what per cent of \$200?
  7.  $\frac{1}{4}$  of 35 are  $\frac{1}{4}$  of how many times 12?
  8. How many square inches on the surface of a 6-in. cube?
  9. At 6% a year what is the interest of \$1,000 for 48 days?
- For 33 days?
10. What will 5 pints of molasses cost at \$.80 a gallon?
  11. How many 4-in. squares can be cut from a 20-in. square?
  12. If I sold a horse for \$105, and gained  $16\frac{2}{3}\%$ , what did the horse cost?
  13. How many times 4 are  $\frac{1}{4}$  of  $\frac{1}{4}$  of 18?
  14.  $\frac{1}{4}$  of 40 are  $\frac{1}{4}$  of what number?
  15. What per cent is lost on goods sold at  $\frac{1}{4}$  of their cost?
- At  $\frac{1}{4}$  of their cost? At  $\frac{1}{4}$  of their cost? At half price?
16. What per cent of 27 is 20% of 45?
  17. What is the ratio of 18 : 9? 35 : 7? 16 : 4? 5 : 15?
- 9 : 12?
18. 48 is the antecedent, and 8 is the consequent; what is the ratio?
  19. 12 is the consequent, and 5 is the ratio; what is the antecedent?
  20. To what sum will \$100 amount when on interest at 6% for 2 yr. 6 mo.?
  21. At 5%, for how much can I insure my store if I pay a premium of \$50?
  22. How much will 8 tons of coal cost at \$5.25 a ton?

1. Place a cube on your desk. How many dimensions has it? How many faces has it?
2. These faces are called its surfaces. A surface is a boundary of a solid. Define surface.
3. How many dimensions has each surface?
4. Surfaces are bounded by edges called lines. A line is the limit of a surface, or it is the path traced by a point as it moves from one position to another. To read a line we usually use two letters, naming the starting-point first.
5. How many dimensions has a line?
6. How are the lines of the cube limited?
7. A point is the limit of a line and has no extent, only position. Define point.
8. In the cube how many faces meet to form a line?
9. Each face is bounded by how many lines? If the cube has six faces, and each face has four lines, how many lines has the cube? Why is not the number 24?
10. How many lines meet at each point? If the cube has 12 lines, and each line has two points, how many points has the cube? Why not twenty-four?
11. In a square prism, how many surfaces, lines, and points are there?
12. By how many lines is each surface bounded? How many surfaces meet in each line? How many lines meet at each point? Are the surfaces the same shape? The same size?
13. Examine in the same way a triangular prism and an hexagonal prism.
14. What kind of lines have you found on these solids?
15. A straight line is a line which has the same direction throughout its entire length.
16. Define a straight line. Draw one.
17. Look at a cylinder. How many edges or lines has it? Are these lines straight? What are they?

1. Define a curved line. Write: A curved line is a line that constantly changes its direction.

2. Fasten a weight to one end of a cord. Hold the other end at rest in the hand. This is a plumb line, and is said to be vertical.

3. Define a vertical line. Draw one.

4. A horizontal line is a line which has the direction of any line in the surface of still water. Practically it is a line that points towards the horizon.

5. Lines neither vertical nor horizontal are called inclined lines or oblique lines.

6. How are horizontal lines represented on paper? Vertical lines?

7. Hold your ruler vertically, horizontally, inclined. Draw on paper lines to represent these three positions.

8. Draw a vertical line, and through it two horizontal lines.

9. Draw two lines which have the same direction, that is, do not meet, however far extended. These lines are said to be parallel. Define parallel lines.

10. Draw two parallel straight lines. Two parallel curved lines.

11. Draw two parallel horizontal lines. Two parallel vertical lines.

12. Draw two lines not parallel. Prolong them till they meet.

13. This point of meeting is called their intersection. Define intersection.

14. Hold two pencils parallel. Hold them so they would intersect.

15. Hold two rulers parallel; not parallel.

16. Are two vertical lines always parallel to each other?

17. Can two horizontal lines ever intersect each other?

18. Give examples of vertical and horizontal lines.



1. By means of a triangle and a ruler draw through a point outside of a given line, a parallel to that line.
2. Draw several parallel lines freehand. Test and correct them with ruler and triangle.
3. Write: When one line meets another line so as to make the adjacent angles equal, the lines are said to be perpendicular to each other.
4. Draw two lines so as to form equal adjacent angles. Define perpendicular lines.
5. Draw a line perpendicular to a vertical line. Draw one perpendicular to an inclined line.
6. Draw three lines: *a*) All parallel; *b*) Two parallel, one perpendicular to them; *c*) No two parallel, all intersecting at one point; *d*) No two parallel, and not all meeting in a point.
7. In case *d*), in how many points do the lines intersect?
8. Draw freehand a horizontal line of any length. Draw a vertical line of equal length. Test your work.
9. Draw two lines whose ratio shall be 1 : 2; 1 : 4; 1 : 6.
10. Draw two lines whose ratio shall be as 2 : 3; 3 : 4; 2 : 6.
11. What is meant by drawing to a scale?
12. How long would you draw a line to represent 20 in., using a scale 1 : 8?
13. How many centers can a circle have? How many circles can have the same center?
14. Can a plane surface and a curved surface be parallel? Draw a straight line and a curved line that shall be parallel.
15. If I use a scale of  $\frac{1}{4}$  in. to a foot, what ratio do I use?
16. What is the standard unit of length in this country?
17. Draw to a convenient scale lines representing 130 ft.; 250 yd.; 75 rd.
18. What scale did you use?
19. How many lines parallel to a given line can be drawn through a point outside of the line?

1. A tank can be filled by one pipe in 15 min., and by another pipe in 30 min. In what time can it be filled by both together?

2. Suppose water runs in through the first pipe and out through the other. In what time then will the tank be filled?

3. A cistern can be filled by one pipe in half an hour, by a second pipe in 45 min., and by a third pipe in an hour. In what time will the cistern be filled if all run together?

4. Suppose water runs out through the second pipe and in through the other two. In what time then will it be filled?

5. The population of a certain town was 35,416 in 1890. If it increases 50% in 10 years, what will it be in 1900?

6. Find the gain or loss per cent:

Cost, \$20; selling-price, \$25.

Cost, \$2.00; selling-price, \$2.12½.

Cost, \$12.40; selling-price, \$10.23.

Cost, \$74.00; selling-price, \$70.30.

7. A house is sold for \$400, and 25% is made. How much profit would be made by selling for \$336?

8. By selling a house for \$7,590 a man gained 10%. What per cent would he have lost if he had sold it for \$6,210?

9. A draper bought 960 yd. of silk at \$2.00 a yard. He sold ¼ at a gain of 25%, ¼ at a gain of 20%, and the remainder at a loss of 15%. For what was it sold?

What is paid for goods marked:

10. \$600, with a discount of 33⅓%?

11. \$1,200, with a discount of 16⅔%?

12. \$1,000, with a discount of 27% and 10%?

13. \$600, with discounts of 20%, 10%, and 1%?

14. \$2,500, with discounts of 20%, 5%, and 1½%?

15. Which is cheaper, to buy goods at a discount of 30% and 5%, or with 33⅓% off? How much cheaper on a bill of \$600?

1. Buy oranges at 15¢ a dozen, and sell them at 2 for 5 cents, and tell me your gain per cent.

2. Buy a horse for \$300, and sell it for \$360, and tell me your gain per cent.

3. A boy bought some marbles for 12 cents, and sold them for 15 cents. Find the gain per cent.

4. A merchant bought muslin at 10¢ a yard, and sold it at a gain of 20%. Find his gain on 25 yards.

5. Eggs cost me 20 cents a dozen, but in selling them I gained 25%. How many dozen eggs must I sell in order to receive \$1.00?

6. \$7 is  $16\frac{2}{3}\%$  more than what?

7. A farmer had 50 sheep, and bought 20% more. How many had he then?

8. If  $\frac{1}{4}$  of an acre of land is worth \$60, how much is  $\frac{3}{4}$  of an acre worth?

9. If a boy lost  $\frac{3}{4}$  of his money, and had 8 cents left, how much had he at first?

10. A boy lost  $\frac{1}{3}$  of his marbles and sold  $\frac{1}{3}$  of them, and then had 21 left. How many had he at first?

11. If  $\frac{1}{4}$  of a yard of cloth cost 30 cents, how much will  $\frac{3}{4}$  of a yard cost?

12. If 5 yd. of cloth are worth 80 cents, how much is  $\frac{3}{4}$  of a yard worth?

13. \$30 is  $\frac{1}{4}$  of the cost of a cow. Find the cost of two cows at the same rate.

14. If 3 bu. of corn cost \$3 $\frac{1}{2}$ , how many bushels can I buy for \$2.50?

15. If a yard of cloth cost \$ $\frac{1}{4}$ , how many yards can be bought for \$10?

16. A man exchanged 5 sheep at \$9 each, and 2 cows at \$30 each, for pigs at \$5 each. How many pigs did he get?

17. If  $\frac{1}{4}$  of A's money is \$50, what is his money?

1. What is the circumference of the largest circle that can be drawn on a 9 ft. square?

2. If it requires 440 ft. of lumber to board up the gable ends of a barn 40 ft. wide, how high is the ridge above the eaves?

3. The area of a triangle is 36 sq. yd., and the base is 36 ft. What is the altitude?

4. Find the convex surface of an equilateral triangular pyramid, the sides of whose base are each 8 ft., and whose slant height is 24 ft.

5. How long a band of iron will it take to surround a cylindrical tank 15 ft. 8 in. in diameter?

6. What is the area of a semicircle whose radius is 24 inches?

7. At \$1.25 a square yard, how much will it cost to pave a triangular space, one of whose sides is 80 yd., and the perpendicular distance from the opposite vertex to that side 180 feet?

8. The diagonal of a trapezium is 18 ft., and the perpendiculars from the opposite vertices are 9 ft. and 8 ft. respectively. What is the area?

9. Find the convex surface of a cone, the radius of whose base is 16 in., and whose slant height is 8 ft.

10. At 25¢ a square yard, it costs \$18.75 to paint a triangular surface. If the base is 60 ft., what is the altitude?

11. How many acres are there in a field in the form of a trapezoid, if the parallel sides are 24 rd. and 16 rd., and the distance between them 18 rd.?

12. What is the surface of a sphere whose circumference is 24 ft.?

13. If a bin is 8 ft. square, how deep must it be to hold 256 bu. of apples?

14. A field in the form of a trapezoid contains  $11\frac{1}{4}$  acres. If the parallel sides are 60 and 40 rd., how far apart are they?

15. How many feet in a board 15 ft. long by 16 in. wide?

1. Make out a correct bill, supplying dates and items.
2. Make out a correct monthly statement.
3. A dealer bought 13 head of young cattle for \$325. He kept them for 4 months at an expense of \$2 a head a month, and then sold them at \$32 each. Did he gain or lose? and how much?
4. It is 40 rd. round a field. At \$22.65 a thousand find the cost of rails for the fence. Each rail is 11 ft. long, and the fence 6 rails high.
5. A farmer raised 8,526 bu. of wheat. He had it ground into flour. If 1 bu. made 40 lb. of flour, how many barrels did he receive?
6. A horse and carriage are valued at \$420;  $\frac{1}{3}$  of the value of the horse is equal to  $\frac{1}{4}$  of the value of the carriage. Find the value of each.
7. A bought pears at the rate of 6 for 5 cents, and B bought peaches at the rate of 3 for 4 cents. How many peaches should B give to A for 120 pears?
8. If telegraph poles cost 25¢ each, and wire  $\frac{3}{4}$  of a cent a yard, how much will the material cost for 2 miles of telegraph line consisting of 6 wires, if the poles are 80 ft. apart?
9. A rectangular field containing 27 A. is 40 rd. wide. What will it cost to fence it at 35¢ a yard?
10. A wagon upon which 4-ft. wood was piled was 12 ft. long. How high was the wood, if there were  $2\frac{1}{2}$  cd.?
11. Find the number of board feet in 14 planks,  $8\frac{1}{2}$  ft. long, 16 in. wide, and  $3\frac{1}{2}$  in. thick.
12. What will it cost to plaster a room 32 ft. long, 18 ft. wide, and 13 ft. high, at 15¢ a square yard, allowing 148 sq. ft. for openings.
13. At what rate will \$652 gain \$440.10 in 15 years?
14. What sum of money on interest at  $4\frac{1}{2}\%$  will yield an annual interest of \$1,200?

1. A company charges \$30.37½ for insuring \$1,350 worth of property. What was the rate of insurance?

2. A mill was insured for \$5,000 in one company at 1½% and for \$6,000 in another company at 1½%. What was the total premium paid?

3. A bankrupt pays 42½¢ on a dollar. How much will a creditor lose whose bill is \$1,460?

4. A bankrupt's liabilities are \$30,000 and his assets \$8,000. How much can he pay on a dollar?

5. A house is insured for  $\frac{3}{4}$  of its value at 1½%. The premium is \$81.00. What is the value of the house?

6. A merchant fails in business, owing \$7,200. His assets are \$3,000. How much will a man receive who is creditor to the amount of \$600?

7. My horse and buggy together are worth \$300, and the horse is worth 4 times as much as the buggy. What is each worth?

8. If the divisor were one-third what it is, the quotient would be 948. What is the quotient?

9. If 63 be added to a certain number it will contain forty-two 246 times. What is the number?

10. How many times must 720 be added to 522 to make 987,642?

11. A man bought an equal number of lemons and oranges for \$6.25. For the lemons he paid 2¢ each, and for the oranges 3¢ each. How many of each did he buy?

12. One train left Boston at 1 P.M. on the B. & A. R. R. A second train left at 3 P.M. The first goes 30 miles an hour, and the second 40 miles an hour. When will the second overtake the first? and how many miles from Boston?

13. What fraction divided by  $\frac{3}{4}$  of 12 will give  $\frac{1}{3}$  for a quotient?

14. If  $\frac{3}{4}$  of a farm is worth \$963, what is the value of  $\frac{1}{4}$  of the farm?

1. A house worth \$3,500 is insured for  $\frac{2}{3}$  of its value at  $\frac{3}{4}\%$ .
2. A man owns  $\frac{2}{3}$  of a store, and sells  $\frac{1}{3}$  of his share for \$492.
3. I insured my barn for  $\frac{2}{3}$  of its value at  $1\frac{1}{4}\%$ , and paid a premium of \$15.
4. I lost \$1,280 on 160 acres of land, which I sold for \$34 an acre.
5. Goods marked \$64 were sold at  $6\frac{1}{4}\%$  discount and 5% for cash.
6. A tree 45 ft. high was broken at such a point that the part broken off was 4 times the length of the part left standing.
7. After deducting his commission of 5%, an agent returned to his employer \$1,436.40.
8. A commission merchant sells goods for \$5,728, and sends to his principal \$5,649.24.
9. A company insured a mill and its machinery for \$117,944, the machinery being worth 15% of the value of the mill. The owner paid 2% on the mill, and  $1\frac{1}{2}\%$  on the machinery.
10. Three sevenths of a certain number exceeds  $\frac{1}{4}$  of the same number by 25.
11. A certain number multiplied by three thousandths will give 3645.
12. The premium at 3% is \$756.
13. My agent sold for me some property for \$1080, and charged me \$81 commission.
14. A man built  $\frac{1}{2}$  of a wall in  $5\frac{1}{2}$  days.
15. It cost 55¢ a cubic yard to dig a well 5 ft. in diameter, and 30 ft. deep.
16. Forty cents were divided among three boys, A, B, and C, in such a way that B and C each had twice as much as A.
17. A farmer sold a horse and cow for \$240. He sold the horse for five times as much as the cow.
18. \$206.25 was the premium paid for insuring a factory at  $1\frac{1}{2}\%$ .

1.  $\frac{3}{4}$  of 20 are  $\frac{3}{4}$  of what number?  $\frac{3}{4}$  of what?  $\frac{3}{4}$  of what?
2. When oranges cost 25¢ a dozen, how many can be bought for  $66\frac{2}{3}\%$  of \$2.25? For 50% of \$1.25? For 3 times  $16\frac{2}{3}$  cents?
3. Charles is 12 years old.  $\frac{3}{4}$  of Charles's age is  $\frac{3}{4}$  of Bessie's.  $\frac{3}{4}$  of Charles's age is  $\frac{3}{4}$  of George's age. How old are Bessie and George?
4. What is the cost of 54 yd. of cloth at \$0.16 $\frac{2}{3}$  a yard?
5. What per cent of 100 is 25% of 80?  $66\frac{2}{3}\%$  of 75?
6. What per cent of 80 is 25% of 100?  $66\frac{2}{3}\%$  of 75?
7. Find the cost of tea a pound when 10% is gained by selling it at 55¢ a pound?
8. A teacher said, "I should have 50 pupils in my room, but 5% are absent." How do you know she made a mistake?
9. At 6% find the interest for 12 days:
 

\$3000	\$840	\$30	\$ 1500
3500	620	10	2400
1200	150	15	10000
10. A ship worth \$60,000 was insured at 4%. What was the premium?
11. A bill of goods amounting to \$500 I bought at a discount of 10%. What did I pay?
12. If 36 is the antecedent and 3 is the ratio, what is the consequent?
13. What is the area of a triangle whose base is 24 ft. and altitude 8 ft.?
14. If the area of a triangle is 48 sq. ft. and the altitude is 8 ft., what is the base?
15. If the area of a rectangular field is 77 sq. rd. and the base is 11 rods, what is the altitude?
16. How many board feet in a plank 20 ft. long, 15 in. wide, and 2 in. thick?
17. What will 8 boxes of oranges cost if  $\frac{3}{4}$  of a box cost \$9?



1. The greater of two numbers is twice the less, and the sum of the numbers is 135. What are the numbers?
2. The sum of the ages of a mother and daughter is 36 years, and the age of the mother is eight times that of the daughter. What is the age of each?
3. A man traveled 320 miles in three days. If he traveled three times as far the first day as he did the third, and four times as far the second day as the third, how far did he go each day?
4. Divide 36 into three parts so that the first part shall be three times the second, and the second two times the third.
5. A farmer bought a horse, a cow, and a calf for \$104. The cow cost three times as much as the calf, and the horse three times as much as the cow. What was the cost of each?
6. One boy has three times as many marbles as his companion. If both boys have 28 marbles, how many marbles has each?
7. Divide 126 into two such parts that one part may be five times as large as the other part.
8. Divide 96 cents between two children so that one child shall have three times as many cents as the other child.
9. If from 8 times a certain number three times the number is taken, the remainder will be 45. What is the number?
10. Five times a certain number and eight times the same number is 156. What is the number?
11. Three men, A, B, and C, had 270 acres of land. B had 3 times as much as A, and C had five times as much as A. How many acres had each?
12. Divide the number 264 into two parts so that one part will be 5 times the other.
13. Three men, A, B, and C, earned \$330. A earned 4 times as much as B, and C as much as both A and B. How much did each earn?

1. The product of 4 and 3 is written  $4 \times 3$ , but the product of  $a$  and  $b$  is not written  $a \times b$ , but  $ab$ . The product of 5,  $x$  and  $y$  is written  $5xy$ .

2. Write the product of 8 and  $c$ ; 4,  $a$ , and  $d$ ; 7,  $x$ ,  $y$ , and  $z$ .

3. In the expression  $5xy$ , 5,  $x$ , and  $y$  are the factors of  $5xy$ . When one of the factors of an expression is a numerical quantity, it is called the coefficient of the remaining factors.

4. The index has the same meaning as in arithmetic.  $3a$  and  $a^3$  are not alike.  $3a$  means  $a + a + a$ .  $a^3$  means  $a \times a \times a$ . If  $a = 4$ , find the value of  $3a$  and of  $a^3$ . *Be careful to distinguish between coefficient and index.*

5. Express in the form of a fraction  $15 \div 5$ . In the same way express  $3x \div 2$ ;  $4x \div 5$ ;  $a \div b$ ;  $2 \div c$ .

6. If  $a = 4$ ,  $b = 3$ ,  $c = 1$ ,  $x = 2$ ,  $y = 5$ ,  $z = 6$ , find the value of:

$4a$ ,	$3c$	$4y^2$ ,	$a^2$ ,	$2b^4$ ,	$5x^3$
$7z$ ,	$c^2$ ,	$3x^2$ ,	$z^3$ ,	$4y^4$ ,	$7x$ ,
$2ab$ ,	$3bc$ ,	$4cx$ ,	$5xy$ ,	$6yz$ ,	$2a^2x^3$ .

7. If  $a = 4$ ,  $b = 1$ ,  $c = 3$ ,  $x = 5$ ,  $y = 7$ ,  $z = 0$ , find the value of:

$3x + 5z - 7b$ ;	$3y - 4z + 7c$ ;	$3a - 5b + c$ .
$5c - 8z + 3a$ ;	$4x - 2y - 3b$ ;	$2x - 3y + 5a$ .
$x^2 - 3a^2 + 2c^2$ ;	$b^3 - 2x^2 + 3a^2$ ;	$xz - zb - 2cz$ .

8. Express four increased by five;  $a$  increased by  $b$ .

9. Express 9 diminished by 5;  $a$  diminished by  $b$ .

10. If the dividend is 12, and the quotient 4, express the divisor.

11. How much does  $b$  lack of being 15?

12. If a man walks 3 miles an hour, how many miles will he walk in  $x$  hours?

13. If a man walks  $x$  miles an hour, how many miles will he walk in  $c$  hours?

1. Reduce  $\frac{1}{2}$  to a fraction whose denominator is 500.
2. Find the value of a mill if  $\frac{1}{2}$  of  $\frac{1}{2}$  of it is worth \$4,690.
3. A man owned  $\frac{1}{2}$  of a large factory. He sold  $\frac{1}{2}$  of his share for \$9,900.90. What is the value of the factory?
4.  $\frac{1}{2}$  of  $\frac{1}{2}$  of 378 is  $\frac{1}{2}$  of  $\frac{1}{2}$  of what number?
5. The product of three numbers is 79. Two of them are  $8\frac{1}{2}$  and  $6\frac{1}{2}$ . What is the other?
6. What is the cost of 3,170 lb. of iron at \$9 $\frac{1}{2}$  a ton, and 10,160 lb. at \$6 $\frac{1}{2}$  a ton?
7. A bought 140 oranges at the rate of 2 for a cent, and 150 at the rate of three for a cent, and sold them all at the rate of 5 for two cents. Did he gain or lose? and how much?
8. A rectangle is 27 ft. wide, and contains 945 square feet. What is its length?
9. If 15 $\frac{1}{2}$  bbl. of flour cost \$63, what will 10.4 bbl. cost?
10. At 7 $\frac{1}{2}$ %, what is the interest of \$864 for 2 yr. 11 mo. 13 days?
11. A and B are together worth \$102,375. How much does each own, if A owns  $\frac{1}{2}$  as much as B?
12. A man bought  $\frac{1}{2}$  of a mill, and sold  $\frac{1}{2}$  of what he bought to one man, and the remainder to another man for \$4,000. What was the value of the mill?
13. How much did a lawyer receive for collecting his bills, one of \$225 at 8 $\frac{1}{2}$ %, and the other of \$789 at 9 $\frac{1}{2}$ %?
14. A man invested  $\frac{1}{2}$  of his money in real estate,  $\frac{1}{2}$  in a mill,  $\frac{1}{2}$  in bank-stock. The remainder, \$2,300, he kept in cash. How much was he worth?
15. A barn is 40 ft. long and 20 ft. wide, with 16-ft. posts, and gables 8 ft. high. Find the cost of painting the barn, if the work costs 80¢ a square, and it takes 20 gal. of paint at \$1.50 a gallon.
16. An agent remitted his employer \$8,775 for the sale of some land. If his commission was 2 $\frac{1}{2}$ %, what was the value of the land sold?

1. What is the difference on a bill of \$1,275 between a discount of 40% and a discount of 30% and 10%?
2. What per cent is gained by selling articles at 21¢ each that cost \$3.36 a dozen less 25% and 14%?
3. If you buy a bicycle at a discount of 25% from the list price, and sell at list price, what is your gain per cent?
4. A lawyer, collecting a note at a commission of 5%, received \$9.75. What was the face of the note?
5. Find the area of a trapezoid when the parallel sides are 84 rd. and 66 rd. and the altitude 38 rd.
6. What must be the height of a pile of wood 32 ft. long and 6 ft. wide, to contain 9 cords?
7. William has a certain number of marbles; Charles has five times as many as William; Henry has as many as twice William's subtracted from Charles's; and Henry's added to William's are equal to 40. How many has each?
8. What was the amount of risk if \$25.20 was paid for insurance at 70¢ on \$100?
9. Find the per cent of loss on a bill of goods bought for \$3,360 and sold for \$2,520.
10. What number decreased by 25% is \$342.60?
11. Find the number of feet, board measure, in a plank 24 ft. long, 9 in. wide, and 3 in. thick.
12. Scale,  $\frac{1}{4}$  in. to a rod. Draw a plan of Mr. Gordy's farm, whose boundary runs as follows: From A east to B, 30 rd.; from B south to C, 12 rd.; from C east to D, 24 rd.; from D south to E, 36 rd.; from E west to F, 54 rd.; from F north to A. Find the whole area of his farm, and the length of his boundary fence. Connect A and C with E, and find the area of each of the three fields.
13. A man after drawing out 20% of his money, and then 10% of the remainder, found that he had in the bank \$1,512. How much had he in the bank at first?

1. A house valued at \$800 was insured for three years at  $1\frac{1}{4}\%$ . What was the premium?
2. A man sold a horse and carriage at a loss of 20%. If he received \$240, what was the cost?
3. A dealer sold a set of books for \$25, and gained 25%. What per cent would he have gained or lost if he had sold for \$21?
4. By selling a carriage for \$180, a man gained 20%. Find the cost.
5. At \$1.25 a rod, find the cost of building a wall one mile long.
6. A man sold a cow for \$60, which was  $\frac{3}{4}$  of the cost. How much did he lose?
7. Make an example like the 6th, using per cent.
8. If your steps are each 2 ft. long, how many steps will you take in walking 2 rd. 1 yd.?
9. How many surface feet in a rectangular piece of marble 4 ft. long, 3 ft. wide, and 2 ft. thick?
10. Find the commission that an agent receives for selling 40 bbl. of flour at \$6 a barrel, if he receives 3%.
11. A bookseller bought a book for \$3.75, and sold it for \$4.50. What per cent did he make?
12. How many acres in a field 80 rd. long and 40 rd. wide?
13. How many cords in a pile of wood, 32 ft. long, 4 ft. high, and 4 ft. wide?
14. At \$1.25 a cubic foot, find the cost of a block of granite 4 ft. long, 3 ft. wide, and 2.5 ft. thick.
15. At \$1 a hundred board feet, find the cost of 6 boards, each 12 ft. long, 10 in. wide.
16. At 20¢ a square yard, what will it cost to paint a ceiling 18 ft. by 24 ft.?
17. If  $\frac{3}{4}$  of a yard of cloth cost \$8, what will  $2\frac{1}{2}$  yd. cost?
18. What will 4 tons of hay cost at \$18.75 a ton?

NOTE. — These rules are for reference, and need not be memorized.

1. In painting and plastering, it is customary to deduct from the whole area of the room one-half of the area of all doors, windows, or openings. This rule is not always observed.

2. PAPERING. American wall-paper is usually  $1\frac{1}{2}$  ft. wide and 24 ft. long for a single roll, 48 ft. long for a double roll.

There are various rules:—

(a) Find the perimeter of the room in feet, and divide by  $1\frac{1}{2}$  ft. (width of paper); the quotient equals the number of strips of paper required. Divide the length of a roll by the height of the room to find the number of strips in a roll. Divide the strips in the room by the strips in a roll to find the rolls required.

In the first and third division, if there is a fraction, take the next higher integer; in the second division, take the next lower integer.

(b) Same as a, except from perimeter of room, deduct the width of doors and windows.

Use this method unless otherwise directed

(c) Find the area as for plastering, divide the square feet in the area by 36 (the square feet in one roll of paper); this will give the number of rolls.

3. SHINGLING. Shingles are put up in bunches consisting of 25 layers on each side, 20 in. wide. Every 4 in. is reckoned as 1 shingle. Four bunches make 1000.

In practice, allowing for waste, a thousand will cover 100 sq. ft. when laid 4 in. to the weather.

Find the number of squares, then compute the number of shingles.

4. LATHS. A lath is 4 ft. long, and  $1\frac{1}{2}$  in. wide, usually nailed  $\frac{3}{8}$  of an inch apart. There are 50 or 100 laths in a bunch. In this work bunches of 50 estimated to cover, allowing for waste, 3 sq. yds. are reckoned.

Find the number of square yds. and divide by 3 to find the number of bunches of laths required.

5. CLAPBOARDS. A clapboard is usually 4 ft. long, 6 in. wide, and 25 are put in a bundle. They are usually laid  $3\frac{1}{2}$  in. to the weather. 1 bunch will cover 25 sq. ft. allowing for waste.

Find square feet in area, and divide by 25 to find the number of bunches required.

NOTE. — Laths and clapboards are sold only by the bunch. One-half of openings is usually deducted in making estimates.

(See note on page 121.)

1. **STONE MASONRY.** All stone work is usually reckoned by the cubic foot.

In measuring for cellars and buildings the distance round the outside of the walls is taken for the length, thus measuring each corner twice. This is considered an offset for the greater labor in constructing the corners. For the same reason no allowance is made for an opening unless it is a large one; then deduct one-half.

2. **BRICK WORK.** In measuring, the same rule applies as in stone work.

(a) Find the number of square feet in the surface, and multiply by 7 if the wall is one brick in thickness; by 14 if 2 bricks in thickness; by 21 if 3 bricks in thickness.

(b) Find the number of cubic feet in the wall, and multiply by 22; for 22 bricks including mortar fill 1 cubic foot.

3. To find capacity of bins. A bushel contains 2150.4 cu. in. This is nearly  $1\frac{1}{4}$  times a cubic foot.

(a) For practical purposes take  $\frac{2}{3}$  of the number of cubic feet in the bin for the required number of bushels.

(b) For accurate purposes divide the number of cubic inches by 2150.4.

NOTE. — Use the first method unless the second is asked for.

NOTE. — In measuring bulky fruits and vegetables, as apples and potatoes, a bushel will fill  $1\frac{1}{2}$  cu. ft.\* Then find  $\frac{2}{3}$  of the number of cubic feet for the number of bushels. In small fruit, as berries, or in grain,  $1\frac{1}{4}$  cu. ft. are used.

4. To find the number of gallons in a cistern. A gallon contains 231 cu. in. Hence  $7\frac{1}{2}$  gal. equals 1 cu. ft.\* and 1 bbl. equals  $4\frac{1}{2}$  cu. ft. 1 gal. of water weighs  $8\frac{1}{2}$  lb. 1 cu. ft. of water weighs  $62\frac{1}{2}$  lb.\*

(a) Multiply the cubic feet in the cistern by  $7\frac{1}{2}$ , to find number of gallons.

(b) Divide the cubic inches in cistern by 231, to find the number of gallons.

5. To measure coal. A short ton of hard or anthracite coal measures about 36 cu. ft. A short ton of soft or bituminous coal measures about 42 cu. ft.

Divide the cubic feet in the bin by 36 or 42 as the case demands.

6. To gauge or find the volume of a barrel or cask. Find mean diameter. This is the head diameter plus two-thirds of the difference between the head and bung diameters.

(a) Square the mean diameter (multiply it by itself); multiply by the length of the cask in inches and that by .0034; the result will be the number of gallons.

(b) When the cask is not full, multiply the square of  $\frac{1}{3}$  of the sum of the head, mean, and bung diameters in inches by the depth of the liquid in inches, and this by .0034.

\* Approximately.

1. At \$4.25 per M., how much will the shingles cost for a double roof, rafters 22 ft. long, house 34 ft. long? The shingles are laid 4 in. to the weather.
2. At \$2.75 per M., how much will the laths cost for a ceiling 18 ft. by 24 ft.?
3. How many barrels of water will a cistern hold that is 6 ft. by 7 ft. by 8 ft.?
4. A cistern 6 ft. wide and 10 ft. long holds 40 bbl. of water. How deep is the water? Approximate measurement.
5. How many barrels of water in a cistern 6 ft. in diameter, if the water is 6 ft. deep?
6. How many barrels of apples will a crib contain that is 8 ft. wide, 9 ft. high, and 30 ft. long?
7. A bin 6 ft. wide and 10 ft. long holds 150 bu. of potatoes. How high is the bin?
8. A house is to be built 40 ft. by 30 ft. If the wall beneath it is to be 6 ft. high and 16 in. thick, how many cubic feet of stone are required?
9. How many bricks are required for the 8-in. walls and bottom of a rectangular cistern, the outside dimensions of which are: length 8 ft., depth 7 ft., width 6 ft.?
10. A room is  $16\frac{1}{2}$  ft. by 15 ft. Carpet is  $\frac{3}{4}$  yd. wide. If the breadths run widthwise of the room, and there is a waste of 6 in. on each breadth for matching, how many yards of carpet will it take to carpet the room?
11. A room  $18\frac{1}{2}$  ft. long and 15 ft. wide is carpeted with Brussels carpeting worth \$1.12 $\frac{1}{2}$  a yard. Find the cost of carpeting the room if the breadths run lengthwise, with an allowance of 8 in. on each breadth for matching.
12. A circular cistern is 5 ft. in diameter. If the water in it is 8 ft. deep, how many gallons are there?
13. A cubical cistern is 6 ft. deep. How many gallons of water will it hold? Approximate measurement.



124      *APPLICATIONS OF THE PRACTICAL RULES.*

1. At \$4.00 per M., find the cost of shingles for a roof 50 ft. long, and each of the two sides  $23\frac{1}{2}$  ft. wide, if you allow 1000 shingles for every 125 sq. ft.

2. How far must a man walk in plowing lengthwise a field 15 rd. long, 8 rd. 12 ft. wide, if each furrow is 18 in. wide?

3. A roof is 36 ft. long, and each side 24 ft. wide. Slates are 16 in. by 12 in., and lap one-half. How many slates will it take to cover the roof?

4. At 25¢ a cut, find the cost of sawing into 4 pieces a pile of wood 56 ft. long, 4 ft. high, and 4 ft. wide.

5. At 18¢ a pound,  $4\frac{1}{2}$  lb. to a square foot, find the cost of lead to line a tank 6 ft. by 5 ft., and  $4\frac{1}{2}$  ft. deep.

6. A house is 38 ft. long, 24 ft. wide, and 20 ft. high, with a gable 8 ft. high. How many bundles of clapboards will it take to cover the house?

7. A hall is 18 ft. by  $6\frac{1}{2}$  ft., and  $7\frac{1}{2}$  ft. high. There is 1 door and 1 window, each  $3\frac{1}{2}$  ft. wide. At 24¢ a roll, find the cost of the paper.

8. At \$4.12 $\frac{1}{2}$  a cord, find the cost of a pile of wood 8 ft. long, 4 ft. wide, and 3 ft. high.

9. A's lot is 400 ft. by 25 ft., and B's is 100 ft. square. Which has the larger area? Which man will pay the more for fencing, and how much more at 32¢ a foot?

10. A building-lot containing  $\frac{1}{4}$  of an acre is 36 ft. wide. How deep is it?

11. What length must be cut off an inch board 8 in. wide to obtain 3 board feet?

12. If a man can walk 1 mile, 15 rd. in 20 min., how many hours will it take him to walk 84 mi. 175 rd.?

13. The walls of a hall are 120 ft. long, 75 ft. wide, 30 ft. high, and 21 in. thick. At \$3 per M., find the cost of brick, deducting one-half for openings; 12 windows, 3 ft. by 6 ft.; 4 doors, 5 ft. by 10 ft.

1. What is the cost of digging a cistern in the form of a cylinder whose diameter is 6 ft., and whose depth is 18 ft., at 62¢ a cubic yard? How many gallons of water will it hold?

2. A grocer placed in his window a pyramid of oranges 5 ft. high, and 6 ft. square at the base. How many dozen oranges did he use, if 9 oranges fill the space of a cubic foot?

3. A grain-box has a base 8 ft. long and 6 ft. wide. The height of the box is 11 ft. How many bushels of grain will fill the box?

4. A cylindrical shaped bin has a diameter at the base of 16 ft., and the height of the cylinder is 24 ft. This cylinder has a cone-like base, with a depth of 9 ft. How many bushels of grain will the bin hold?

5. A lot was bounded as follows: beginning at the north-west corner, the line ran east 35 ft. to B; thence south 90 ft. to C; thence west 60 ft. to D; thence to A. How many loads of gravel will it take to raise this lot 4 ft.?

6. The following represents a cellar 6 ft. deep. Scale, 1 in. to 10 ft. Draw, starting from the northeast corner, A, south 1 in., west 1 in., south 1 in., east 1 in., south  $\frac{3}{4}$  in., east  $\frac{1}{2}$  in., south  $\frac{1}{2}$  in., west  $1\frac{1}{8}$  in., north  $\frac{1}{2}$  in., west 2 in., north  $2\frac{1}{8}$  in., east 3 in. Find the cost of excavating this cellar at 45¢ a cubic yard.

7. Scale, 1 in. to 8 rd. Find the cost, at \$75 an acre, of a field whose boundary runs as follows: beginning at A, the north-west corner, running eastward 26 rd. to B; thence to the south-east to a point C, 12 rd. east of D, a point 24 rd. directly south of B; thence southwesterly to E, 24 rd. south of D; thence west 26 rd. to F; thence northwest to G, which is 12 rd. west of H, a point 24 rd. north of F; thence to A.

8. How many board feet in 100 boards, each 8 in. wide and 12 ft. long?

9. Find the area of a circle whose diameter is 77 ft.

1. By selling goods for \$47.50 a man lost 5%. What would he have gained if he had sold them for \$57?

2. A man sold his house and lot for \$8,304, at a profit of 15½%. If the lot cost  $\frac{2}{3}$  as much as the house, find the cost of each.

3. Find an agent's per cent of commission, when he received \$340.20 on a sale, the net proceeds of which were \$9,379.80.

4. If you buy oranges at 30¢ a doz., and sell them at \$2.80 a hundred, what will be your gain per cent?

5. The premium on an insurance of \$7,440 is \$44.64. What is the rate?

6. If by selling an organ for \$30, I lose 25%, at what price should I have sold it to gain 15%?

7. Find the cost of flour a barrel, when a commission merchant receives \$323.00 for selling 1,360 bbl. at 5%.

8. A man bought 2 houses for \$3,450, paying 30% more for one than he did for the other. Find the cost of each.

9. Find a man's income when 45% of \$1,800 is 18% of  $\frac{1}{2}$  of his income.

10. A man having \$9,600 in the bank, drew out 14% of it at one time, and then deposited a sum equal to 150% of what he had drawn out. How much had he in the bank then?

11. 66⅔% of the cost of my horse is 65% of the cost of my carriage. If the horse cost \$156, find the cost of both.

12. A boy sold two balls for 35 cents. This is a loss of 12½%. Find the cost of a ball.

13. Mr. Williams sold a piano to Mr. George at a gain of 14⅔%. Mr. George sold it to Mr. Bruce for \$320, thus losing 20%. What did Mr. Williams pay for the piano?

14. A man received \$24 as 20% of the price of a bicycle, sold at a gain of 20%. What did the wheel cost in the first place?

1. A man lost  $\frac{3}{4}$  of his money in business one year, and gained the next year  $\frac{1}{4}$  of what he had left. How much had he at first, if at the end of the second year he had \$70,000?

2. How many marbles has a boy if the difference between  $\frac{3}{4}$  and  $\frac{1}{2}$  of the number he has is 135?

3. How many  $2\frac{1}{2}$ -in. cubes can be cut from a large cube  $2\frac{1}{2}$  ft. on a side?

4. One man can do a piece of work in  $7\frac{1}{2}$  days; his brother can do it in 6 days. In how many days can both do it working together?

5. What part of 10 A. 140 sq. rd. is 7 A. 80 sq. rd.?

6. Change .0033 $\frac{1}{3}$ , .0875, and .066 $\frac{2}{3}$  to common fractions.

7. A can do a piece of work in 12 days, A and C in 9 days, and A and B in 6 days. In how many days can A, B, and C do it? How many days will it take B and C?

8. A man owned  $\frac{3}{4}$  of an acre. How much has he left after selling  $28\frac{3}{4}$  square rods?

9. A man had \$4,200 of a fortune left after having put  $\frac{1}{2}$  of it in a bank, and spending  $\frac{1}{4}$  and  $\frac{1}{8}$  of it.

10. If  $\frac{3}{8}$  of a yard of silk cost \$1.20, how many yards will cost \$51.60?

11.  $\frac{1}{4}$  is what part of  $\frac{7}{8}$ ?

12. What is the cost of  $64\frac{1}{2}$  bushels of seed at  $\$2\frac{1}{2}$  a bushel?

13. A man had  $13\frac{3}{4}$  acres of land, which he divided into building-lots each containing  $\frac{1}{4}$  acres. How many lots did he have?

14. Change .968 $\frac{2}{3}$  to a common fraction.

15. Change  $\frac{1}{3}$  to 225ths;  $\frac{1}{15}$  to 1,309ths.

16. If  $8\frac{1}{2}$  tons of coal cost  $\$37\frac{1}{2}$ , what will  $27\frac{1}{2}$  tons cost?

17. George lost  $\frac{3}{4}$  of his marbles, then bought  $\frac{1}{2}$  as many as he had at first. If he then had 84, how many had he at first?

18. Change  $\frac{1}{4}$  of a gallon to pints.

1. The surface of a lake is  $4\frac{1}{2}$  sq. mi. How many gallons of water will it take to raise the surface  $\frac{1}{2}$  in.?

2. How many cubes whose sides are 4 in. are equal in volume to a cube whose side is  $2\frac{1}{2}$  ft.?

3. If George has  $33\frac{1}{3}\%$  more marbles than his brother, what per cent less than George has his brother?

4. If by selling land at \$80 an acre I lose 25%, how must I sell it to gain 40%?

5. A man sold  $\frac{3}{8}$  of a barrel of flour for what  $\frac{1}{8}$  cost, and the rest of the barrel for what  $\frac{1}{2}$  cost. What was the gain or loss per cent on the whole barrel?

6. How many blocks  $7\frac{1}{2}$  in. by  $4\frac{1}{4}$  in. will it require to pave a lot 100 ft. by  $12\frac{3}{4}$  ft.?

7. If  $\frac{1}{11}$  of an acre of land cost \$37.75, what will  $43\frac{1}{2}$  acres cost?

8. What is the expense for carpeting a room 17 ft. 6 in. long, 14 ft. wide, with carpet  $\frac{1}{8}$  yd. wide, at \$1.25 a yard; breadths to run lengthwise?

9. The ratio is  $3\frac{1}{4}$  and the consequent 12. What is the antecedent?

10. A room is 20 ft. square and 10 ft. high. If each side of the room were 10 ft. longer, how much greater would the entire surface of the room be?

11. A merchant marked his goods at 75% above cost, and sold them at  $33\frac{1}{3}\%$  below his marked price, deducting 10% for cash. What per cent of profit did he make?

12. Divide two thousandths by eight millionths.

13. At 16¢ a square foot, what will it cost to paint the ceiling of a room 15 ft. 6 in. long, and 12 ft. 6 in. wide?

14. What is the acreage of a rectangular field whose length is 234 rd. and whose breadth is 165 rd.?

15. If three times a certain number is added to the number the sum will be 12. What is the number?

1. If the sod  $2\frac{1}{2}$  in. thick is removed from a field containing  $\frac{1}{2}$  of an acre, how many cubic yards are taken?
2. A garden whose breadth is 4 rd., and length  $2\frac{1}{2}$  times its breadth, has a wall 2 ft. thick and 3 ft. high around it, outside of the line. Find the cost of the wall at 5¢ a cubic foot (exact measurement).
3. Outside of the wall in example 2 is a ditch 3 ft. wide and 4 ft. deep. What did it cost to dig it at  $1\frac{1}{2}$ ¢ a cubic foot?
4. How many bricks will be required to build a house 36 ft. long, 28 ft. wide, and 20 ft. high? The wall is  $1\frac{1}{2}$  ft. thick, and has four doors 4 ft. by 8 ft., 32 windows, 3 ft. by 6 ft.
5. If it takes 3 days to dig a cellar that measures 8 ft. each way, how long will it take to dig one of the same depth, but the other dimensions  $1\frac{1}{2}$  times as large?
6. What is the area of a circular pond which contains 15 times as much area as one 25 rd. in diameter?
7. What is the value of a lot of land  $4\frac{1}{2}$  rd. long, 50 ft. wide in front, 42 ft. wide in the rear, at 65¢ a square foot?
8. How many board feet in 15 sticks of timber 27 ft. 9 in. long, and the other dimensions 8 in. and 10 in.?
9. A built a square house 40 ft. on each side. B built a house containing the same area, but 80 ft. long. The perimeter of A's house is what per cent of the perimeter of B's house?
10. A general placed 4800 men in three regiments so that the 2d regiment had twice as many as the 1st regiment, and the 3d regiment had as many as both the others. How many were placed in each regiment?
11. Three times, eight times, and four times a number is 360. What is the number?
12. In what time will \$165 amount to \$179.85 at 6%?
13. If 15% is lost by selling an estate for \$10,200, for what must it be sold to gain 20%?
14. Reduce 563,147 inches to feet, etc.

1. Find 20% of 50 men. 70% of 120 yr. 25% of 120 bu. 62½% of 64 days.

2. Find 50% of 600 yd. 8½% of 72 bu. 40% of 80 tons. 5% of \$40.

3. A farmer raised 4,200 bu. of grain, and sold 20% of it. How much did he sell?

4. A house was bought for \$4,400, and sold at a gain of 25%. Find the selling-price.

5. 40 is 25% of what number? 60 is 10% of what number? 80 is 40%? 12 is 16⅓%?

6. 64 is 20% of what number? 125 is 50%? 70 is 33⅓%? 30 is 12½%? 200 is 66⅔%?

7. A farmer owns 420 acres of land, which is 25% of what his neighbor owns.

8. What per cent of 75 is 15? Of 80 is 40? Of 80 is 32? Of 120 is 30? Of 72 is 27?

9. What per cent of 120 is 90? Of 400 is 160? Of 200 is 120? Of 64 is 40? Of 90 is 30?

10. What per cent of 63 is 27? Of 56 is 49? Of 48 is 6? Of 90 is 15? Of 160 is 10?

11. If you buy an article for \$210, and sell it for \$42 gain, what per cent do you gain?

12. A regiment entered battle with 960 men, and came out with 912 men; what per cent were missing?

13. A merchant buys lead-pencils at ½¢ each, and sells them at 3¢ each; what per cent does he gain?

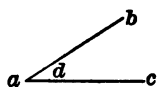
14. If a merchant buys goods for ¾ of his selling-price, what per cent does he gain?

15. A man is 72 years old, and 12½% of his age is 25% of his son's age. What is the son's age?

16. My house is insured for \$4,000 at 1½% premium. Find the premium.

17. How many inches in 25% of one foot?

1. Take the dividers, and open the points one inch.
2. Do the legs of the dividers now point in the same direction or in different directions?
3. Draw lines to represent the legs of the dividers.
4. These lines are said to make an angle.
5. What is an angle? An angle is the difference in direction of two lines.



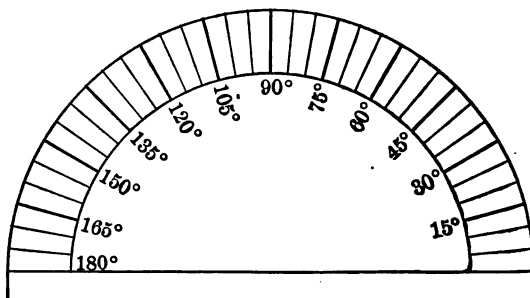
6. In this figure the lines  $ab$  and  $ac$  are called the sides of the angle. The point where these lines meet, as at  $a$ , is the vertex.
7. Define vertex.
8. The angle in 6 is named by reading the angle  $d$ , or the angle  $c a b$ .
9. Draw two lines perpendicular to each other. The angle you have formed is called a right angle.
10. Define a right angle.
11. Draw two lines that shall meet but not be perpendicular to each other. The angle formed is an oblique angle.
12. Define an oblique angle.
13. Is the angle you have formed less than a right angle? If so, it is an acute angle.
14. Define an acute angle.
15. Is the angle you have formed greater than a right angle? If so, it is an obtuse angle.
16. Define an obtuse angle.
17. At 2 o'clock what angle do the hands of a watch make?
18. At 9 o'clock what angle do the hands make?
19. What kind of an angle does a vertical line make with a horizontal one?
20. Starting at 12 o'clock, in what time will the hands of a clock be at right angles to each other?
21. What kind of angles do you find on a cube? On a square prism? On a triangular prism?



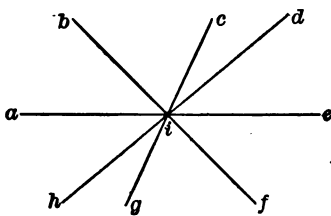
1. For convenience, every circle is supposed to be divided into 360 equal parts called degrees.

2. By means of an instrument called a protractor any angle can be measured.

3. From cardboard cut out a figure like this, and mark it in the same way. This is a protractor sufficiently accurate for all practical purposes.



4. Draw a circle and two diameters at right angles to each other. If there are 360 degrees in a circle, how many are there in  $\frac{1}{4}$  of a circle? In  $\frac{1}{2}$  of it?



5. Using your protractor, measure each of the angles in this figure. What is their sum?

6. In the wheel of my carriage there are 12 spokes. How many degrees between two spokes? Between the first and fifth?

7. Between the first and eighth? The third and seventh?

8. When the hour hand of the clock is at 12, where must the minute hand be that the two hands may be  $30^\circ$  apart?  $15^\circ$ ?  $150^\circ$ ?  $75^\circ$ ?  $45^\circ$ ?  $60^\circ$ ?  $120^\circ$ ?  $90^\circ$ ?  $180^\circ$ ?

9. Using your protractor make an angle of  $85^\circ$ .

1. Draw several angles, and measure them by using a protractor. First estimate their size before measuring.
2. Using your protractor, make an angle of  $45^\circ$ ;  $60^\circ$ ;  $90^\circ$ ;  $30^\circ$ ;  $150^\circ$ ;  $135^\circ$ ;  $100^\circ$ ;  $40^\circ$ ;  $20^\circ$ .
3. Draw several angles, and by means of the protractor make other angles equal to each of them.
4. Draw an angle. Using protractor, make another angle twice the size of the given angle. Do the same, using dividers.
5. Draw an angle. Make another angle of  $\frac{1}{4}$  the size; of four times the size.
6. Draw a horizontal line,  $AB$ . At a point  $C$  in the line, draw an oblique line that shall not cut  $AB$ . Measure the angles. Add their result. What is the sum?
7. Do the same after drawing one vertical and two oblique lines.
8. Draw three lines cutting each other at one point. How many angles are formed? Measure them. Add results. What is the sum? What ought it to be?
9. Draw a four-sided figure. Estimate the angles of the figure. Record your estimate in one column; the true value, found with a protractor, in another; and the error in a third column.
10. Substituting a five-sided figure, repeat 9.
11. Substituting a six-sided figure, repeat 9.
12. Using a protractor, erect a perpendicular at each extremity of a horizontal line; of an oblique line; of a vertical line.
13. Draw a vertical line. Using protractor and ruler, construct a square upon this line.
14. How many degrees are passed over by the hour hand of a clock in an hour? in 3 hours? In how many hours will the hour hand pass over  $90^\circ$ ?
15. Make any angle. Using your protractor, make an angle three times as large.

NOTE. — A study of imports should precede this topic.

1. Have you ever been in Washington, D.C.?
2. What can be seen there?
3. Name some of the persons engaged in making and executing our laws.
4. Who pays these men for their work?
5. Where does the Government get all its money for all expenses?

Ans. I. From an Internal Revenue, — a tax on the right to make or sell liquors, tobacco, etc. II. From Customs or Duties, — taxes on imported goods.

6. What are imported goods?
7. What goods do we import?
8. Make a list of the more important things that are imported, and the name of the country from which they are imported.

9. Who determines the amount of Duties to be paid?

Ans. The Government, by an Act of Congress, usually called, "The Tariff Act." This is a list of goods on which duties must be paid, with the rate of duty assessed on each.

10. What is Tariff?
11. Where are these duties collected?

Ans. At Custom Houses, — buildings owned by the United States, where the Collector and other officers do business.

12. The Government designates certain places called Ports of Entry, where Custom Houses are built.

13. What are Ports of Entry?

14. Are Ports of Entry ever found except on the sea-coast? Why?

15. What is smuggling?

16. Duties are either specific or *ad valorem*. Specific duties are based on the number, quantity, weight of the merchandise. *Ad Valorem* duties are based on the value of the merchandise.

17. What are Specific Duties?
18. What are *Ad Valorem* duties?

1. Importers are required to submit to the collector an invoice. This is a description of the goods, and their cost in the country from which they are imported.

2. What is an invoice?

3. Allowances, called Tare, Leakage, and Breakage, are deducted before estimating duties.

Tare is a deduction from the gross weight because of the weight of the box, etc. Leakage is an allowance made on liquids in casks or barrels. Breakage is an allowance made on liquids in bottles.

4. Invoices are made out in the money of the country from which the goods are imported. When changed to United States money *the duty is computed on the nearest dollar.*

5. What is the duty at 35% on 75 pieces of satin, each piece containing 47 yd. at \$1.65 a yard?

6. What is the duty on 20 casks of wine containing 40 gal. each, invoiced at \$1.12½ a gallon, at 28%, leakage, 5%?

7. A grocer imports 360 bags of cocoa, gross weight 145 lb. each, tare 3½%, invoiced at 13¢ a pound. What was the duty at 2½¢ a pound?

8. What is the duty upon merchandise invoiced at 120 *lira*, allowing 7½% for breakage; rate of duty, 27%? A *lira* is equal to \$.193.

9. If the rate of duty is 50%, and tare 2%, find the duty on merchandise invoiced at 4,670 *guilders*. A *guilder* is equal to \$.402.

10. Find the duty on 1,150 gal. of brandy, leakage 2%, rate of duty, \$1.75 a gallon.

11. At 35% *ad valorem*, what is the duty on 150 doz. pairs of kid gloves invoiced at 68 *francs* a dozen? A *franc* is equal to \$.193.

12. What is the duty at 2½¢ a pound on 500 sacks of cocoa, each containing 85 lb., the tare being 1½% and 600 sacks each containing 75 pounds?

1. What is the duty at 30¢ a dozen, and 15% *ad valorem*, on 750 doz. linen collars valued at 88½¢ a dozen?

2. If velvet cloaks cost 600 *francs* each in Paris, and the duty is 50%, what will be the duty on a dozen cloaks?

3. By the last Tariff Act the duty on varnish is 35%, and \$1.32 a gal. Find the duty on 6 bbl. of varnish, 31½ gal. to a barrel, invoiced at \$6 a gallon, leakage 10%.

4. If the duty on Brussels carpets is 44¢ a square yard, and 40% *ad valorem*, find the total cost to me of 300 yd., ¾ yd. wide, invoiced at 6s. a yard. A shilling is \$.243.

5. The duty on an importation of lace at 60% was \$624. How many yards were there if the lace was invoiced at 80¢ a yard?

6. What is the duty at 20% on 25 oil-paintings, averaging \$1,375, and on 15 pieces of statuary averaging \$978 each?

7. What is the duty at 2¢ a pound on 375 boxes of figs, weighing 138 lb. each, tare 16 lb. on each box?

8. What is the duty on 12 casks of molasses, 63 gal. each, at 3¢ a gallon, leakage 15 gal.?

9. The duty on plate glass is 35¢ a square foot. Find the duty on 416 plates, each plate 9½ ft. by 12½ ft.

10. Find the duty at \$.65 a cubic foot on a block of Italian marble 2½ by 3½ by 8 ft., invoiced at 3,450 *lira*.

11. Merchandise invoiced at 7,689 *florins* pays a duty of 35%. A *florin* is equal to \$.359.

12. At 44¢ a square yd., and 40% *ad valorem*, find the duty on 2,468 yd. Brussels carpet 27 in. wide, invoiced at 3½ shillings.

13. A merchant imported 26 hhd. of sugar, invoiced at 1,045 lb. each. What will be the duty, tare being 12½%, and the rate 2¢ a pound?

14. What is the duty on 40 bales of peanuts invoiced at 80 lb. each, and 25 bales invoiced at 70 lb each, tare being 6%, and the duty ½¢ a pound?

1. A butcher bought  $7\frac{1}{2}$  doz. turkeys for \$108, and found 20% of them spoiled. How must each one of the remainder be sold to gain  $33\frac{1}{3}\%$  on the lot?

NOTE. — Most merchants choose some word of 10 letters as their private key, thus: *p r e c a u t i o n*. Now, if a merchant wishes to mark an article 25%, he uses the letters *r a*.

2. A merchant's private key for marking goods is "abridgment." If he buys goods at *bd* a yard, how must he mark them so as to gain 20%?

3. Using the same key, how must shoes that cost \$*b.dt* be marked to gain 50%?

4. What is the duty on 224 chests of tea, each weighing 67 lb., tare being 4 lb. to a chest, at \$1.50 a hundred weight?

5. \$475 *Holyoke, April 1, 1903.*

*Three months after date, we jointly and severally promise to pay J. A. Dickinson, or order, Four Hundred seventy-five Dollars, with interest. Value received.*

*John French.*

*James Fiske.*

Is this a negotiable note? What is the meaning of jointly and severally? Find the interest due when the note is legally due.

6. A grocer borrowed \$400 at 6% interest, and bought flour at \$4 a barrel. He kept the flour 1 yr. 3 mo., when he sold it all at an advance of 25%. After paying his note, how much had he gained?

7. What will it cost to build a half mile of road at \$4.75 a rod?

8. How many screws in  $5\frac{1}{2}$  gross and  $4\frac{3}{4}$  doz.?

9. What will be the cost of 8 bu. 3 pk. 6 qt. of nuts at \$3.20 a bushel?

10. How many feet in  $7\frac{1}{2}$  rd.,  $11\frac{1}{2}$  yd.?

(Review page 117.)

1. If one part of 12 is 7, what is the other part?
  2. If one part of 15 is  $x$ , what is the other part?
  3. If one factor of 25 is 5, what is the other factor?
  4. If one factor of 18 is  $x$ , what is the other factor?
  5. If a pear costs  $2x$  cents, and a peach  $3x$  cents, what will represent the cost of both?
  6. What will 8 yards of cloth cost at  $2x$  cents a yard?
  7. Draw a rectangle. Call the length  $x$  in. and the width  $y$  in. Express the perimeter of the rectangle. Express one-half of the perimeter. Express the difference between the length and the breadth.
  8. Express the area of the rectangle;  $\frac{1}{2}$  of the area.
  9. Express the square of  $x$ ; the cube of  $y$ ; the fourth power of  $a$ .
  10. If  $x$  and  $y$  represent two numbers of which  $x$  is the greater, what will represent their sum? their difference? their product? their quotient?
  11. The expressions  $6 + 4 = 10$ , or  $6a + 4a = 10a$  are called equations. The parts at the left and right of the sign  $=$  are called members, or sides. They are distinguished as first member, or left side, and second member, or right side.
  12. In the equation  $6 + 4 = 10$ , add 2 to the first side. What must you do to the other side to preserve the equation?
  13. Learn: If anything is added to one side of an equation, an equal amount must be added to the other side.
- NOTE. — A self-evident statement like the above is called an Axiom.
14. If  $3x = 9$ , what does  $3x + 5$  equal?
  15. If  $x - 4 = 8$ , what does  $x$  equal.
  16. If  $x = 2$ , how can you change the equation so that the right side shall be 6?

NOTE. — The last three examples are illustrations of Axiom 1. Prove it.

(Review pages 117, 138.)

1. In the equation  $6 + 4 = 10$ , subtract 4 from the left side. What must you do to the other side to preserve the equation?

AXIOM 2. — If anything is subtracted from one side of an equation, an equal amount must be subtracted from the other.

2. If  $x = 7$ , what will  $x - 3$  equal?

3. If  $x + 4 = 6$ , what does  $x$  equal?

4. If  $x = 9$ , how can you change the equation so that the right side shall be 5?

5. In the equation  $6 = 6$ , multiply the left side by 3. What must you do to the right side to preserve the equation?

6. Write the statement as Axiom 3.

7. If  $x = 4$ , what will  $7x$  equal?

8. If  $\frac{4x}{5} = 20$ , what does  $4x$  equal?

9. In the equation  $6 = 6$ , divide the left side by 2. What must you do to the right side to preserve the equation?

10. Write the statement as Axiom 4.

11. If  $4x = 12$ , what does  $x$  equal?

Express in the form of equations the following statements:

12.  $a$  is equal to  $b$  added to  $c$ .

13. 25 exceeds 19 by 6.

14. The excess of 17 over 8 is 9.

15. The excess of  $x$  over  $y$  is  $z$ .

16. Write three times the expression three plus four.

17. Write two times the expression  $x$  minus  $y$ .

18. What number is less than 16 by 4?

19. Learn: We may add the same number to both members of an equation; subtract the same number from both members; multiply both members by the same number, or divide both members by the same number, and not affect the value of the equation.



1. How many cubic feet in a rectangular block 2 ft. square at the end and 6 ft. long?
2. How many times larger would a block be that was twice as long, twice as wide, and twice as thick?
3. How many blocks  $\frac{1}{4}$  of an inch on a side can be sawed from a 2-in. cube?
4. When a number is used twice as a factor, or multiplied by itself, the product is called the square of a number.
5. Name the squares of the numbers from 1 to 10.
6. When the number is used three times as a factor, the product is called the cube of the number.
7. Name the cubes of the numbers from 1 to 10.
8. When 100 shares of bank-stock are sold for \$17,650, what is the price per share?
9. Divide 24,584 by 10; by 100; by 1,000.
10. Divide 16,485 by 10,000; by 1,000; by 100; by 10.
11. At \$7.50 each how much will 100 trunks cost?
12. At \$2.50 each, how many chairs can be bought for \$50?
13. Separate \$20 into 5 equal piles. What is the answer? Since you are asked to find  $\frac{1}{5}$  part of \$20, examples like this are called Partitive Division.
14. Separate \$20 in piles of \$4 each. What is the answer? Since here the size of the pile is given, and we are asked to measure the larger pile by it to find the number of piles, this is called Measuring Division.
15. Name the terms used in Division.
16. In the partitive form of division, which of these terms are alike?
17. In the measuring form of division which of the terms are alike?
18. Make an example to illustrate Partitive Division.
19. Make an example to illustrate Measuring Division.
20. Define quotient. Divisor. Dividend.

1. A man sold some property at a profit of 20%, and with the proceeds bought some more property, which he sold for \$4,860, at a loss of 55%.

2. A man sold a bicycle for \$93.10, thereby losing 5%.

3. A teacher's salary after being increased 20% was \$3,000.

4. I received \$1,642.60 for some property which my agent sold for \$1,720.

5. As agent, working on  $4\frac{1}{2}\%$  commission, I received \$129.51.

6. I bought a barrel of flour for \$5.94. The dealer gained 44 cents.

7. I sold a house for \$638 less than I paid for it. I sold it for  $92\frac{3}{4}\%$  of its cost.

8. I sold goods on 2% commission, and remitted to my employer \$4,777.50.

9. A man bought 20 sheep for \$250, and 20 cows for \$1,050. He sold the sheep at a gain of  $12\frac{1}{2}\%$ , and the oxen at a gain of  $7\frac{1}{2}\%$ .

10. A man left by will \$4,500 to his wife, and the remainder of his property to be equally divided among his three children. It was found that the share of each child was one-fifth of the whole property.

11. A bill of goods amounted to \$5,650. It was offered for sale with 20% and 5% discount and 10% off for cash. The offer was accepted.

12. On a bill of \$600 a dealer offered me a discount of  $33\frac{1}{3}\%$ .

13. A lawn-tennis court is 78 ft. long and 27 ft. wide. At each end there is a margin of grass 12 ft. wide, and at each side a margin 6 ft. wide. It cost 50¢ a square yard to turf the margin, and 85¢ a square yard to gravel the court.

14. A rectangle 150 ft. by 120 ft. has in the center a rectangular grass-plot 80 ft. by 60 ft. Cover the rest with gravel 8 in. deep at a cost of 62¢ a cubic yard.

1. How many cubic inches in a sugar-loaf in the form of a cone, the diameter of the base being 8 in., and the height 18 in.?

2. How many cubic feet in a cylinder 60 ft. long, and 8 ft. in diameter?

3. Find the contents of a pyramid whose base is 9 ft. square, and whose altitude is 79 ft.

4. How many board feet in 15 2-in. planks 12 ft. long, 18 in. wide at one end, and 12 in. wide at the other end?

5. How many acres in a field in the form of a triangle whose base is 965 rd., and altitude 576 rd.?

6. A man placed \$4,200 insurance on his house, \$2,400 on his furniture, and \$700 on his library, for 3 yr., paying a premium of \$109.50. What was the rate per annum?

7. I paid \$50.12½ as premium for insuring my house at 2½%. What was the value of the house?

8. A store valued at \$10,000, and a stock of goods valued at \$15,000, were insured for 75% of their value at ¾%. If there were a total loss by fire, what would be the owner's loss? Company's loss?

9. A man bought a horse, carriage, and harness for \$240. He gave three times as much for the carriage as for the harness, and as much for the horse as he did for both the carriage and the harness. How much did he give for each?

10. A farmer, when asked how many cows he had, replied that if he had twice as many more he would have 60. How many had he?

11. If the interest on \$960 at 5% is \$54.40, what is the time?

12. A gave his note Aug. 6, 1902, for \$670, interest at 7%. He paid the note and interest May 17, 1903. How much did he pay?

13. Find the interest on \$9 for 9 yr. 9 mo. 9 da. at 9%.

1. The product of three numbers is 2,090; one is  $28\frac{1}{2}$  and another 22. What is the third number?
2. Find the sum, difference, product, and quotient of  $\frac{1}{3}$ ,  $\frac{4}{5}$ , using the larger fraction as minuend and dividend.
3. Divide the least common multiple by the greatest common divisor of 18, 48, 72, 66.
4. If  $61\frac{1}{2}$  lb. of tea cost  $\$55\frac{7}{10}$ , what is the cost of 217  $\frac{1}{10}$  lb.
5. If an agent receives  $2\frac{1}{2}\%$  for collecting, and is paid \$739, how much will his employer receive?
6. A sold some goods to B at a profit of 10%. B sold them to C at a profit of 10%. C sold them to me at a profit of 20%. Now, if I paid \$1,452, what did they cost A? Had A sold directly to me at the price I paid C, what per cent would he have made?
7. If a broker sells goods that cost \$4,800 at a profit of 6%, and charges 5% on the amount received as commission, how much does the owner of the goods receive as profit?
8. How many cubic yards of earth will be thrown out in digging a ditch 3 ft. deep and 4 ft. wide just within the boundary of a rectangular field 12 rd. long and 8 rd. wide?
9. Express .035, .625, .12288 as common fractions in their lowest terms.
10. If a piece of cloth contains 246 yd. 1 ft. 6 in., how many times can you cut from it 14 yd. 1 ft. 6 in.?
11. Reduce  $\frac{1}{3}$ ,  $1\frac{2}{3}$ , and  $\frac{4\frac{1}{2}}{3\frac{1}{2}}$  to decimals.
12. If 7 men build  $6\frac{1}{2}$  rd. of wall in  $15\frac{1}{2}$  days, in how many days can 12 men do as much?
13. A father is 30 years older than his son. The sum of their ages is 42 years. How old is each?
14. If eight times a number exceeds five by as much as 17 exceeds three times the number, what is the number?
15. A horse and cart cost \$115. The horse cost \$5 less than twice the cost of a cart. Find the cost of each.

1. Mention several things for which a city or town needs money.

2. Where does the money for schools, roads, sewers, lights, police, etc., come from? *Ans.* From taxes.

3. What is a tax? *Ans.* A tax is a sum of money assessed on persons and owners of property to meet the expenses of a town, city, county, or State.

4. Find out what need a county has for money.

5. Find out what need a State has for money.

6. Poll means head. What, then, is a poll-tax?

7. Who pays a poll-tax?

8. Does a man need to own any property in order to be assessed a poll-tax?

9. How many kinds of property are there? *Ans.* *Real Estate* and *Personal Property*.

10. *Real estate* consists of lands, houses, or, in general, immovable property, and is taxed in the place where it is situated.

11. *Personal property* consists of horses, money, merchandise, etc., or, in general, movable property, and is taxed in the place where the owner lives.

12. Any tax upon real estate or personal property is called a *property tax*.

13. *Assessors* are persons appointed to make an inventory of all taxable property and estimate its value.

14. How large a poll-tax is assessed in your town or city?

15. How large a tax on property is assessed in your town or city?

Taxes are usually assessed and collected as follows:

16. The State determines the amount to be expended for State purposes, and divides that amount among the counties according to their valuation, previously determined.

17. The county adds to this sum the amount it will need for county expenses, and divides the total among the towns of the county according to their valuation.

1. Each town adds to this amount whatever it needs for schools, police, roads, salaries, etc., and thus finds its total tax, or *tax levy*.

2. The assessors now find the number who must pay a poll-tax, and multiply this by the tax on one poll. This is the *poll-tax*, and is subtracted from the whole tax. The amount of the tax that is left must be assessed on property, and is called the *Property Tax*.

3. The assessors now find the entire valuation of the town by adding all the Real Estate and Personal Property. This is called the *Total Valuation*.

4. If the tax that is to be raised on the property is now divided by the property, what will the result be? This tax on \$1 is called the *tax-rate*.

5. The assessors had to find the amount of property each man owned when they found the valuation of the town. If the sum of money that each man owns is multiplied by the tax-rate, what will be the result?

6. In some States the county and State tax are found separately, and are not united with the town tax.

7. Write the different steps taken in computing taxes, and tell the reason for each step.

8. Do you see the reason why all persons are not taxed the same amount?

9. Sometimes there is assessed an Income tax; that is, a tax on a man's income.

10. Sometimes, to provide for abatements and uncollected taxes, the property tax is increased by a small percentage.

11. There are 4,120 persons, each of whom pays \$1.50 in the city of B. The total valuation is \$5,864,528, and the total tax \$94,147.92. What must Mr. Philips pay, who owns a mill valued at \$4,500, and who owns personal property valued at \$1,240?

1. The property valuation of a town is \$1,500,000, and the tax levy is \$12,500. There are 250 male adults, each paying \$2. Mr. Dunbar's real estate is valued at \$7,500, and his personal property at \$4,500. Find his tax.

2. In the town of C there are 2,500 polls, each taxed \$2. The tax levy is \$245,000, and the taxable property \$12,000,000. Find Mr. J.'s tax on property worth \$125,000.

3. A tax of \$50,000 is levied in a town, valued at \$3,200,000. There are 1,000 persons who pay a poll-tax of \$2. What is my tax if my property is valued at \$9,000?

4. In the city of H there are 1,200 male adults, each poll taxed \$1. The taxable property is \$30,000,000, and the tax levy \$151,200. What tax does Mr. Sims pay, who is assessed \$12,000 for real estate, and \$2,500 for personal property?

5. The real estate of a city is \$3,099,500; personal property, \$1,487,280; tax levy, \$66,023.92. 1,340 polls are each assessed \$1.35. What does Mr. A pay, whose property is valued at \$18,000?

NOTE. — In all these examples a poll-tax is included in finding the whole amount of a man's tax.

6. What is the rate of taxation in a city when \$125 is the tax on a house assessed at \$6,000?

7. How much tax will a man pay on \$6,250, if the rates are  $1\frac{1}{8}\%$  for a city tax, and  $\frac{3}{8}\%$  for State and county tax?

8. If the tax-rate is 21 mills on a dollar, what is the assessed value of a property that pays \$62,622 tax?

9. The valuation of the town of H is \$2,432,500, and the tax-levy is \$48,650. Mr. Smith owns \$150,000 worth of property, assessed at  $\frac{3}{4}$  of its value. What is his whole tax?

10. If a tax-rate of  $2\frac{1}{4}\%$  on a dollar produces \$130,000, what is the assessed valuation of the property?

11. What is the rate of taxation when \$2,285.10 is the tax upon \$152,340?

1. Find the area of a walk, 9 ft. wide, round the outside of a park that is 9 rd. square.

2. What is the duty, at 15%, on 640 yd. of silk invoiced at \$2.25 a yard?

3. What is the duty, at 12¢ a cwt., on 36 bags of salt, each containing 115 lb., tare 2 lb. a bag?

4. How much tax will a person pay whose property is assessed \$246,500, if he pays  $1\frac{3}{4}\%$  city tax, 1% school tax,  $\frac{3}{4}\%$  county tax?

5. Real estate of a town, \$1,105,843; personal property, \$249,031; tax to be raised, \$21,258.11. There are 850 polls, each assessed \$1.10. What tax will Mr. Martin pay if his property in the town is assessed for \$8,650, and he, not being a resident of the town, pays no poll-tax?

6. An insurance company took a risk of \$375,000 at  $\frac{1}{2}\%$ . It reinsured \$175,000 in another company at 1%. How much premium did it clear above what it paid?

7. When the rate of insurance is  $3\frac{1}{2}\%$  and the premium \$14.63, what is the value of the property insured?

8. The premium for insuring a house at  $\frac{3}{4}$  of its value was \$54.60. If the rate was  $1\frac{1}{2}\%$ , for what amount was the house insured?

9. A note for \$1,460 was given, to be paid in 1 yr. 7 mo. 21 da., with interest at 6%. What was the amount due at the expiration of the time?

10. What number increased by 22% of itself is 23,546?

11. A horse and carriage were sold at a profit of 20%, which was a gain of \$100. The horse cost 50% more than the carriage. What was the cost of each?

12. I sold 75 bu. of wheat at \$1.05 a bushel. This was 5% more than I paid for it. How much did I gain?

13. What part of 6 rd. is 14 yd. 2 ft.?

14. Write a promissory demand note.



1. Find the area of a parallelogram 15 in. by 9 in.
  2. Find the area of a rhomboid whose base is 42 ft. and whose altitude is 16 ft.
  3. Find the area of a trapezoid whose parallel sides are 14 ft. and 10 feet, and whose altitude is 12 ft.
  4. Find the cost of gilding a ball 50 in. in diameter at 5¢ a square inch.
  5. Find the volume of a cone if its height is 18 in. and the diameter of its base 20 in.
  6. A bin 24 ft. long, 6 ft. wide, and 2 ft. deep is  $62\frac{1}{2}\%$  full of oats. How many bushels are in the bin?
  7. Find the volume of a cone whose base contains 57 sq. in., and whose height is 28 in.
  8. Find the number of bushels of grain required to fill a bin  $14\frac{1}{2}' \times 8\frac{1}{2}' \times 4'$ .
- NORM. — The signs ' and " are sometimes used in place of feet and inches.
9. Find the number of gallons of water in a well  $4\frac{1}{2}$  ft. in diameter if the water is 9 ft. deep.
  10. How many feet board measure in 20 beams  $8'' \times 10''$  and 24' long?
  11. How many feet board measure in 48 boards  $\frac{7}{8}$  in. thick, 4 in. wide, and 15 ft. long?
  12. What is the height of a pile of wood containing 50 cords, if it is 400 ft. long and 4 ft. wide?
  13. A canal was dug 300 ft. long, 15 ft. wide, and 10 feet deep. How many cubic yards of earth were removed?
  14. Estimate the number of bricks required for the walls of a building  $80' \times 50' \times 22'$ , if the walls are  $1\frac{1}{2}$  ft. thick, and if 500 cu. ft. are allowed for doors and windows.
  15. The rafters of a house are 21 ft. in length. If the house is 36 ft. long, how many shingles, laid 4 in. to the weather, will be needed?
  16. Find the area of a rectangle 24 ft. by 36 ft.

1. Reduce to whole or mixed numbers:  $\frac{249}{100}$ ;  $\frac{148}{88}$ ;  $\frac{128}{13}$ ;  
 $\frac{441}{17}$ .
2. Reduce to improper fractions:  $47\frac{3}{4}$ ;  $96\frac{1}{3}$ ;  $18\frac{7}{9}$ ;  $41\frac{17}{100}$ .
3. Find product of:  $\frac{1}{9} \times 549$ ;  $\frac{3}{5} \times 108$ ;  $\frac{1}{3} \times 384$ ;  
 $\frac{4}{25} \times 500$ .
4. Find product of;  $\frac{1}{3} \times \frac{3}{4}$ ;  $\frac{2}{3}$  of  $\frac{1}{5}$  of  $\frac{3}{4}$ ;  $22 \times 6\frac{1}{11}$ .
5. Find the value of:  $\frac{2}{3}$  of  $3\frac{1}{2}$  of  $\frac{2}{5} \div 5\frac{1}{2}$ .
6.  $97\frac{1}{2}$  is  $\frac{1}{2}$  of what number?
7.  $89\frac{3}{4}$  is  $\frac{1}{2}$  of what number?
8. 99 is  $\frac{1}{2}$  more than what number?
9. If  $\frac{1}{10}$  of a cord of wood is worth \$5, find the cost of  
 $9\frac{3}{4}$  cd.
10. Find the sum of  $\frac{1}{15}$ ,  $\frac{1}{10}$ ,  $\frac{2}{3}$ ,  $\frac{1}{5}$ .
11. Find the sum of  $26\frac{1}{3}$ ,  $37\frac{2}{10}$ ,  $16\frac{1}{2}$ ,  $48\frac{3}{4}$ .
12. From  $41\frac{3}{8}$  take  $11\frac{1}{8}$ .
13. What number must be added to  $18\frac{1}{2}$  to make  $24\frac{1}{2}$ ?
14. A can do a piece of work in 15 days, B in 18 days, and  
 C in 12 days. How many days will it take the three together  
 to do the work?
15. If a man walks  $3\frac{3}{4}$  miles an hour, how many hours will it  
 take him to walk  $187\frac{1}{2}$  miles?
16. A man raised  $187\frac{1}{2}$  bu. of barley on  $7\frac{1}{2}$  acres of land.  
 How many bushels an acre did he raise?
17. From 120 acres of land,  $44\frac{1}{2}$  A. were sold to one man, and  
 $\frac{1}{3}$  of the remainder to another. Find the value of the unsold  
 acres at \$65 an acre.
18.  $\frac{4}{5}$  of  $\frac{3}{4}$  of the value of a pair of horses is \$210. What is  
 the value of the pair?
19. A can do a piece of work in 6 days, B in 8 days, and C  
 in 10 days. How much of it can they do in 2 days, working  
 together?
20. At  $16\frac{1}{2}$ ¢ a yard, what will  $415\frac{3}{4}$  yd. of cloth cost?
21. Divide  $12\frac{1}{2} \times 16\frac{3}{4}$  by  $6\frac{3}{4} \times 5\frac{1}{2}$ .

1. What is the result of an example in subtraction called?
2. (a) A man had \$25, and lost \$10. (b) You have \$25, and I have \$10. In (a) is the answer a difference or remainder? In (b)?

3. What kind of numbers can be subtracted?

4. When numbers are unlike, what must be done before subtracting?

5. When fractions are unlike, what must be done before subtracting?

6. What are the terms in subtraction called?

In the following four examples, give the answer in terms of both the minuend and subtrahend:

7. 3 lb. — 32 oz.      4 hr. — 180 min.      4 yr. — 36 mo.
8. 5 yd. — 12 ft.      5 ft. — 48 in.      9 ft. — 2 yd.
9. \$12 — 350¢.      2 T. — 2000 lb.      5 dimes — 20 ct.
10. 4 bu. — 8 pk.      6 pk. — 24 qt.      10 qt. — 8 pt.

11. Find the missing term:

Minuend,	45	$x$	43	53	$x$	61	$x$	95	74
Subtrahend,	$x$	36	27	$x$	34	29	64	$x$	39
Remainder,	24	15	$x$	11	42	$x$	19	32	$x$

12. In the morning I had \$2, during the day I spent 35 cents, a half-dollar, a quarter, and a dime. How much money had I at night?

13. What day of what month is the 75th day of every year not a leap year? What change does leap year make?

14. What number is 12 less than  $76 - 38$ ?

15. What kind of numbers can be compared?

16. What are the terms of a ratio called?

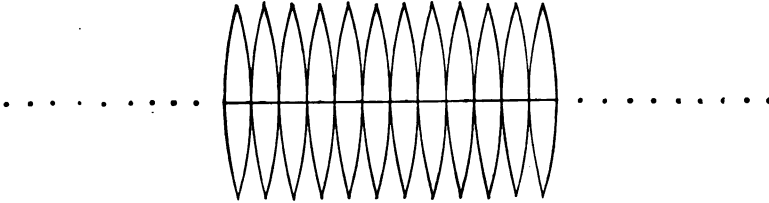
17. When do you have a fraction for your ratio? When an integer?

18. What effect on an integer is made by annexing a cipher? By dropping a cipher?

19. Name the terms used in multiplication.

1. Find the interest of \$1,378 for 3 yr. 8 mo. at 5%.
2. Find the interest of \$426.25 for 20 yr. 9 mo. 24 da. at  $6\frac{1}{2}\%$ .
3. Find the interest of \$417.16 for 4 yr. 11 mo. 17 da. at 4%.
4. A has \$16,000, and loans  $12\frac{1}{2}\%$  of it to B for 2 yr. 3 mo. 7 da. at  $4\frac{1}{2}\%$ . How much does B owe him at the expiration of the time?
5. Find the interest of \$900 for 93 days at 7 %.
6. A man bought 43 cows at \$38.50 each, and paid half cash and the remainder in a note for 2 yr. 4 mo. at  $2\frac{1}{2}\%$ . What was due on the note at maturity?
7. Find the amount of a note for \$916.84, given Aug. 16, 1902, at  $6\frac{1}{2}\%$ , and due Mar. 11, 1905.
8. A man bought a horse Oct. 3, 1902, for \$186, and gave in payment his note at 7%. On June 13, 1903, he sold the horse for \$192.50, and paid his note in full. How much did he gain or lose?
9. What principal on interest at 6% will gain \$105 interest in 3 yr. 6 mo.?
10. Interest \$21.69 $\frac{2}{3}$ , time 2 yr. 4 mo. 9 da., rate 5%. Find the principal.
11. At what rate will \$3,470 give \$1,408.82 interest in 6 yr. 9 mo. 6 da.?
12. A man gave his note Jan. 11, 1903, for \$6,000 at 5%. Sometime afterward he canceled the note by paying \$9,326.66. What was the date of cancellation?
13. Write a note supplying all the data.
14. John James paid Amos White, of Salem, Mass., Sept. 1, 1902, \$112.24, amount due for meat. Write the receipt in full.
15. I loaned a friend \$8,000 June 16, 1903. If money is worth  $6\frac{1}{2}\%$ , how much is due me Sept. 1, 1906?
16. Find the interest of \$267 for 2 yr. 10 mo. at 4%.

1. What is 138% of 64.6?
2. 210 is  $16\frac{2}{3}\%$  more than what number?
3. \$174.72 is 28% of what sum?
4. A merchant dropped \$4.65 from his price on a chamber-set by taking off 15% from his price. What was the price asked?
5. Find the net amount of a bill of \$642 with  $\frac{1}{3}$  and 10% off.
6. What is paid for an article listed at \$2.40, and sold at 25% and 10% off?
7. A man offered for sale a piece of land that cost him \$660 so as to gain 40%. He discounted his price  $16\frac{2}{3}\%$ . How much did he receive for it?
8. A man built two houses at a cost of \$2,785 each. He sold one at a gain of 16%, and the other at a loss of 5%. What was his gain?
9. A commission merchant sold 7,500 bu. of wheat at 65¢, charging  $2\frac{1}{2}\%$  commission. How much money should he remit to his principal or employer?
10. Find the cost of insuring a stock valued at \$4,685, risk being taken at  $1\frac{1}{2}\%$  on  $\frac{2}{3}$  of its value.
11. What would be the tax on \$1 if \$5,838 must be raised on \$486,500?
12. Find the duty at 20% on 348 doz. of olives, invoiced in Italy at 4 *liras* per dozen. (*Lira* = \$.193.)
13. Find the interest on \$476.50 for 3 yr. 7 mo. 4 da. at 8%.
14. Find the amount of \$726.50 for 3 yr. 4 mo. at 4%, compound interest.
15. An agent paid \$58.50, at 2%, for insuring 1,000 bbl. of flour. If the flour was insured for 75% of its cost, what was the cost?
16. An agent sells buggies for \$76 each. If the rate of his commission is 15%, how many must he sell to earn \$1,026?



**1ST ILLUSTRATION.** — Draw a line 6 in. long, and divide it into 12 equal parts. Make 9 dots at each extremity of the line. Take the compass, and with a radius of five in. draw arcs intersecting as in the figure. Cut out, and fold into a sphere. This is not quite exact.

**2D ILLUSTRATION.** — Take a wooden hemisphere, and drive a tack in the center of its curved surface. Commencing at the tack, carefully wind a cord about the curved surface, as a boy winds a top. When the surface is covered, cut the cord. Drive a tack into the center of the base of the hemisphere, and wind the cord tightly about the tack. When the surface is covered, just  $\frac{1}{2}$  of the string will have been used. This proves that the hemisphere is two times greater than the circle in surface. Hence the whole surface of the sphere is four times greater than the circle. How do the circumference and diameter of the circle compare with those of the sphere? How do you find the area of the circle? By what would you multiply this to get the area of the sphere?

If you multiply  $\frac{1}{2}$  the circumference by  $\frac{1}{2}$  of the diameter, and then multiply by 4, you will find the area of the sphere. But since  $\frac{1}{2}$  times  $\frac{1}{2}$  times 4 equals 1, we see that the area of the sphere is found by multiplying the circumference by the diameter.

**3D ILLUSTRATION.** — Place a sphere on the table. Holding it between the palms of your hands, roll it over once. How long a space has been passed over? Continue until the pupil sees that it is a space as long as the circumference of the sphere. Let him next determine how wide a space has been passed over. Continue until he discovers that it is a space as wide as the distance between the hands, or the diameter of the sphere.

Lead the pupils to see that you have covered a rectangle, as long as the circumference of the sphere and as wide as the diameter. With the area of this form all are familiar.

Find the surfaces of the spheres, when the following dimensions are known :

1. Radius 5 in.
2. Diameter 8 in.
3. Circumference 31.416 ft.
4. Diameter 2 ft. 6 in.
5. Diameter  $4\frac{1}{2}$  ft.
6. Radius 6 in.
7. Diameter 12 ft.
8. Circumference 62.832 in.
9. Radius  $5\frac{1}{2}$  ft.
10. How much leather will it take to cover a base-ball if its diameter is  $3\frac{1}{2}$  inches?
11. At 15¢ a square foot, what will it cost to gild a sphere 14 in. in diameter?
12. Find the surface of a sphere 4 ft. 6 in. in diameter.

1. I asked \$184 for my horse. This was a gain of 15%. I sold him for \$150. What per cent did I lose?
2. A speculator sold two farms at \$4,000 each. On the one he gained 20%, and on the other he lost 20%. Did he gain or lose? and how much?
3. An agent sold goods to the value of \$13,656, and received as his commission \$307.26. What was the rate of commission?
4. What tax must I pay if my property is valued at \$5,600 in a town where a tax of \$3,240 is to be raised? There are 398 polls, each paying \$1.50, and the taxable property is valued at \$440,500.
5. A house worth \$16,000 was insured for  $\frac{1}{2}$  of its value at  $1\frac{1}{2}\%$ . If it was totally destroyed by fire, find the loss to the owner and to the company.
6. Find the interest on \$1,248 from Mar. 4, 1901, to Aug. 12, 1904, at 7%.
7. Find the interest on \$3,020 from Apr. 14, 1902, to June 10, 1905, at  $5\frac{1}{2}\%$ .
8. A town whose valuation is \$4,900,000 raises by taxation \$63,210. There are 980 poll-tax payers, each taxed \$2. What is Mr. Brown's tax, who owns a farm valued at \$3,200, personal property amounting to \$1,800, and who pays one poll-tax?
9. A man imported from France 12 doz. pairs of gloves, invoiced at 6 francs a pair. The duty was \$2.25 a dozen and 50% *ad valorem*. What did the gloves cost him? A franc is \$.193.
10. Find the duty on 1,000 boxes of cigars weighing 2,480 lb., invoiced at \$3.50 a box; tare, 8 oz. a box; duty, \$2.50 a pound and 25% *ad valorem*.
11. Eight times a certain number minus 11 equals five times the number plus 25. Find the number.
12. My sister is 4 years younger than I am, and the sum of our ages is 26 years. What are our ages?
13.  $2x + 3 = 13$ . Find  $x$ .

1. What is the duty, at 22¢ a gallon, on 600 qt.-bottles of oil, breakage 5%?

2. A merchant imported 600 yd. of dress goods, invoiced at 8 francs a yard. What is the duty at 40% *ad valorem*?

3. What is the duty at 18¢ a sq. yd., and 30% *ad valorem*, on 1,000 yd. of carpet  $\frac{3}{4}$  yard wide, invoiced at 10 francs a yard?

4. If a man travel 720 miles in 30 days of 7 hr. each, how far will he travel in 7 days of 10 hr. each?

5. Brooks raised five times as many bushels of potatoes as Avery, and Maxfield 10 bushels more than both of the others. If all together they raised 328 bushels, how many did each raise?

6. Find the entire surface of a cylinder whose altitude is 8 ft. and the radius of the base 4 ft. Find contents.

7. The slant height of a cone is 50 ft., and the diameter of the base 18 ft. Find its convex surface.

8. A room measures  $18' \times 12' \times 10'$ . At 37¢ a square yard, find the cost of plastering it, allowing 80 sq. ft. for doors and windows. At \$1.12 $\frac{1}{2}$  a yard, find the cost of Brussels carpet laid lengthwise, allowing 8 in. loss on each breadth for matching.

9. What is the cost of 10 girders each 40 ft. long, 14 in. wide, and 12 in. thick, at \$28 per M.?

10. A barn is 80 ft. long, 50 ft. wide, with 20-ft. posts. The gable is 10 ft. high, and the rafters are 28 ft. long. How many boards 16 ft. long and 15 in. wide will it take to roof and board it?

11. An irregular stone was thrown into a cylinder 3 ft. 4 in. in diameter, which was partly filled with water. After the stone was thrown in, the water in the cylinder rose 10 in. Find the contents of the stone.

12. A bin is 11 ft. 6 in. long, 4 ft. wide, 6 ft. 3 in. deep. How many bushels of wheat will it hold? How many of apples?



$$\begin{array}{ll}
 1. & x - 4 = 6 \quad (1) \\
 & \quad + 4 = 4 \quad (2) \\
 & x = 6 + 4 \quad (3) \\
 & x = 10 \quad (4)
 \end{array}$$

In this equation we desire to get rid of  $-4$  in the first member. This we can do by adding 4 to both members. What difference do you notice between the first and third equation?

$$\begin{array}{ll}
 2. & 2x + 6 = 14 \quad (1) \\
 & \quad - 6 = -6 \quad (2) \\
 & 2x = 14 - 6 \quad (3) \\
 & 2x = 8 \\
 & x = 4
 \end{array}$$

In this example we desire to eliminate  $+6$  in the first member. Why do we add  $-6$  to both members? What difference do you notice between the first and third equations?

NOTE. — In the first and second examples we notice in each case that the terms that we wish to eliminate from one member appear in the other member with their sign changed. This operation is called *transposition*. According to this *any term can be removed from one side, provided we put it on the other side with the opposite sign.*

Find the value of  $x$  in the following:

$$3. \quad x - 7 = 4 \qquad 5x + 2 = 12.$$

$$4. \quad 2x + 6 = 8 \qquad 4x + 5 = 13.$$

$$5. \quad 3x - 7 = 11 \qquad 2x - 9 = 21.$$

$$6. \quad 2x - 5 = 7 \qquad 3x - 7 = 20.$$

$$\begin{array}{ll}
 7. & 2x = x + 5 \quad (1) \\
 & -x = -x \quad (2) \\
 & 2x - x = 5 \quad (3) \\
 & x = 5
 \end{array}$$

In this example we must eliminate  $x$  in the second member. Why do we add  $-x$  to both members? Compare the first and third equations.

$$8. \quad 2x + 7 = 15 \qquad 7x - 70 = 5x - 20.$$

$$9. \quad 3x - 18 = 7 - 2x \qquad 8x + 12 = 2x + 18.$$

$$10. \quad 7x - 13 = 7 - 5x \qquad 16x + 14 = 8x + 54.$$

$$11. \quad 8x - 15 = 6x - 3 \qquad 9x + 12 = 3x + 80.$$

$$12. \quad 5x - 4 = 6 + 3x \qquad 3x - 6 = 2x - 2.$$

$$\begin{array}{ll}
 13. & \frac{x}{4} = 3 \quad (1) \\
 & \frac{4x}{4} = 12 \quad (2) \\
 & x = 12 \quad (3)
 \end{array}$$

Compare the first and second equations. What change has taken place? Why did we multiply both sides by 4? How is the third equation formed from the second?

NOTE. — This operation is called *clearing the equation of fractions.*

$$14. \quad x + \frac{x}{2} + \frac{x}{4} = 7. \qquad \frac{x}{2} + \frac{x}{3} = x - 3.$$

1. Find two numbers whose sum is 180, and whose difference is 40.
2. Divide the number 133 into two parts such that one part is 15 more than the other.
3. Three men together have \$68,400. If the second has \$3,000 more than the first, and the third \$2,400 more than the second, how much has each?
4. If to three times a number I add 38, I shall obtain 98. What is the number?
5. A man and his two sons sawed 25 cd. of wood. The elder son sawed 5 cd. less than three times as many as the younger son, and the father sawed twice as many as the elder son. How many cords did each saw?
6. A man sold three houses of equal value, and a barn for \$24,400. If the barn brought \$1,600 less than a house, what was the price of each?
7. One number is 4 times another, and their difference is 30. Find the numbers.
8. John is two times as old as Henry, and the sum of their ages is 18 years. What is the age of each?
9. Alice had some money, and earned twice as much. After spending 9 cents, she had 21 cents left. How much money did she earn?
10. The sum of two numbers is 155, and the greater is 4 times the less. What are the numbers?
11. The difference of two numbers is 132, and the greater is 4 times the less. What are the numbers?
12. Samuel has a certain number of marbles, and John has 15 more than Samuel. If together they have 103 marbles, how many has each?
13. The sum of two numbers is 58, and the greater is 7 less than 4 times the smaller. What are the numbers?
14.  $3x + 4 = x + 10$ . Find the value of  $x$ .

1. Draw a horizontal line,  $AB$ . Draw two lines,  $CD$  and  $EF$ , perpendicular to  $AB$ . Where will  $CD$  and  $EF$  meet? Why?
2. Draw a horizontal line,  $AB$ , and at each extremity draw an oblique line, making with  $AB$  an angle of  $45^\circ$ .
3. Draw a vertical line,  $AB$ . At each extremity draw two oblique lines, each making an angle of  $75^\circ$  with  $AB$ . Where will the oblique lines meet? Why?
4. Draw an oblique line,  $AB$ . Draw two lines,  $CD$  and  $EF$ , making with  $AB$  an angle of  $65^\circ$ . Will these oblique lines meet?
5. Draw a horizontal line,  $AB$ . Draw two lines,  $CD$  and  $EF$ , making respectively angles of  $45^\circ$  and  $75^\circ$ . If you produce the two oblique lines, will they meet?
6. Draw a horizontal line,  $AB$ . Draw an oblique line,  $CD$ , making with  $AB$  an angle of  $60^\circ$ . Draw  $EF$  parallel to  $AB$ , and cutting  $CD$ . Draw  $GH$  parallel to  $CD$ , and cutting  $AB$ .
7. What is the ratio of one rt.  $\angle$  to 4 rt.  $\angle$ ?
8. What is the ratio of one-fourth of a rt.  $\angle$  to 2 rt.  $\angle$ ?
9. How can you test whether two intersecting lines are perpendicular to each other?
10. Make an angle. Make another angle 3 times as large. Make another  $\frac{1}{4}$  as large.
11. An angle formed by a vertical line meeting a horizontal line is an angle of — degrees?
12. Draw an angle that is equal to one-half of a right angle.
13. Draw a right angle. Cut from it an angle of 25 degrees. How many degrees are there in the remaining angle?
14. Draw a quadrilateral that is not a parallelogram.
15. Draw an equilateral rectangular parallelogram.
16. Draw a rhombus having an angle of  $75^\circ$ . Write the number of degrees in each of the other angles.
17. Using protractor, draw two lines perpendicular to each other.

1. What per cent of 12 bu. is 6 pk.?
2. A horse was sold for \$225 at a gain of  $12\frac{1}{2}\%$ . What did it cost?
3. Two boys have 567 apples. One has  $33\frac{1}{3}\%$  more than the other. How many has each?
4. What fraction increased by 20% of itself equals  $\frac{3}{4}$ ?
5. One farmer has 306 sheep; this is 66% less than B has. How many have both?
6. If I lose 13% by selling an article for 52 cents less than cost, how should it be sold to gain 12%?
7. I gained 23% by selling 24 bbl. of apples for \$8.28 more than cost. What was the cost a barrel?
8. A commission merchant charged \$17.28 for selling 320 bu. of potatoes at 60¢ a bushel. What was the rate of his commission?
9. Find the premium paid for insuring buildings for \$3,500 at  $1\frac{1}{4}\%$ , and furniture worth \$3,000 at  $\frac{3}{4}\%$ .
10. If I pay \$468 to insure property worth \$10,400, what is the rate?
11. The valuation of a town is \$175,600. There are 276 polls, each assessed \$1. The town wishes to raise \$2,910. What tax will a man pay who owns a house valued at \$3,000?
12. What is the interest on \$1,785 from Aug. 16, 1898, to Mar. 28, 1901, at 7%?
13. A and B received \$3,159 as the interest on their money invested for 6 yr. 6 mo. at 6%. If B's money equals  $\frac{2}{3}$  of A's, how much money has each?
14. A furniture dealer sold a parlor-set for \$70, and gained  $16\frac{2}{3}\%$  by so doing. What per cent would he have gained had he sold it for \$72?
15. A bookseller sold a set of encyclopedias for \$26, at a profit of  $8\frac{1}{3}\%$ . What per cent would he have made by selling the set for \$28?

1. In 840 in. how many feet?
2. At 2¢ a foot, find the cost of a rope 720 in. in length.
3. At 1¢ an inch, find the cost of three pieces of ribbon. In one piece there are 27 in., in another  $\frac{1}{4}$  yd., and in the third  $1\frac{1}{2}$  yards.
4. If you earn \$25, and then spend \$17.25 for a suit of clothes, \$3.25 for a pair of shoes, how much of your money will you have left?
5. If two desks are worth \$1 $\frac{1}{2}$ , what are two dozen desks worth?
6. At 40¢ a peck, what will a farmer receive for 96 qt. of beans?
7. Find the cost of 4 $\frac{1}{2}$  lb. of beefsteak at 16¢ a pound, and four chickens at \$1 $\frac{1}{4}$  each.
8. If a box of butter contains 25 lb., what will 4 boxes cost at 25¢ a pound?
9. Find the cost of an ounce of tea when  $\frac{1}{2}$  lb. costs \$.32.
10. If a man's salary for 8 months is \$840, what will be his salary for 6 months?
11.  $250 \div \frac{2}{3}$  of 25 = ?
12. How many times can you sell a quart of chestnuts if you sell 3 bu.?
13. The divisor is 9, the quotient 16. What is the dividend?
14. If I have \$3.50 at first, how many peanuts can I buy at 5¢ a quart and have 30 cents left?
15. Find the cost of 4 $\frac{1}{2}$  gal. oil at 8¢ a gallon.
16. Find the cost of 1 lb. of candy if 2 oz. cost 12¢.
17. What is the seventh part of 574?
18. If 2 bu. of apples cost \$2.40, what will a peck cost?
19. How many sevens must be added together to make 224?
20. \$18 pays for how much insurance at  $\frac{3}{4}\%$  premium?
21. What will 6 acres of land cost at \$60 $\frac{1}{2}$  an acre?

(Review pages 11 to 19.)

1. What is a savings bank?
2. How many of you have money in a savings bank?
3. Why is it a good plan to put money in a bank?
4. Suppose you each put \$200 in a savings bank Jan. 1, 1902, and the bank pays 4% interest. How much interest will be due Jan. 1, 1903?
5. To whom does this \$8 belong?
6. If you go to the bank Jan. 1, 1903, and the bank pays you \$8, how much money will you have in the bank for the next year?
7. If you do not call for the \$8, what will be done with it by the bank officials?
8. If the \$8 is added to your \$200, how much money will you have on interest the second year?
9. What will be the interest of \$208 for the second year?
10. When the interest is added to the principal, so as to find interest on principal and interest, it is called Compound Interest.
11. Interest at savings banks is usually compounded semi-annually. Sometimes quarterly. Compound annually unless otherwise directed.
12. In the 9th question what will be done with the \$8.32?
13. How much will you have on interest then?
14. How much will the interest be the third year?
15. What will be the new principal for the fourth year?
16. This example is worked like the following:

\$200.00	1st Principal.
8.00	Interest for 1st year.
\$208.00	Principal for 2d year.
8.32	Interest for 2d year.
\$216.32	Principal for 3d year.
8.653	Interest for 3d year.
\$224.973	Amount in bank at end of 4th year.
200.00	1st Principal.
\$24.973	Compound Interest for 3 years.

1. When there is a part of a period remaining, find the interest for the part period, and add it as for the whole period.

2. What is the compound interest of \$450 for 4 yr. 3 mo. 15 da. at 4% ?

3. What is the difference between the simple and compound interest of \$500 for 5 yr. 6 mo. 18 da. at 6% ?

Find the compound interest of the following :

4. \$4,500 for 2 yr. 6 mo. at  $3\frac{1}{2}\%$ .

5. \$1,278 for 3 yr. 9 mo. at 4%.

6. \$2,576 for 4 yr. 2 mo. 12 da. at 5%.

7. \$1,563 for 5 yr. 3 mo. 21 da. at  $4\frac{1}{2}\%$ .

8. \$6,793 for 3 yr. 6 mo. 15 da. at 4%.

9. \$728 for 2 yr. 9 mo. 24 da. at 3%.

10. \$1,560 for 6 yr. 6 mo. 6 da. at 6%.

11. Find the compound interest of \$600 for 1 yr. 9 mo. at 4% annually, compounded semi-annually.

NOTE. — 4% is the annual rate. The rate for 6 mo. would be what ?

12. Find the compound interest of \$550 for 1 yr. 4 mo. 12 da. if compounded quarterly at 1% a quarter.

Find the difference between the simple and compound interest of :

13. \$1,678 for 2 yr. 5 mo. 19 da. at 6%.

14. \$2,768 for 3 yr. 3 mo. 3 da. at 3%.

15. \$1,248 for 4 yr. 4 mo. 4 da. at 4%.

16. \$678 for 3 yr. 11 mo. 21 da. at  $4\frac{1}{2}\%$ .

17. \$624 for 5 yr. 7 mo. 21 da. at 5%.

18. \$4,635 for 2 yr. 4 mo. 15 da. at  $3\frac{1}{2}\%$ .

19. \$6,745 for 3 yr. 9 mo. 17 da. at 4%.

20. \$5,476 for 4 yr. 5 mo. 13 da. at 5%.

21. \$4,124 for 3 yr. 3 mo. 3 da. at 6%.

22. \$2,146 for 4 yr. 4 mo. 4 da. at 4%.

23. \$1,486 for 5 yr. 5 mo. 5 da. at 5%.

24. \$4,238 for 3 yr. 8 mo. 18 da. at 6%.

1. At \$1.75 a cubic foot, find the value of a block of marble in the shape of a square pyramid, whose height is 18 ft., and the sides of whose base are each  $1\frac{1}{2}$  ft.
2. At 8¢ a square foot, what will it cost to paint the gable-ends of a house 25 ft. wide, if the height of the gable is  $8\frac{1}{2}$  ft.?
3. A plank contains  $40\frac{1}{2}$  board feet. If it is 20' 2" long, and 3" thick, how wide is it?
4. What must I pay for 6 pieces of broadcloth, 150 yd. each, at 80¢ a yard, 10% off for cash?
5. Bought a bill of goods for \$640, but by paying cash I was allowed a discount of 10% and 5%. What was the discount? and what did I pay?
6. What is the compound interest of \$3,460 for 4 yr. at 6%?
7. A man placed a certain sum of money at simple interest, at 6%, when his son was born, and when the son became of age the amount was \$2,260. What was the sum?
8. I borrowed \$500 May 6, 1902, at 6% interest. I returned it when it amounted to \$600; when was the money returned?
9. A house valued \$3,600 rents for \$20 a month; what rate per cent does it yield?
10. What is the interest of \$877.50 for 50 d. at 8%?
11. What is the interest of \$1,250 for 54 d. at 6%?
12. What is the interest of \$650.30 from May 10, 1901, to July 16, 1904, at 7%?
13. What is the duty on 60 hhd. of 63 gal. each, and 25 gal. of syrup, worth 70¢ a gallon, leakage 3%, duty  $12\frac{1}{2}$ %.
14. Mr. Snow owns property, valued at \$16,480, in a town whose valuation is \$950,000. What will be his share of a tax of \$33,250?
15. If the cost of insuring property at  $2\frac{1}{2}$ % is \$121.50, what is the value of the property?
16. The area of a triangular lot is  $\frac{1}{2}$  A. If the base is 200 ft., what is the altitude?



1. A farmer raised in four fields the following quantities of corn: 65 bu. 3 pk. 2 qt.; 98 bu. 1 pk. 3 qt.; 110 bu. 5 qt.; 176 bu. 3 pk. 5 qt. How much did he raise in the four fields?

2. If a man travels 97 m. 120 rd. every day for 15 days, how much does he lack of having traveled 1,600 miles?

3. At 12¢ a pound, how much sugar can be bought for \$1,128.96?

4. Reduce 25,480 m. to weeks, etc.

5. At \$6½ a cord, find the cost of a pile of wood 20 ft. long, 4 ft. wide, 6 ft. high.

6. Bought 8 A. of land at \$175 an acre, and sold it at 22¢ a sq. ft. What was the gain or loss?

7. What is the value of 16,840 lb. of wheat at 98¢ a bushel? A bushel of oats weighs 60 lb.

8. Find the value of a load of straw weighing 1,860 lb. at \$18 a ton, and a load of hay weighing 2,520 lb. at \$18.50 a ton.

9. Divide 4.7151 by .604½.

10. If 5½ yd. of carpet cost \$7½, how much will 12½ yd. cost?

11. A man's income in 3 years was \$4,020. If his income the second year was 15% more than the first, and his income the third year was 20% more than the first, what was his income each year?

12. Find the volume of a cone whose altitude is 36 ft. and the radius of the base 15 ft.

13. At \$12.75 a sq. ft., what will it cost to gild the hemispherical dome of an observatory 40 ft. in diameter?

14. At each corner of a square 50 ft. on a side, with a radius of 25 ft., segments of circles are drawn. Find the area within the square not included in the segments.

15. Three boys had 85 marbles. The second boy had 4 more than 3 times as many as the first, and the third had 5 times as many as the first. How many had each?

Use the wooden sphere to illustrate this point, or take a sphere (a round potato or apple) and cut it into several small pieces, each shaped like a pyramid. To do this the cut must in each case reach the center of the sphere.

Examine each piece. The base of each piece is a part of what? The sum of all the pieces is what?

The altitude of each piece is what of the sphere?

How do you find the volume of each piece?

State the rule for finding the volume of a sphere.

If we multiply the surface of a sphere (the sum of all the bases of the pyramids) by  $\frac{1}{3}$  of the radius (the height of the pyramids), what shall we have?

LEARN: To find the volume of a sphere, multiply its surface by  $\frac{1}{3}$  of its radius.

Can you tell why you multiply by  $\frac{1}{3}$  of the radius?

1. Find the volume of a sphere whose radius is 5 in.
2. If the diameter of a sphere is four inches, what is its volume?
3. The radius of a sphere is 15 in. Find its surface; its volume.
4. How does the volume of a sphere 4 in. in diameter compare with the volume of a cylinder 4 in. in diameter and 4 in. in altitude?
5. In the fourth example, if the sphere was cut out of the cylinder, what part would be cut away?
6. A man cut a cylinder of the largest possible size out of a cubical block of wood measuring 2 ft. What part of the cube did he cut away?
7. If a sphere is cut out of a cube, what part of the cube must be cut away?
8. How does the volume of the sphere compare with the volume of the cube?
9. Find the volume of a sphere whose diameter is 4 ft.
10. From a sphere 15 in. in diameter two inches in thickness were cut off. How many cubic inches were cut off?
11. Find the volume of a sphere whose radius is 6 in.

1. If at  $2\frac{1}{2}\%$  premium, I pay \$177.50 for insurance on my property, what is the value of the property?

2. I paid a lawyer \$115.65 for collecting a bill at  $4\frac{1}{2}\%$ . What was the amount of the bill?

3. An agent sold goods to the amount of \$6,680.00. He paid \$19.40 for storage, \$60 for freight, and returned \$6,300 to me. What was the rate per cent of his commission?

4. An auctioneer sells goods to the amount of \$1,920, and charges \$48. What per cent does he receive as commission?

5. An agent sold 75 bales of cotton, each bale weighing 350 lb., at  $12\frac{1}{4}\%$  a pound, on a commission of  $2\frac{3}{4}\%$ . What was his commission?

6. A dealer sold goods at \$1.84 a yard, and gained 15%. At what should he have sold a yard to gain  $18\frac{3}{4}\%$ ?

7. Sold two houses at \$4,968 each, gaining on one 8%, and losing 8% on the other. What did I gain or lose?

8. By selling goods at \$237.60 less than cost I lose 44%. At what should I sell them to gain 16%?

9. A merchant having \$10,000 worth of goods, lost 25% by fire, and sold the remainder at a gain of 40%. What was the gain or loss per cent?

10. If the cost of coal at the mine is \$2.60 a short ton, and the freight \$1.30 a ton, at what price must it be sold to gain 30%?

11. Bought a house for \$3,300 when property was depreciated 40% in value. What was the value of the house?

12. The number of the pupils enrolled in the schools of a city is 5,805, which is 35% more than the number in attendance. What is the number in attendance?

13. Find the amount of duty on the following invoice: 80 hhd. molasses, 63 gal. each @ 26¢; 38 hhd. sugar, 380 lb. each @  $4\frac{1}{2}\%$ ; 250 boxes raisins, 20 lb. each @  $7\frac{1}{2}\%$ . The duties are: molasses, 20%; sugar, 30%; raisins, 10%.

1. Two of the boundary lines of a field run north and south, and are 60 rd. and 48 rd. in length. The distance between them is 36 rd. Find the area of the field.
2. A cylinder is 8 in. in diameter and 28 in. in length. Find the volume of the largest cone that can be cut from it. How many cubic inches of the cylinder must be cut away?
3. Find the volume of a square prism the perimeter of whose base is 44 ft., and whose altitude is 42 ft.
4. At \$1.87½, what will it cost to carpet a room 18 ft. long, 15 ft. wide, with carpet 27 in. wide, if the breadths run lengthwise?
5. At \$30 per M., what shall I pay for 3 boards, each 12 ft. long, 16 in. wide at one end, and 10 in. at the other?
6. What will a pile of wood cost at \$7.50 a cord, if it is 16 ft. 8 in. long, 4 ft. wide, and 6 ft. 3 in. high?
7. A piece of land 35 rd. long and 7 rd. wide is divided into 5 square lots of equal size. What will be the cost of boundary and cross fences at \$2.12½ a rod?
8. My house is on a corner lot, — 150 ft. on one street, and 60 ft. on the other. The sidewalk is 6 ft. wide. How many cubic feet of snow do I shovel in clearing my walk after a 15-in. snowstorm?
9. How many gallons in a tub having a base of 3½ sq. ft. and a depth of 15 in.?
10. How many bricks are necessary for the 12-in. walls of a house 40 ft. long, 28 ft. wide, 22 ft. high, deducting 56 cu. ft. for openings?
11. How many feet (board measure) in 48 joists, each 18 ft. long, 10 in. wide, 2½ in. thick?
12. How many bunches of shingles laid 4 in. to the weather will cover a roof, each half of which is 40 ft. × 20 ft.?
13. What will it cost, at 25¢ a cut for every cord, to saw into four pieces 4-ft. wood, piled 60 ft. long, and 6 ft. high?

$$1. \quad \frac{x}{2} - 8 = \frac{x}{4} - 4 \quad (1)$$

$$\frac{4x}{2} - 32 = \frac{4x}{4} - 16 \quad (2)$$

$$2x - 32 = x - 16$$

$$2x - x = 32 - 16$$

$$x = 16$$

Compare the second equation with the first. Why did we multiply by 4 instead of 2? What is the least common denominator? Carefully note and explain each step in the solution of this problem.

NOTE. — In the following examples multiply by the least common denominator to clear the equation of fractions.

$$2. \quad \frac{x}{3} + 4 = \frac{2x}{4} + 3$$

$$3. \quad \frac{3x}{9} + 7 = \frac{4x}{6} + 4$$

$$4. \quad 2x + \frac{6x}{4} - \frac{2x}{4} = 24$$

$$5. \quad \frac{7x}{2} + \frac{x}{4} + \frac{x}{8} = \frac{31}{2}$$

$$6. \quad \frac{3x}{4} - 7 = \frac{2x}{3} + 8$$

$$7. \quad \frac{2x}{3} + \frac{x}{4} - 5 = \frac{x}{12} - \frac{x}{6} + 7$$

$$8. \quad \frac{x}{3} - \frac{x}{4} + 2 = 3$$

$$9. \quad \frac{3x}{4} + 16 = \frac{x}{2} + \frac{x}{8} + 17$$

$$10. \quad x + \frac{x}{2} - 40 = \frac{7x}{10}$$

$$11. \quad \frac{2x}{3} + 12 = \frac{4x}{5} + 6$$

$$12. \quad \frac{2x}{2} + 4 = 10 - \frac{4x}{4}$$

$$13. \quad 3x - 3 = 7x - 15.$$

$$\frac{2x}{4} - 2 = \frac{x}{3}.$$

$$\frac{7x}{3} - 5 = \frac{5x}{3} - 3.$$

$$\frac{6x}{4} - \frac{10x}{8} = 44.$$

$$\frac{x}{4} - \frac{33}{2} = \frac{35}{2} - \frac{x}{4}.$$

$$2x + \frac{5x}{6} - 8 = 9.$$

$$\frac{x}{2} - 3 + \frac{x}{3} = 5 - 3.$$

$$\frac{x}{4} + \frac{x}{8} - \frac{x}{6} = \frac{5}{12}.$$

$$x + \frac{x}{2} + \frac{x}{4} = 7.$$

$$\frac{x}{2} + \frac{x}{4} + \frac{x}{8} = 28.$$

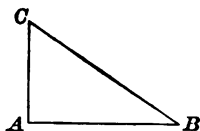
$$3x + \frac{3x}{4} + 15 = \frac{x}{2} + 41.$$

$$\frac{3x}{4} - 2 = 3 - \frac{5x}{10}.$$

$$5x - 10 = 3x + 4.$$

(Review page 158.)

1. Draw a horizontal line,  $AB$ , 2 in. long. At point  $A$  make an angle of  $60^\circ$  by drawing  $AC$  3 in. long. Join  $BC$ .
2. What is the figure  $ABC$  called?
3. How many sides and angles has it?
4. What kind of angles are  $A$ ,  $B$ , and  $C$ ?
5. Since all the angles are acute, the figure is called an acute triangle.
6. Draw the line  $AB$ . At  $A$  make an angle of  $110^\circ$  by drawing  $AC$ . Connect  $BC$ .
7. This figure is called an obtuse triangle. Why? What is an obtuse triangle?
8. Draw  $AB$ . At  $A$  make a right angle by drawing  $AC$ . Connect  $BC$ .
9. What name is given this figure? Why?
10. What is a right triangle?
11. In this right triangle the line  $AB$  is the base, — the line on which it rests; the line  $AC$  is the perpendicular; the line  $BC$  is the hypotenuse.
12. The hypotenuse is always opposite what angle?
13. Define base, perpendicular, hypotenuse.
14. How many right angles can a triangle have? Obtuse angles?
15. Draw two lines,  $AB$  and  $AC$ , each 3 in. long, and forming an angle at  $A$ . Join  $BC$ .
16. When two sides of a triangle are equal, the triangle is called an isosceles triangle.
17. Define an isosceles triangle.
18. Draw an isosceles triangle that shall contain an obtuse angle.
19. Draw an isosceles triangle that shall contain a right angle.



1. Two towns are 150 miles apart. If the railway fare is \$3.00, what is the rate a mile?
2. What is the cost of 5 suits of clothes at \$15 each, and 4 hats at \$1.50 each?
3. At 3¢ an ounce, what is the cost of 2 lb. of pepper?
4. What is the cost of  $5\frac{1}{2}$  lb. of cheese at 18¢ a pound?
5. If 16 sheep cost \$96, what will 30 sheep cost?
6. How many yards long is a piece of cloth that is 720 in. long?
7. Find the cost of  $1\frac{1}{2}$  bu. of potatoes at 15¢ a half-peck.
8. How many quarter-inch squares can be cut from a 2-in. square?
9. Give answers:
 

$8,100 \div 100.$	$66,000 \div 22,000.$	$13 \times 300.$
$9,300 \div 100.$	$39,000 \div 13,000.$	$24 \times 400.$
$11,200 \div 100.$	$56,000 \div 12,000.$	$16 \times 200.$
10. At 32¢ a pound, find the cost of 2 lb. 5 oz. of butter.
11. What is the total cost of 2 lb. tea at 75¢ a pound, and  $\frac{1}{4}$  bbl. flour at \$6 a barrel?
12. At 80¢ a pound, what will 6 oz. of tea cost?
13. A boy had 64 stamps. How many had he after he had sold 28 and bought 16?
14. A grocer sold 15 lb. 8 oz. of tea on one day, and 17 lb. 8 oz. on another day. How much tea did he sell on both days?
15. If you have 75 cents, how much will be left after paying for  $\frac{1}{4}$  lb. of 80-cent tea, and a pound of 35-cent coffee?
16. A man paid \$50 for a parlor-set, and \$30 for a bedroom-set. He paid  $\frac{1}{4}$  in cash, and the rest in 6 monthly payments.
17. Find the cost, at 18¢ a pound, of two hams, one weighing 4 lb. 8 oz., and the other 5 lb. 8 oz.
18. When apples are worth a half-dollar a bushel, how many bushels can be bought for \$15?
19. How many quarts in 25% of a peck?

\$690. *New Haven, Conn., Feb. 3, 1901.*

*On demand, for value received, I promise to pay  
to Charles W. Ashley, or order, Six Hundred Ninety  
Dollars, with interest at 6%. M. E. Martin.*

1. In the above note who is the Maker? The Payee? What is the Face, or Principal?
2. Promissory notes can be bought and sold like other forms of property, when they are negotiable.
3. What words in the above note gives Charles Ashley a right to sell this note?
4. If Charles Ashley should sell this note, he would write his name on the back as an Indorser, and thus be held responsible for its payment to the holder in case the maker fails to pay it when due.
5. Is this a demand or a time note?
6. Substitute "three months after date" for the words "on demand." What kind of a note is it now?
7. Sometimes two persons sign the note, when it would read, "we, jointly and severally, promise to pay," or "we, or either of us, promise," etc.
8. If the words "with interest" are in the note, it draws interest from date to payment. If these words are omitted from a demand note, the note will not bear interest; if omitted from a time note, the note will not draw interest until after it becomes due.
9. Write a non-negotiable demand note.
10. Write a time, joint and several, negotiable note.
11. Substitute "bearer" for "order." Does this change the meaning?
12. In each of the notes that you have written, name the maker and payee.
13. Write a time, interest-bearing, negotiable note.



1. Instead of paying the note in full, it frequently happens that part payments are made at different times.

2. A record of each partial payment, with date of payment, is made on the back, and called an Indorsement.

3. These indorsements were made on the foregoing note : Dec. 3, 1901, \$40 ; April 3, 1902, \$60 ; Dec. 3, 1903, \$150.

4. If Martin, on Dec. 3, makes a payment, how long will he have used Ashley's money ? How much will he owe as interest ? How much will be due Dec. 3, 1901 ? *Ans.* \$724.50.

5. If Martin then pays \$40, how much will he still owe ?

6. If Martin comes again April 3, 1902, how long will he have used \$684.50 ? How much interest will he then owe ? What is the interest of \$684.50 for 4 mo. ? What is done with this interest ? How much will Martin owe Ashley, April 3, 1902 ? If he pays him \$60, what balance will still be due ?

7. On Dec. 3, 1903, how long has \$638.19 been on interest ? To what has it amounted ? What payment was then made ? What balance was left due ?

8. If the note was paid Feb. 27, 1904, what sum was paid ?

9. The following is a good form to use :

Original principal . . . . .	\$890.00
Interest from Feb. 3, 1901, to Dec. 3, 1901 (10 months) . . . . .	34.50
Amount due Dec. 3, 1901 . . . . .	<u>\$724.50</u>
First payment . . . . .	40.00
Balance due Dec. 3, 1901, or Second Principal . . . . .	<u>\$684.50</u>
Interest from Dec. 3, 1901, to April 3, 1902 (4 months) . . . . .	13.69
Amount due April 3, 1902 . . . . .	<u>\$698.19</u>
Second payment . . . . .	60.00
Balance due April 3, 1902, or Third Principal . . . . .	<u>\$638.19</u>
Interest from April 3, 1902, to Dec. 3, 1903 (20 months) . . . . .	63.82
Amount due Dec. 1903 . . . . .	<u>\$702.01</u>
Third payment . . . . .	150.00
Balance due Dec. 3, 1903 . . . . .	<u>\$552.01</u>
Interest from Dec. 3, 1903, to Feb. 27, 1904 . . . . .	7.72
Amount due at settlement . . . . .	<u>\$559.73</u>

1. Write a composition on Partial Payments. Carefully describe each step taken in working a problem. Write a rule for another's guidance.

2. \$1,000. *Springfield, Dec. 13, 1901.*  
*For value received, I promise to pay Clarence Rogers, One Thousand Dollars, with interest at 6%.*  
*G. A. Morse.*

Indorsements: \$231, June 19, 1902; \$350, Oct. 1, 1902; \$125, Feb. 19, 1903. How much was due June 25, 1903?

3. \$2,500. *Boston, Mass., Dec. 14, 1900.*  
*On demand, for value received, I promise to pay Charles Conway, or order, Two Thousand Five Hundred Dollars, with interest.*

*Martin O. Sikes.*

Payments: March 26, 1901, \$50; Nov. 1, 1901, \$500; Dec. 19, 1902, \$1250. What is due Dec. 31, 1903?

4. \$850.75. *Worcester, Mass., Jan. 1, 1900.*  
*For value received, I promise to pay Flora Jordan, or order, Eight Hundred Fifty  $7\frac{3}{8}$  Dollars, with interest at 6%.*  
*E. S. Smith.*

Indorsements: July 16, 1900, \$150.00; July 30, 1902, \$450.00; April 9, 1903, \$342.39. If this note is settled Dec. 17, 1903, what amount will pay it?

5. Face, \$480.50. Date, June 15, 1900. Indorsements: Nov. 30, 1900, \$175.75; Sept. 2, 1901, \$140.00; Oct. 9, 1902, \$85.00; May 18, 1903, \$90.00. What is due June 1, 1904?

6. Face, \$1600. Date, Sept. 16, 1900. Indorsements: June 18, 1901, \$400; Oct. 15, 1902, \$500; Jan. 15, 1903, \$300. With interest at 5%, what is due May 10, 1903?

7. Supplying names, write the 5th example as a note.

1. \$336.

*Boston, March 26, 1901.*

*On demand, we promise to pay J. C. Stephens,  
or bearer, Three Hundred Thirty-six Dollars, with  
interest. Value received. Rogers & Brown.*

Indorsements: July 20, 1901, \$55; April 7, 1902, \$8,  
Sept. 26, 1902, \$6.00; Jan. 7, 1903, \$160. What is due  
May 1, 1903?

Principal . . . . .	\$336.00
Interest to July 20, 1901 (3 months, 24 days) . . . . .	6.38
Amount due July 20, 1901 . . . . .	<u>\$342.38</u>
First payment . . . . .	55.00
Second Principal . . . . .	<u>\$287.38</u>

NOTE. — We find that the interest due at the next payment is \$12.56. As the payment was less than the interest, we make no use of it on that date, but consider it as paid Sept. 26, 1902. We find that the interest from July 20, 1901, to Sept. 26, 1902, is \$20.40. As the united payments only amount to \$14.00, we must again consider that no payment was made on the principal until Jan 7, 1903.

Interest from July 20, 1901, to Jan. 7, 1903 . . . . .	\$25.29
Amount due Jan. 7, 1903 . . . . .	<u>\$312.67</u>
Payments (\$8 + \$6 + \$160) . . . . .	174.00
Balance due, or Third Principal . . . . .	<u>\$138.67</u>
Interest from Jan. 7, 1903, to May 1, 1903 . . . . .	2.63
Amount due May 1, 1903 . . . . .	<u>\$141.30</u>

NOTE. — Unless the payment and the interest are very nearly equal, you can mentally calculate whether the payment exceeds the interest or not.

2. Face, \$1,650. Date, May 12, 1900. Indorsements:  
Jan. 24, 1901, \$140.50; Dec. 6, 1901, \$20.10; Aug. 15, 1902,  
\$136.87; Dec. 6, 1902, \$75. What was due April 24, 1903?

3. Face, \$165. Date, April 15, 1900. Indorsements:  
May 24, 1901, \$24.18; July 18, 1902, \$5.25; Sept. 6, 1902,  
\$45.00; Jan. 24, 1903, \$40.00. What was due April 15, 1904?

4. A note of \$720, dated Aug. 14, 1897, has the following  
indorsements: Dec. 26, 1898, 200; Sept. 14, 1901, \$175;  
Dec. 31, 1902, \$400. Settled Dec. 31, 1903. Find the sum  
paid at time of settlement.

5. Write the rule for Partial Payments.

MERCHANT'S RULE.

This rule is usually used when settlement is made within a year.

A note of \$850 was dated Jan. 2, 1903. The indorsements were: March 18, \$200; May 2, \$150; Aug. 18, \$300. What was due Dec. 2, 1903?

Amount of \$850 from Jan. 2, 1903, to Dec. 2, 1903 . . .	\$896.75
Amount of \$200 from March 18 to Dec. 2 . . .	\$208.47
Amount of \$150 from May 2 to Dec. 2 . . .	155.25
Amount of \$300 from Aug. 18 to Dec. 2 . . .	305.20
	<u>668.92</u>
	\$227.83

1. What is the face of this note? For how long a time was it on interest? To what does it amount Dec. 2, 1903?

2. What was the first payment? On what date was it paid? For how long a time did the payee have the use of this \$200? To what did it amount?

3. Answer the same questions for each of the other payments.

4. To what do all the payments amount?

5. What is the difference between the amount of the note and the amount of the payments?

6. Write a clear analysis of an example in Partial Payments performed by the Merchant's Rule.

7. A note of \$1,250.60, dated July 6, 1901, with interest at 7%, was indorsed as follows: Sept. 21, 1901, \$260; Nov. 22, 1901, \$325; March 6, 1902, \$120; May 17, 1902, \$250. What was due at settlement, July 6, 1902?

8. A note of \$2,000, dated Jan. 20, 1902, had the following indorsements: May 20, 1902, \$100; July 20, 1902, \$175; Dec. 23, 1902, \$250. Find the balance due Jan. 15, 1903.

9. A note for \$3,000, dated Jan. 1, 1902, had indorsements as follows: March 1, 1902, \$300; Oct. 1, 1902, \$150; Nov. 1, 1902, \$1,500. What is due Jan. 1, 1903?

10. Note, \$460. Date, May 9, 1902. Settled, Feb. 24, 1903. Payments: July 1, 1902, \$120; Sept. 16, 1902, \$150; Jan. 2, 1903, \$100.

1. A note of \$1,200, dated Aug. 15, 1902, was indorsed as follows: Dec. 15, 1902, \$20; Sept. 15, 1903, \$150. Find the amount due Aug. 15, 1904.

2. By selling a farm for \$4,400 I lost  $8\frac{1}{2}\%$ . What had I paid for the farm?

3. A farmer offered a cow for sale for \$40. He sold her at 10% discount. and yet made 25%. What was the cost of the cow?

4. I bought a chair for \$8, 20% off, and sold it for \$10, 15% and 6% off. How much did I gain?

5. A dealer sold two horses for \$300; on one he gained  $12\frac{1}{2}\%$ , and on the second he lost 20%. Did he gain or lose if for the second horse he received  $\frac{3}{4}$  as much as for the first?

6. A furniture dealer had 800 chairs, worth \$6 each. A fire destroyed 30% of them, and he sold the remainder at \$8.50 each. How much did he lose?

7. If a man starts in business with \$8,000, and each year gains  $12\frac{1}{2}\%$  of his capital, what will he have at the end of three years?

8. Find the interest of \$651 for 16 days at 9%.

9. A debtor owed me \$1,560. A lawyer collected 75% of the debt, and charged 5% commission. How much did I receive?

10. A man who owned 75% of a ship, sold 40% of his interest for \$30,000. At that rate, what was the value of the whole ship?

11. What is the value of 75% of a farm, if  $\frac{5}{8}$  of it is worth \$4,000?

12. Bought  $67\frac{1}{2}$  yd. of carpet at \$1.87 $\frac{1}{2}$ , receiving a discount of 15%. What was my bill?

13. 1,320 is 12% less than what number?

14. Mr. S. has a salary of \$1,800, and pays \$378 rent. What per cent of his salary does he pay for rent?

1. If a merchant's gain at retail is 35%, and he sells at wholesale for 10% below his retail price, what is his gain at wholesale?

2. Find the per cent of lighting surface to floor surface in a room  $28' \times 32'$ , with 8 windows, each  $3' 6'' \times 8'$ .

3. The owner of  $66\frac{2}{3}\%$  of a ship sold 50% of his interest for \$32,000. Find the value of the whole ship at the same rate.

4. A man bought a horse for \$400, which was 20% less than its real value, and sold it at 20% above its real value. Find selling-price.

5. What is  $62\frac{1}{2}\%$  of a sum of money, if 75% of it is \$1,200 more than  $66\frac{2}{3}\%$  of it?

6. What premium must a man pay on furniture worth \$1,800, insured at  $87\frac{1}{2}\%$  of its value, at  $1\frac{1}{3}\%$  premium?

7. By selling an article at \$6.65 a man lost 5%. For how much must he sell it in order to gain 5%?

8. A man sold a paper-mill, receiving 45% of the price in cash. He invested  $\frac{3}{4}$  of the sum received in a farm worth \$2,160. For how much was the mill sold?

9. A man, who had been paying \$25 a month rent, borrowed \$4,000 at 5%, and bought a house. Instead of rent he now pays interest on the borrowed money, \$50 a year taxes, \$8 water-tax, \$12 insurance, and \$25 for repairs. Find his gain or loss a year.

10. Leaving  $87\frac{1}{2}\%$  of my money at home, I spent 5% of the rest for butter at 29¢ a pound. I bought 40 lb. of butter. How much money had I at first?

11. If a lawyer retained \$9.08 for collecting \$181.60, at the same rate, what would he need to collect to receive \$20,000 a year?

12. What is the compound interest of \$236 for 1 yr. 6 mo. at 8% per year, interest compounded semi-annually?

13. What is the duty on 46,080 pencils at 2¢ a gross?

1. A physician, whose charges are \$1.50 a visit, made an average of 6 visits a day in the year 1903. He collected 65% of his charges, and saved 40% of the sum collected. At that rate how much could he save in 3 yr. 6 mo.?

2. Owing to a deficiency in the appropriation bill, the salaries of the clerks in the post-office were reduced 16% for the last quarter of the fiscal year. How much did a clerk who was paid \$336 for the last quarter receive during the whole year?

3. The cost of insuring a store at  $1\frac{1}{2}\%$  is \$108 a year, and the cost of insuring its contents at  $2\frac{1}{4}\%$  is \$175.50. What is the whole amount of insurance?

4. The ice on a circular pond is 18 in. thick. If the pond is 800 ft. in circumference, how many cubic feet of ice does it contain?

5. Two merchants offer the same quality of goods at the same list-price. The first offers a discount of 10% and 5%, and the second offers a discount of 15%. Of whom will it be more advantageous to buy? and how much will be saved on a bill, the list-price being \$1,050?

6. A gross amount of a bill is \$570.35, and the discounts are 10%, 10%, and 5%. What net cash will pay the bill?

7. Find the cost of papering the walls and ceiling of a hall 36 ft. long, 24 ft. wide, and 18 ft. high, if 64 sq. yd. are allowed for openings. The paper costs  $37\frac{1}{2}$ ¢ a roll.

8. Subtract 23 ten-millionths from 2 hundredths of 6 thousandths.

9. Multiply four hundred thousandths by four hundred-thousandths, and divide the product by four tenths.

10. A man's farm is 120 rd. wide. He sells 12 A. off one end. How much shorter is his farm than it was before?

11. A suit of clothes cost \$17. The trousers cost \$1 less than 3 times as much as the vest cost, and the coat cost 2 times as much as the trousers. Find the cost of each.

1. Charles rode 5 hours on his bicycle, going  $11\frac{1}{2}$  miles the first hour,  $7\frac{1}{6}$  miles the second,  $9\frac{1}{2}$  the third,  $7\frac{1}{2}$  the fourth, and  $7\frac{1}{2}$  the fifth. How many miles did he ride?

2. A man owning a vessel gave  $\frac{3}{4}$  of it to his son, and sold 25% of the remainder for \$2,000. What was the value of the vessel?

3. Mr. S. started to walk  $21\frac{1}{2}$  miles. After walking 5 h. at the rate of  $3\frac{1}{2}$  miles an hour, how many miles of his journey remained?

4. Change to common fractions: 2.00875, 76.88, 15.0125.

5. 3 men reap  $\frac{1}{2}$  of a field of wheat in  $1\frac{1}{2}$  days. How many days will it take one man to reap the whole field?

6. From 2 orchards 120 bbl. of apples were picked. If one orchard produced  $\frac{3}{4}$  as many barrels as the other, how many barrels were picked from each field?

7. A cubic foot of water weighs  $62\frac{1}{2}$  lb. If copper is  $8\frac{1}{2}$  times as heavy as water, what is the weight of a cubic foot of copper?

8. If a man can row  $4\frac{1}{2}$  miles an hour in still water, how many miles can he row in  $3\frac{1}{2}$  h. up a river that flows at the rate of  $1\frac{1}{2}$  miles an hour?

9. How many miles can he row in  $2\frac{3}{4}$  h. down the same river?

10. How many hours will it take him to row 15 miles down the river? 15 miles up the river?

11. If a train runs  $40\frac{1}{2}$  miles in an hour, what part of a mile does it travel in a minute?

12. If a man's debts amount to \$10,500, and his property is worth \$4,650, how many cents on a dollar can he pay?

13. A can build a wall in  $12\frac{1}{2}$  days, and A and B together can build  $\frac{1}{2}$  of the wall in a day. In how many days can B build it alone?

14. The divisor is  $7\frac{1}{2}$ , the quotient  $1\frac{1}{2}$ , what is the dividend?



1. If 3 men can do a piece of work in 4 days, how long will it take 24 men to do it?

2. At what rate will \$400 gain \$40 in 1 yr. 8 mo.?

3. Seven is three-eighths of what number?

4. Sold a cow for \$24, losing thereby 40%. Had I sold her for 20% advance on the cost, what would I have received for her?

5. What is the effect on the value of a decimal, if the decimal point is moved two places to the right?

6. What is the effect of multiplying the numerator and denominator of a fraction by 4? Why?

7. Divide .006 by 100. Multiply the same numbers.

8. If the denominator of a fraction is divided by 3, what is the effect upon the value of the fraction?

9. In every fraction, what is shown by the denominator? By the numerator?

10. What is meant by a decimal fraction?

11. What is a factor?

12. If a cipher is added at the right of a decimal, what effect has it on the value of the decimal?

13. Reduce  $\frac{1}{2}$  to a decimal fraction.

14. What is meant by the ratio of one quantity to another?

15. What is meant by minuend? By quotient? By multiplicand?

16. How many board feet in a plank 14' long, 6" wide, and 3" thick?

17. What is the volume of a square pyramid whose altitude is 15 in., and each side of the base 10 in.?

18. A room is  $\frac{3}{4}$  as wide as it is long. Its length is 15 ft. Find the square feet in the floor.

19. What will  $\frac{3}{4}$  of a yard cost, if 5 yd. cost 90 cents?

20. How many days from May 18 to July 4?

21. How many hours in  $33\frac{1}{3}\%$  of a day?

To find the sum to be invested, after deducting the per cent commission from the amount remitted.

1. A merchant sent \$9,180 to his agent in Chicago with which to buy wheat. If the agent charges 2% for buying, how many bushels of wheat can he buy at 90¢ a bushel?

(a) If an agent is expending money for another, on what has he a right to take a commission?

(b) Did he spend all of the \$9,180 for his employer? Has he a right to take a 2% commission on that sum?

(c) Does this \$9,180 include the agent's commission?

(d) Does it include the sum spent for wheat?

(e) What per cent of any number is the number itself?

(f) If \$9,180 includes the agent's commission, 2%, and the sum spent for wheat, 100%, what per cent is it of the sum spent for wheat?

(g) If \$9,180 is 102% of the sum spent for wheat, what is 100%, or the sum spent for wheat?

$$102\% = \$9,180$$

$$1\% = \frac{1}{102} \text{ of } \$9,180 = \$90$$

$$100\% = 100 \times \$90 = \$9,000$$

(h) At 90¢ a bushel how many bushels of wheat can be bought for \$9,000?  
 $\$9,000 \div 90¢ = 10,000$  times, i.e. 10,000 bu.

(i) 2D EXPLANATION. — If the agent keeps 2%, 2¢ on a dollar, how much money must be sent him to allow him to buy \$1 worth of wheat? If he buys \$1 worth of wheat with every \$1.02 sent him, how many dollars' worth will he buy with \$9,180 sent him?

2. If \$10,250 includes the amount expended for wool and 2½% commission to the agent, how much money does the agent spend in wool?

3. If \$3,549 are remitted to an agent to buy cotton, after deducting 4% commission how much will be invested in cotton?

4. How many barrels of flour at \$5 each can be bought with a remittance of \$2,575, after deducting 3% commission?

5. A country merchant forwarded 800 bbl. of apples to be sold at \$1.25 a barrel, the agent to receive a commission of 3% for selling. After paying \$5.75 for cartage, and deducting his commission of 1½% for investing, he invested the proceeds in sugar at \$9½ a hhd. How many logsheads did he buy?

1. When \$9,823 are sent an agent, whose commission is  $4\frac{1}{2}\%$ , how much is spent for goods?
2. An agent is paid  $1\frac{1}{2}\%$  for purchasing goods. What amount does he purchase from a remittance of \$1,258.60?
3. An agent is paid 6% for buying goods, what amount can he buy with \$2,650, after deducting his commission?
4. A merchant remitted to an agent \$1,412.45, with instructions to buy apples at \$2.16 $\frac{2}{3}$  a barrel after deducting his commission of  $2\frac{1}{2}\%$ . How many barrels did he buy?
5. A merchant shipped 240 bbl. of flour to be sold at \$6 $\frac{1}{2}$  a barrel at 3% commission. After paying \$15.90 for cartage, he buys hay at \$18 a ton, commission  $2\frac{1}{2}\%$ . How many tons of hay does he buy?
6. I remit to my agent in Chicago \$169,302 to purchase flour. After deducting his commission of  $1\frac{1}{2}\%$  and \$48 for other expenses, how many barrels of flour at \$4 a barrel, will the money purchase?
7. A dealer shipped \$40,000 worth of goods to his agent with instructions to buy groceries with the proceeds. The agent charged  $2\frac{1}{2}\%$  for selling and 2% for buying. What sum did the agent receive as commission?
8. A commission merchant sold 1,000 bbl. of apples at \$2.50 a barrel at  $3\frac{2}{3}\%$  commission, and invested the net proceeds in cloth at 25¢ a yard. How many yards did he buy, commission 5%?
9. An agent sold goods for a merchant to the amount of \$1,200, and invested the net proceeds in apples less a commission of  $2\frac{1}{2}\%$  in both cases. What was his whole commission?
10. I sent \$12,300 to my agent, with which to purchase flour at \$5 a barrel, after deducting his commission of  $2\frac{1}{2}\%$ .
11. A grain dealer in Chicago received \$2,460 with directions to purchase corn at 60¢ a bushel, after deducting his commission of  $2\frac{1}{2}\%$ . How many bushels of corn did he purchase?

1. A and B have together \$1,053. If  $\frac{3}{4}$  of A's money is equal to  $\frac{1}{2}$  of B's, how much has each?
2. The driving-wheels of a locomotive are 15 ft. 9 in. in circumference. How many revolutions will they make in a mile? If the wheels revolve  $2\frac{1}{2}$  times a second, what is the rate of speed a mile?
3. How much will it cost to fence  $3\frac{1}{4}$  miles of railroad at the rate of  $62\frac{1}{2}$ ¢ a rod?
4. How many bushels of grain will a bin 6 ft. long, 4 ft. wide, and  $5\frac{1}{2}$  ft. high, hold?
5. From a pile of wood 16 ft. long, 4 ft. wide, and 8 ft. high, there were sold at one time  $2\frac{1}{4}$  cd., and at another time 12 cd. ft. What is the remainder worth at \$5.75 a cord?
6. A fruit-dealer bought  $202\frac{1}{2}$  crates of peaches for \$225, but was obliged to sell them at a loss of 20%. For what were they sold a crate?
7. How many posts 8 ft. 6 in. apart will it take to inclose a rectangular field 23 rd.  $10\frac{1}{2}$  ft. by 179 ft. 6 in.?
8. A cubic foot of water weighs  $62\frac{1}{2}$  lb. If pine wood is 60% as heavy as oak wood, and 40% as heavy as water, how much will a cord of oak wood weigh?
9. If oranges cost me 20¢ a dozen, which is the better offer and what per cent better: 3 cents each, or 20% profit?
10. A clerk who received \$100 a month, paid for living expenses \$800 a year. When his salary was increased 25% he increased his expenses 30%. Did he save more or less than before his increase? and how much?
11. Make out and receipt a bill of goods sold to-day to Mary R. Sullivan, as follows:  $17\frac{1}{2}$  yd. cloth @  $15\frac{3}{4}$ ¢; 2 pr. shoes @ \$3.75;  $1\frac{1}{2}$  yd. silk @ \$2.25.
12. A bushel of corn weighs 56 lb., and a bushel of wheat 60 lb. How many bushels of wheat will weigh as much as 445 bu. of corn?

1. A rectangular field contained 40 acres. Each corner was cut off, forming a triangular lot 50 rd. by 20 rd. What per cent of the field remained?

2. I paid  $\$37\frac{1}{2}$  for a carpet at  $\$1.25$  a square yard. The width of the floor was 15 ft. What was its length?

3. At what rate will  $\$800$  gain  $\$62.50$  in 1 yr. 3 mo.?

4. Sold a span of horses at 30% gain, and with the money bought another span, which I sold for  $\$364$  and lost 12½%. What did each span cost?

5. A man paid  $\frac{3}{4}$  of his money for a farm; had he paid  $\$75$  more he would have paid  $\frac{1}{4}$  of his money. Find the cost of the farm.

6. A horse was sold for  $\$184$  at an advance of 15%. What would it have brought at a gain of 20%?

7. 75% of a farm is cultivated; 80% of the remainder is pasture; and the remainder, 2 A. 80 sq. rd., is woodland. What is the area of the farm?

8. What is the difference between specific and *ad valorem* duties?

9. Why does moving the decimal point to the left two places give two months' interest at 6%.

10. A dealer obtained  $\$360$  for a piano on the list-price of which he had discounted 50%. He still made a profit of 20%. Find the cost and list-price of the piano.

11. Carpeting  $\frac{3}{4}$  yd. wide is used for a room 18 ft. square. The waste in matching is 8 in. to a strip. What is the cost at  $\$1.37\frac{1}{2}$  a yard?

12. The last reading of my gas-meter was 54,700 cu. ft. The previous reading was 47,900. At  $\$1.50$  a thousand, with a discount of 12½%, what was the amount of my gas-bill?

13. If the valuation of a town is  $\$6,400,000$ , and my property is assessed at  $\$11,200$ , how much of a tax of  $\$40,000$  ought I to pay?

1. What is a bank? A bank is an institution chartered by the Government; i.e., given permission to do business.

2. What business is done by banks? They furnish a safe place of deposit for money, they exchange money, issue notes for circulation, borrow and lend money, and collect money on notes and drafts.

3. Suppose Mr. R. J. Bartlett has some money on deposit in the Home National Bank, but is owing \$100 to C. R. Hooker of New York. Instead of sending the money, Mr. Bartlett fills out a blank check like the following:

Holyoke, Mass. .... 19..... No.....

HOME NATIONAL BANK.

Pay to the order of..... \$.....

..... Dollars.

This check is sent to Mr. Hooker, who takes it to any national bank in New York, and they will pay it or collect it for him. All banks have dealings with one another, so that through a "Clearing House" the check comes back to the Home National Bank, and the amount is placed on the book against Mr. Bartlett. In this way banks help in exchanging money.

4. Fill out a blank check.

5. We have learned also that banks can issue notes or bills for circulation. Examine carefully some bank-notes, and see how they read.

NOTE. — Before a bank can issue bills of its own it must deposit with the Treasury Department in Washington Government Bonds equal in amount to the bills issued. The bills are printed by the department and the bonds are held in trust for the security of the bill holders.

6. Who are the stockholders of a bank? They are men who own all the property, and, like partners in other kinds of business, share in the gains and losses.

7. How does the Government try to protect those who deposit their money in the banks? By having the banks examined at stated intervals by Bank Examiners.

8. From what does a bank derive its income? From loaning money, discounting notes, etc.

9. How may one get a note discounted at a bank?

A. C. Bardwell has the following note for \$500.00, taken in business, which he wishes to get discounted at a bank.

\$500. Holyoke, Mass., Sept. 10, 19.....  
 Sixty days ~~~~~ after date I promise to pay  
 to the order of ~~~~~ A. C. Bardwell, ~~~~~  
 ~~~~~ Five Hundred ~~~~~ Dollars,  
 payable at Home National Bank.  
 Value received. A. C. Jones.

(a) A. C. Bardwell must write his name across the back ; i.e., indorse it, and thus become responsible for its payment, if Mr. Jones should fail to pay it when due.

(b) Mr. Bardwell can now take the note to the bank, and if the officials are satisfied that the note is good, they may accept it and loan the money.

(c) The time when the note is due will then be ascertained by adding the time specified in the note to the date, which will make it due Nov. 9th. In some States three days of grace are allowed, in which case the above note will be due Nov. 12th.

NOTE. — Days of grace have been abolished in many States. In Massachusetts and some other states they are still in force on sight drafts. Count them or not according to your location.

(d) As Mr. Bardwell presented the note to the bank Sept. 10, the bank finds the interest on \$500 for 60 days, which is \$5.00, and keeping this as their discount, gives Mr. Bardwell the rest, \$495, called Proceeds or Avails.

(e) If Mr. Bardwell had not taken the note to the bank until a later day, say Oct. 1, the bank would have found the interest on \$500 from Oct. 1 to Nov. 9, or for 39 days, which would be \$3.25, and the proceeds \$496.75.

1. Bank Discount is the interest retained by a bank for advancing money on notes before they become due.

2. The Proceeds, or Avails, is the amount received by the borrower, and is equal to the Face of the note less the Discount.

3. The Term of Discount is the time a note has to run from the date of discount to the date of Maturity ; i.e., the day when the note is due.

4. Notes for discount are usually without interest. Sometimes a man may receive an interest-bearing note. At the time he receives it, or at any time before it is due, he may wish to obtain the money on it. If this note is discounted, the amount at maturity, and not its face, will be the sum discounted.

1. In the note on Page 186, who is the maker? The payee? What is the face? What is the date? Is it a demand or a time note? When is it due?

2. Who is the Indorser of this note? By indorsing it he makes himself liable to what? Under what circumstances will Mr. Bardwell be called upon to pay the note?

3. If the note is not paid by Mr. Jones on Nov. 9, the note is said to have gone to protest, and a notary public notifies the indorser. This notice must be made within 24 hours after the note is due.

4. Give a good reason why a business man is very careful never to allow his note to be protested.

5. How much money belonging to the bank did Mr. B. use for 60 days?

6. What is the interest of \$495 for 60 days?

7. Why does Mr. B. pay the bank \$5 interest instead of \$4.95?

8. What is the difference between simple interest and bank discount?

Practice varies in estimating the time of maturity and term of discount. Some banks reckon the time in exact number of days, others in months and days, others the exact number of days when the time is less than two months, but in months and days when the time is more than two months. It is best to conform to the custom of your own locality. In this book the exact statement, whether months or days, is used in finding the date of maturity. In finding the term of discount the exact number of days is found. Days of Grace will not be used.

9. \$595  $\frac{100}{100}$ .

*Boston, Feb. 10, 1903.*

*Three months after date, I promise to pay to the order of James McKenzie Five Hundred Ninety-five and  $\frac{100}{100}$  Dollars at the City National Bank.*

*Value received.*

*William Kenny.*

Discounted at date at 6%. Find proceeds.

Find the bank discount and proceeds of the following notes:

| FACE.           | DATE.   | TIME.  | DAY OF DISCOUNT. | RATE. |
|-----------------|---------|--------|------------------|-------|
| 10. \$ 846.     | Feb. 7. | 60 da. | Feb. 22.         | 6%.   |
| 11. \$1,450.50. | Mar. 6. | 2 mo.  | Apr. 2.          | 5%.   |
| 12. \$ 375.40.  | Apr. 9. | 3 mo.  | May 6.           | 6%.   |
| 13. \$ 248 60.  | May 12. | 4 mo.  | June 2.          | 5%.   |



Find the bank discount and the proceeds in the following:

| FACH.         | DATE.     | TIME.   | DAY OF DISCOUNT. | RATE. |
|---------------|-----------|---------|------------------|-------|
| 1. \$1,234.   | Sept. 10. | 60 da.  | Sept. 30.        | 6%.   |
| 2. \$2,345.   | Nov. 13.  | 90 da.  | Dec. 10.         | 5%.   |
| 3. \$3,456.   | Aug. 11.  | 45 da.  | Aug. 17.         | 4%.   |
| 4. \$4,567.   | Jan. 5.   | 75 da.  | Feb. 6.          | 6%.   |
| 5. \$5,678.   | July 7.   | 60 da.  | July 14.         | 4%.   |
| 6. \$6,789.   | Nov. 21.  | 2 mo.   | Nov. 21.         | 6%.   |
| 7. \$7,890.   | Oct. 2.   | 3 mo.   | Nov. 1.          | 5%.   |
| 8. \$8,901.   | Feb. 21.  | 4 mo.   | March 11.        | 6%.   |
| 9. \$9,012.   | May 20.   | 100 da. | June 30.         | 8%.   |
| 10. \$9,876.  | July 15.  | 96 da.  | Aug. 1.          | 6%.   |
| 11. \$3,765.  | Dec. 24.  | 80 da.  | Jan. 2.          | 5%.   |
| 12. \$7,654.  | Nov. 18.  | 75 da.  | Dec. 6.          | 6%.   |
| 13. \$6,543.  | Feb. 4.   | 70 da.  | March 17.        | 7%.   |
| 14. \$5,432.  | March 6.  | 60 da.  | March 6.         | 6%.   |
| 15. \$4,321.  | April 17. | 90 da.  | May 5.           | 5%.   |
| 16. \$3,210.  | June 16.  | 3 mo.   | June 16.         | 6%.   |
| 17. \$2,109.  | Sept. 13. | 4 mo.   | Oct. 13.         | 8%.   |
| 18. \$1,098.  | Dec. 30.  | 1 mo.   | Jan. 2.          | 6%.   |
| 19. \$276.60  | June 19.  | 63 da.  | July 5.          | 7%.   |
| 20. \$796.70. | March 20. | 75 da.  | March 29.        | 6%.   |
| 21. \$548.30. | July 21.  | 3 mo.   | July 21.         | 5%.   |
| 22. \$274.    | July 22.  | 3 mo.   | Aug. 11.         | 6%.   |
| 23. \$382.    | Aug. 20.  | 4 mo.   | Oct. 13.         | 4%.   |
| 24. \$496.    | Sept. 18. | 2 mo.   | Sept. 30.        | 5%.   |
| 25. \$518.    | Oct. 16.  | 30 da.  | Oct. 16.         | 7%.   |
| 26. \$736.    | Dec. 12.  | 45 da.  | Jan. 5.          | 6%.   |
| 27. \$448.    | Jan. 10.  | 75 da.  | Feb. 7.          | 7½%.  |
| 28. \$569.    | Feb. 8.   | 90 da.  | March 13.        | 6%.   |
| 29. \$224.    | Sept. 5.  | 2 mo.   | Oct. 1.          | 6%.   |

30. Making yourself the payee, and your teacher the maker, write notes, using data given in the first ten examples.

1. After taking out his commission of 4% and \$80.80 for other charges, an agent remitted to his employer \$1,820, the amount due him on wheat sold at \$.60 a bushel.

2. A man put 16% of his money in the bank, and spent 40% of the remainder. If he had \$1,008 left, how much did he have at first?

3. A man willed 30% of his money to his wife, 20% of the remainder to his children,  $12\frac{1}{2}\%$  of what was left to the city library, and the remainder, \$4,900, to benevolent institutions. How much was the whole property and each share?

4. A father bequeathed \$5,580 to his son. This sum was 25% of what the son already had. How much did the son have after receiving his father's bequest?

5. A man spent \$100 a year for 3 years in repairs on his house, and then sold it for \$100 less than its first cost, and his entire loss was 4% of its cost. Find its cost.

6. S owned a half interest in a manufacturing industry. He sold 12% of his share for \$4,500. At that rate what is the value of my share if I own  $12\frac{1}{2}\%$  of the other half?

7. A widow received 36% of her husband's estate, each of two daughters 22% of it, and the son the remainder. If the widow received \$12,600 less than all the children, what was the share of each?

8. B bought some land for \$2,500, and sold it immediately for \$3,000, taking in exchange a six months' note without interest. How much did he make if he had the note discounted at a bank at 6%?

9. Find the value of  $x$ :

|     | COST.    | SELLING-PRICE. | GAIN OR LOSS. | GAIN OR LOSS %. |
|-----|----------|----------------|---------------|-----------------|
|     | \$ 20.00 | \$ 16.00       | $x$           | $x$             |
| 10. | \$ 40.00 | \$ 44.00       | $x$           | $x$             |
| 11. | \$400.00 | \$ $x$         | $x$           | 10              |
| 12. | \$ 2.50  | $x$            | \$ 1.00       | $x$             |

1. If I pay \$10.50 for having my house insured at  $\frac{1}{2}\%$ , for what amount do I get it insured?
2. I paid \$9.75 for a load of coal at \$6.00 a ton. How much did it weigh?
3. A commission merchant received \$25.00 for selling butter at  $2\frac{1}{4}\%$ . How much did the butter bring?
4. I bought a bill of goods amounting to \$25, with a trade discount of 20%, and 5% off for cash. What was the net amount of the bill?
5. A broker sold cotton to the amount of \$620 at  $2\frac{1}{4}\%$  commission. How much did he receive for his services?
6. A sleigh that cost \$28 was sold at a loss of \$4. What per cent was lost?
7. Smith & Murray sell lace curtains at \$10 a pair, and thereby gain 25%. What did the curtains cost them?
8. When flour is selling at \$4.50 a barrel, a merchant loses 10%. What would be his gain per cent if he sold at \$5.50 a barrel?
9. A grocer bought eggs at 36¢ a doz., and sold them at the rate of 6 eggs for 21 cents. What per cent did he make?
10. A sleigh which cost \$50 was sold for \$40. What per cent was lost?
11. A farmer sold a cow for \$40, which was 80% of the cost. What was his loss?
12. If dress goods sell at 60¢ a yard, a gain of 20% is made. How much is the gain per cent when sold for 70¢ a yard?
13. A grocer, by selling flour at \$6.25 a barrel, gains 25%. What did the flour cost him a barrel?
14. A dealer made \$20 on a buggy by selling at an advance of 20%. For what did the buggy sell?
15. If 20% was gained by selling a parlor chair for \$6.00, what per cent would be gained by selling it for \$7.00?
16. How many sheets of paper in 75% of a quire?

1. A merchant bought, Aug. 2, 1902, 12 bales of cloth (15 pieces in a bale, 45 yd. in a piece), at 7¢ a yd., for which he gave his note on interest at 6%. On Dec. 8, 1902, he sold 3 bales at  $12\frac{1}{4}$ ¢ a yard, and gave the money in part payment of his note. On the 20th of June, 1903, he sold 2 bales at 14¢, and paid it all as part payment on his note. Oct. 26, 1903, he sold the remainder at 15¢ a yard, and settled the note. How much did he gain?

2. A person takes a 3-months' note for \$217.80 in payment for a horse. On getting the note discounted at a bank at 6%, he finds that he has lost 20% of what the horse originally cost him. Find the cost.

3. What is the compound interest on a note for \$500, dated Sept. 18, 1902, and paid Dec. 2, 1903, interest at 6% per annum, payable semi-annually?

4. A mill was insured for  $\frac{3}{4}$  its value at  $\frac{1}{4}$ %. If the premium was \$123.75, what was the value of the mill?

5. Find the proceeds of a note of \$1,440, dated Oct. 14, payable in 90 days, and discounted Nov. 21.

6. I bought a farm of 80 acres of land for \$3,000, and sold it at a profit of \$7.50 an acre. What was the gain per cent?

7. A car-load of peaches was bought at 80¢ a basket, and sold at a loss of  $12\frac{1}{2}$ %. If the loss was \$43.20, how many baskets were in the car?

8. A grocer mixed 30 lb. of 25¢ tea, with 20 lb. of 60¢ tea, and sold the mixture at 52¢ a pound. Did he gain or lose? and what per cent?

9. A merchant has policies of insurance on his goods as follows: \$6,000, at  $\frac{3}{4}$ %; \$11,800, at  $\frac{1}{2}$ %; \$16,200, at 1%; \$8,000 at  $\frac{1}{4}$ %. The goods cost him \$48,000. If a fire should totally destroy the goods, what would be his loss, including the sum paid for insurance?

1. Three boys have together 160 marbles. The first has 50 less than the second, and 40 more than the third. How many has each?

2. Edith is 6 years more than  $\frac{1}{2}$  the age of her brother. If their united ages amount to 42 years, how old is each?

3. Divide 80 cents between two girls so that one shall have  $\frac{3}{4}$  as many as the other.

4. Divide 164 into three parts, such that the first shall be 12 greater than the second, and the second 16 greater than the third.

5. What number multiplied by 8, and then diminished by 13, is equal to 11?

6. Find two numbers which differ by 14, and one is three times the other.

7. A horse and wagon are worth \$300, and the horse is worth 3 times as much as the wagon. What is each worth?

8. Divide \$500 among A, B, and C so that B and C may each have twice as much as A. How many dollars will each have?

9. The sum of three numbers is 140. The second is four times the first, and the third is  $\frac{1}{2}$  the second. What are the numbers?

10. If to a certain number, itself, one-fifth of itself, and 5 be added, the sum will be 104. What is the number?

11. A farmer sold two-fifths of his farm to one man, one-third to another, and had 20 acres left. How large was his farm at first?

12. Three-eighths of what number is 30 less than the number itself?

13. A and B went into business together with a cash capital of \$2,400. If A put in four times as much as B, how much did each invest?

14.  $5x - 4 = 3x + 2$ . Find value of  $x$ .

1. a)  $8 + (3 + 2)$       a) To 8 add 3, and to this sum add 2.  
      b)  $8 + (3 - 2)$       What is the result? Add the two numbers  
                                  within the parenthesis. Add their sum to 8.

Do you get the same result as at first?

b) To 8 add 3, and from their sum subtract 2. What is the result?  
 Subtract the numbers within the parenthesis, and add the difference to 8.  
 Do you get the same result as at first?

2. If an expression within a parenthesis is preceded by the sign +, the parenthesis can be removed without any change.

3. a)  $8 - (3 - 2)$       a) From 8 take 3, and from their differ-  
      b)  $8 - (3 + 2)$       ence take 2. What is the result? Subtract  
                                  the numbers within the parenthesis, and take

their difference from 8. Do you get the same result? If  $a$  is written  $8 - 3 + 2$ , would you get the same result as at first?

b) Add the numbers within the parenthesis, and subtract the sum from 8.  
 What is the result? If  $b$  is written  $8 - 3 - 2$ , would you get the same result?

4. If an expression within a parenthesis is preceded by the sign -, the parenthesis can be removed, provided the sign before each term within the parenthesis is changed, the sign + to -, and the sign - to +.

5. Numbers are grouped by using different forms of the bracket, ( ), [ ], { }, and the vinculum. The line also between the numerator and denominator of a fraction acts as a vinculum.

Remove the parenthesis and unite the terms :

$$6. \quad 16 + (7 - 5). \quad 15 - (7 - 2). \quad 17 - (4 + 3).$$

$$7. \quad 21 - (11 - 7). \quad 18 + (7 - 4). \quad 15 - (9 + 2).$$

$$8. \quad 12 - (7 + 5). \quad 11 - (6 - 2). \quad 14 + (7 - 5).$$

If  $a = 4$ ,  $b = 3$ ,  $c = 2$ , find the value of :

$$9. \quad a - (b + c). \quad 2b - (c + a). \quad 3c - (a - b).$$

$$10. \quad 3a - (2b - c). \quad 3a - (bc - a). \quad 4b - (c^2 + 2a).$$

Find the value of  $x$ :

$$11. \quad 2x - (3 + 4x - 3x + 5) = 4.$$

$$12. \quad (2x - 5) - (x - 4) + (x - 3) = x - 4.$$

$$13. \quad 7x - 5 - (6 - 8x) + 2 = 3x - 7 + 94.$$

$$14. \quad 15x - (6x + 3) = 30 + (5x + 15).$$

1. Draw a triangle that shall contain three acute angles.
2. Draw a triangle that shall contain a right angle. How many acute angles will it contain?
3. Can you draw a triangle that shall contain an acute, an obtuse, and a right angle?
4. Draw a triangle. With your protractor measure each angle. Add the sum. What is the result?
5. Cut a triangle out of paper. Cut off the three angles, and place them so as to show their sum. Fold to show the same result.
6. Write: The sum of three angles of a triangle is equal to two right angles or  $180^\circ$ .
7. Draw the line  $AB$ . At  $A$  make an angle of  $50^\circ$ , and at  $B$  an angle of  $60^\circ$ . Prolong the lines until they meet at  $C$ . How many degrees in the angle at  $C$ ?
8. Substitute  $80^\circ$  and  $40^\circ$  in 7. How many degrees in  $\angle C$ ?
9. Draw a triangle having two angles of  $40^\circ$  each. Are the sides equal? What kind of a triangle is it?
10. How do the angles at the base of an isosceles triangle compare?
11. Draw a triangle having two angles of  $60^\circ$  each. How many degrees in the third angle? How do the sides of this triangle compare?
12. In 11 what kind of a triangle was drawn?
13. If all its sides were equal, it was an equilateral triangle.
14. Define an equilateral triangle.
15. What is true of the angles of an equilateral triangle?
16. Write: An equilateral triangle is equiangular.
17. Each angle of an equilateral triangle is of what magnitude?
18. Why could it never be other than  $60^\circ$ ?
19. Draw a triangle, no two of its sides to be equal. This is called a scalene triangle.

1. Cut from paper a scalene triangle. Compare its angles by folding.

2. Is it equiangular? Can it be so? Are any of its angles equal? Can they be?

3. Can a scalene triangle be a right  $\triangle$ ? If so, draw one.

4. Can it be an isosceles triangle? If so, draw one.

5. Show why it cannot be equiangular.

Find the third angle of a triangle when two angles are:

6.  $30^\circ 18'$  and  $20^\circ 45'$ ;  $40^\circ 20' 30''$  and  $34^\circ 45' 50''$ .

7.  $65^\circ 41' 35''$  and  $74^\circ 16' 44''$ .

8.  $79^\circ 10' 30''$  and  $11^\circ 44' 12''$ .

9. Find the other angles when one angle of a right triangle is  $35^\circ$ ;  $40^\circ$ ;  $62^\circ$ ;  $30^\circ 30'$ ;  $20^\circ 20' 20''$ .

Take the proposition: "The sum of the angles of a triangle is equal to two right angles." Fill the blanks in the following:

10. The acute angles of a right triangle are — angles.

11. Each angle of an equilateral triangle must be —.

12. In a triangle there can be but one —.

13. In an isosceles triangle the angle at the vertex is  $40^\circ$ ; find the angles at the base.

14. One acute angle of a right triangle is  $40^\circ$ ; what is the other acute angle?

15. The angle at the base of an isosceles triangle is  $60^\circ 30'$ . Find the angle at the vertex.

16. The angle at the vertex of an isosceles triangle is  $50^\circ 40'$ . Find the angle at the base.

17. Make five similar questions for class to solve.

18. What true statement can you make about a triangle, knowing that its three sides are equal?

19. When the three angles of a triangle are equal, what is true of the sides?

20. The two acute angles of a right triangle are equal. How many degrees are there in each angle?



**NOTE.** — If possible show the pupils a stock certificate or a bond.

1. Many business undertakings are so large that many persons must unite to provide the money necessary to carry on the business. If these individuals secure a charter, and elect such officers as a president, secretary, treasurer, and board of directors, the association is called a Corporation, or Stock Company.

2. Banks, railroads, insurance companies, and many manufacturing companies are illustrations of Corporations.

3. The Charter is the certificate given to the corporation, usually by the Legislature of the State, stating its name, object, amount of capital, etc.

4. The amount of money and other property owned by a corporation is called its Capital, or Capital Stock.

5. The capital is divided into equal shares, usually of \$100 each.

6. A person who owns one or more shares is called a Stockholder.

7. Each stockholder receives a certificate of stock, giving the number and value of his shares.

8. A stockholder cannot demand of the corporation the return of his money, but he may sell his shares.

9. If the company is prosperous, these shares will sell for more than they originally cost; that is, above par or at a premium.

10. If the company is not prosperous, the shares will sell for less than cost; that is, below par or at a discount.

11. The profits of the company are called dividends, and are usually distributed annually, semi-annually, or quarterly among the stockholders.

12. Losses are in like manner divided among the stockholders.

13. The dividends or assessments of a stockholder do not depend on the price at which the shares were bought, but on their par value.

1. When Corporations, or National, State, or City Governments, borrow large sums of money, they give bonds; that is, interest-bearing promissory notes.

2. Coupons are certificates of interest attached to the bonds. There are as many coupons attached as there are payments to be made. These coupons are detached, and presented to the corporation when due.

3. Shares of stock are bought and sold in the market like other property.

4. Persons who make a business of purchasing and selling stocks and bonds are called Brokers.

5. Brokers are really commission agents, and are entitled to a commission called Brokerage. This brokerage is always on the par value of stock.

6. Bonds are usually named according to their rate of interest and date of maturity. Thus Mass. 5's '05 means Mass. bonds bearing 5% interest, and payable in 1905.

7. What is the cost of 25 railroad shares at 92, brokerage  $\frac{1}{2}\%$ ?

$\$92 + \$\frac{1}{2} =$  cost of one share. Such statements as "at 92" mean  
 $\$92\frac{1}{2} \times 25 = \$2,312.50.$  *Ans.* \$100 stocks cost \$92.  $\frac{1}{2}\%$  of the  
 the agent pays \$92 for a share, and asks  $\$92\frac{1}{2}$  a share for his work, what will one  
 share cost you? par value of one share is  $\$92\frac{1}{2}$ . If

8. What is the income from the above stock, if it yields an annual dividend of  $5\frac{1}{2}\%$ ?

$\$100 \times 25 = \$2,500.$  Par value. Why do we multiply \$100  
 $5\frac{1}{2}\%$  of \$2,500 = \$137.50. *Ans.* by 25? See 13, page 196.

9. A man invests \$36,000 in bank-stock at 90. It yields  $3\frac{1}{2}\%$  semi-annual dividends. Find his annual income.

$\$36,000 \div \$90 = 400.$  No. of shares. What is the 90? If you  
 divide the amount invested  
 $\$100 \times 400 = \$40,000.$  Par value. by the cost of a share, what  
 will be the result? Where  
 $7\%$  of \$40,000 = \$2,800. *Ans.* does the 7% come from?

1. What must be the price of stock, in order that \$18,400 stock may be bought for \$16,928?

\$18,400 stock costs \$16,928.

\$1 stock costs  $\frac{1}{18400}$  of \$16,928 = \$.92.

The price will be 92.

2. How much 4% stock must be bought to give an income of \$1,280?

\$1,280 : .04 =  $x$  : \$1.

$x$  = \$32,000.

At 4%, what income will be derived from \$1 stock? The income at  $x\%$  is to the income of \$1, as  $x$  is to \$1.

3. U. S. 4's are bought at 133 $\frac{1}{2}$ . What is the rate of income?

Income of \$100 stock at 4% = \$4. Cost of \$100 stock = \$133 $\frac{1}{2}$ . \$4 is what per cent of \$133 $\frac{1}{2}$ ?

4. What amount of bonds at 97 $\frac{3}{4}$  can be bought for \$7,790?

5. I have \$19,971 invested in U. S. 4's. What is my income if I paid 118 $\frac{1}{2}$ ?

6. How much shall I receive from the sale of 85 shares of New England Central at 100 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ ?

7. I sold 8,000 U. S. 4's at 112 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ . What did I receive for them?

8. How many shares at 110 $\frac{3}{4}$  can be bought for \$12,265.50, brokerage  $\frac{1}{2}\%$ ?

9. What annual income shall I receive by investing \$5,765.50 in 6% stock bought at 110 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ ?

10. A man paid \$21,978 for Boston and Maine 6's at 110 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ . What was his income?

11. A man's income is \$1,428. What amount did he invest in 4 $\frac{1}{2}\%$  stock at 103 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ ?

12. What is my rate of income if I buy 7% stock at 139 $\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ ?

13. If you should sell \$14,400 worth of U. S. 5's at par, and invest the proceeds in New York Central 7's at 120, what change would you make in your income?

NOTE. — Give many examples from stock quotations in daily papers.

1. What sum must be invested in stock at 112, which pays 10% annually, to obtain an income of \$5,500?

2. When bonds with a face value of \$10,000 sell for \$9,250, at what per cent below par are they selling?

3. Which will give you the larger income, one share of 7% stock bought at 108, or a 6% stock bought at 105?

4. How much will be gained on 1,000 shares of Pacific Railroad bought at 54½, and sold at 57½, brokerage ½ for each transaction?

5. Find the cost of 10,000 Atchison, Topeka, and Santa Fé R. R. 4% bonds quoted at 99½, 8,000 Central Pacific R. R. 5% bonds quoted at 101½, and 6,000 Erie R. R. 6's quoted at 75, brokerage ½% in each case.

6. Find the income derived from the bonds in the fifth example.

7. A has a farm of 120 acres, which yields him an annual income of \$2.62½ an acre. A real-estate agent sells the farm for \$75 an acre, charging him 3% commission. A invests the net proceeds in 3½% R. R. stock at 87, brokerage ⅓%. How much did he increase his income?

8. How much will you receive for 75 shares of stock in a silver mine, if sold at 53½, brokerage ¼%?

9. I own some 4½% bonds. They yield me annually \$1,800. What is their par value? If my rate of income is 3%, what did they cost me?

10. What is the market-price of railroad stock when \$11,600 stock costs \$14,529, including brokerage ¼%?

11. I bought 118 shares of stock at 52, and sold it at 64, paying ½ brokerage for each transaction. What was my gain?

12. What is the price of stock when \$8,729 will purchase \$11,600 worth of stock?

1. What is the duty on 400 lb. of coffee, at 4¢ a pound?
  2. What is the duty at 25% on a bill of goods, invoiced at \$4,000?
  3. In the first question, is the duty specific or *ad valorem*? In the second question?
  4. If a man owns \$4,500, what will be his tax, if the rate is 2%?
  5. What premium must you pay at 2% for insuring goods worth \$750?
  6. If a man insures his life for \$4,000 at  $2\frac{1}{2}\%$  per annum, what will be his annual premium?
  7. In what time will \$500 double itself at 4%?
  8. In what time will \$400 gain \$48 at 6%?
  9. At what per cent will \$300 gain \$72 in 4 years?
  10. A man sold a cow for \$33, and gained 10%; what did she cost?
  11. A man bought a sleigh for \$35, and sold it so as to gain 20%. Find the selling-price.
  12. A man bought a bicycle for \$100, but sold it at a loss of 16 $\frac{2}{3}\%$ . For what did he sell it?
  13. If it costs a manufacturer \$50 to make a bicycle that he sells for \$75, what is his per cent of profit?
  14. At  $\frac{1}{2}$  of 1%, what is the discount on a bill of \$1,000?
  15. Find the cost of  $3\frac{1}{2}$  yd. of cloth at  $\$3\frac{1}{2}$  a yard.
- NOTE. — Call the integral part of the cost 1 more. Multiply by 3 and add  $\frac{1}{4}$ .
16. Find the cost of  $6\frac{1}{2}$  yd. of cloth at  $6\frac{1}{2}\%$  a yard.
  17. What will  $12\frac{1}{2}$  doz. eggs cost at  $12\frac{1}{2}\%$  a dozen?
  18. What will  $9\frac{1}{2}$  lb. nails cost at  $9\frac{1}{2}\%$  a pound?
  19. What will  $2\frac{1}{2}$  oz. of candy cost at  $2\frac{1}{2}\%$  an ounce?
  20. At  $\$8\frac{1}{2}$  a yard, find the cost of  $8\frac{1}{2}$  yd. of silk.
  21. At  $7\frac{1}{2}\%$  a yard, find the cost of  $7\frac{1}{2}$  yd. of gingham.
  22. Find the quantity of which \$5 is  $12\frac{1}{2}\%$ .

1. What is the discount on a piano, list-price \$800, at  $88\frac{1}{2}\%$  off for cash?

2. A note dated May 5, for \$764.48 payable in 3 mo. with interest at 8%, was discounted June 17, at 6%. Find the proceeds. See 4, page 186.

3. Face of note, \$2,160; date, July 12; time, 5 mo.; discounted Sept. 9. Find the proceeds.

4. Find the interest on \$647.40 for 1 yr. 7 mo. at 7%.

5. If \$43.75 is the premium paid for insuring \$3,500, what is the rate of insurance?

6. My agent purchased for me 9,400 bu. of wheat. His commission on the purchase at 2% amounted to \$141. What did he pay a bushel?

7. I sent an agent \$2,550 with which to purchase wheat, after reserving his commission at  $6\frac{1}{4}\%$ . How much will he invest in wheat?

8. A 90-days' note for \$125 was dated March 5, and discounted March 21. Find the proceeds.

9. A man bought 75 shares of bank-stock at 108 $\frac{1}{2}$ , received a dividend of  $5\frac{1}{4}\%$ , and then sold the stock for 107. How much did he gain?

10. An agent received \$1,507.50 to purchase cloth, after deducting  $\frac{1}{2}\%$  commission. How many yards did he buy at \$.62 $\frac{1}{2}$  a yard?

11. A man borrowed the money at 7%, and bought \$1,875 bu. of wheat at 75¢ a bushel, Sept. 5, 1902. On June 15, 1903, he sold the lot for 87 $\frac{1}{2}$ ¢ a bushel. After paying back the money he had borrowed, and the interest, how much had he left?

12. A man bought stock at 15% below par, and sold it at 10% above par. How much did he make on 115 shares?

13. I sold goods at 25% gain, and bought other goods with the proceeds, and sold them at 20% loss. Did I gain or lose by the operation? and what per cent?

1. Find the cost of painting, at 35¢ a square yard, a church spire whose base is a hexagon 6 ft. on a side, and whose slant height is 65 ft.
2. The area of a triangle is 270 sq. yd., and the perpendicular is 45 ft. Find the base.
3. The distance round a circular park is  $1\frac{1}{2}$  miles. How many acres does the park contain?
4. One of the side walls of a brick building measures 2 rd. long, 22 ft. high, 18 in. thick. How many bricks did it take to build it?
5. If the building above was in the form of a rectangle whose width was one-half of the given length, how many bricks were required for the whole building?
6. A rectangular field is 60 rd. long, and its width is 60% of its length. How many boards 12 ft. long and 8 in. wide will it take to inclose it with a fence 5 ft. high? The boards are placed 4 in. apart and 4 in. from the ground.
7. Find the surface of a sphere 22 in. in diameter.
8. The perimeter of one square field is 400 ft. and of another 320 ft. How many square feet in a field equal in area to both square fields?
9. How many square yards in the sides of a square pyramid whose slant height is 100 ft., and the perimeter of whose base is 54 ft.?
10. How many cubic feet in a stone  $7\frac{1}{2}$  ft. long,  $5\frac{1}{2}$  ft. wide, and  $4\frac{1}{2}$  ft. thick? How many square feet in its surface?
11. A circular field is 60 rd. in diameter. How many acres does it contain?
12. At \$6.50 a cord, a pile of 4-ft. wood 32 ft. long cost \$35 $\frac{1}{4}$ . How high was the pile?
13. If every person needs on an average 28 cu. ft. of air an hour, how many hours will the air in a room  $18' \times 14' \times 9\frac{1}{2}'$  last 9 men?

1. The duty on 625 yd. of silk, at 40% *ad valorem*, is \$550. For how much a yard must the importer sell the silk to clear 15%?

2. The amount of tax to be assessed in a certain city is \$44,382; the taxable property is \$2,850,800; the number of polls, each assessed \$1.50, is 1,080. What is the rate of taxation?

3. A block of buildings worth \$186,000 is insured for  $\frac{3}{4}$  of its value in three companies. The first company takes  $\frac{1}{4}$  of the risk at  $\frac{3}{4}$ % premium; the second,  $\frac{1}{4}$  of the remainder at  $\frac{1}{4}$ % premium; and the third, the remainder at 1% premium. Find the entire premium.

4. If the above block is damaged by fire to the amount of \$80,000, find the amount that each company will be obliged to pay.

5. A company with a capital of \$250,000 declares a dividend of 3% with a surplus of \$6,750. What were the net earnings of the company?

6. A commission merchant in Savannah received \$23,548, with which to purchase cotton after deducting his commission of  $1\frac{1}{2}$ %. Find his commission and the amount expended for cotton.

7. What number less  $16\frac{3}{4}$ % of itself equals 1,017.90?

8. A merchant sold 8% of a piece of cloth. If 128.34 yd. were left, how many yards were there in the piece at first?

9. How many yards of carpeting  $\frac{3}{4}$  yd. wide will carpet a room  $18\frac{3}{4}$  ft. long, and  $16\frac{1}{4}$  ft. wide, if the strips run lengthwise, and there is a loss of 7 in. on each breadth for matching?

10. How many bushels of wheat will fill a bin 6 ft. long,  $3\frac{1}{2}$  ft. wide, and 3 ft. 8 in. deep? Approximate measurement.

11. How many cubic feet in a round timber 8 ft. long, and 2 ft. in diameter?

12. What is the ratio of 6 rd. to 3 yd.?



(Review Ratio and Proportion.)

1. David Jones is in business with a capital of \$2,000. He takes Joseph Smith into the business with him with a capital of \$2,000. Such an association of two or more men is called a partnership. The association is called a firm, or company. The persons so associated are called partners.

2. What is partnership? What are the persons associated in business called?

3. In the firm of Jones and Smith, what is the whole capital? What part of the capital does each furnish?

4. Suppose the firm gains \$800 the first year, how ought this gain to be divided among the partners? Why should each receive one-half of it?

5. Suppose that for the second year Oscar Brown is admitted into the company with a capital of \$4,000. What will the entire capital be now? How much of the \$8,000 will be furnished by Jones? By Smith? By Brown? What part will be furnished by each?

6. If Jones furnishes  $\frac{1}{2}$  of the capital, what part of a gain of \$1,200 will he receive? How many dollars will he receive?

7. How do you divide the gain among the partners?

8. Write: Take the same share of the gain or loss as each partner's capital is of the whole capital.

9. Partnership is also called Distributive Proportion, and the examples can be performed by Proportion.

NOTE. — The whole capital is to a partner's share of the capital as the whole gain is to the partner's share of the gain.

\$8,000 (whole capital) : \$2,000 (Jones's capital) = \$1,200 (whole gain) :  $x$  (Jones's gain).

$$\frac{2,000 \times 1,200}{8,000} = \$300 \text{ Ans.}$$

The capital is the cause; the gain the effect.

10. Three men purchase a store paying as follows: A, \$2,000; B, \$4,000; C, \$3,000. They gain \$6,000. How much does each gain?

11. Three men buy a house for \$5,000. A pays \$1,000; B, \$2,400; C, \$1,600. They rent it for \$600. What is each one's share of the rent?

**NOTE.** — Be sure that the preceding page is understood.

**NOTE.** — It sometimes happens that the capital of the different partners is invested for periods of time of unequal lengths. In this case the profit of each partner depends on two elements, the amount of his capital and the time it is employed. The element of time must be eliminated before the principals of partnership can be applied.

1. A and B enter into partnership; A furnishes \$300 for 2 mo., and B \$200 for 6 mo. They gain \$150. What is each one's share of the profit?

$$\begin{array}{rcl}
 \$300 \times 2 & = & \$ 600 \\
 \$200 \times 6 & = & \$1,200 \\
 & & \hline
 & & \$1,800 \\
 \$1,800 : \$600 & = & \$150 : x \\
 \frac{600 \times 150}{1,800} & = & \$50
 \end{array}$$

It is obvious that \$300 in business for 2 mo. is the same as \$600 in business for 1 month. And \$200 for 6 mo. is the same as \$1,200 for 1 month. \$600 and \$1,200 can now be considered as the respective capitals, and then proceed as on Page 204.

2. A, B, and C engaged in business together. A put in \$6,000, B \$2,000 more than A, and C \$2,000 less than B. The profits were \$6,000. What was each partner's share?

3. A, B, and C entered into partnership. A put in \$4,000 for one year, B \$3,000 for 9 mo., and C \$2,500 for 6 mo. Their profits were \$1,612.50. What was each partner's share?

4. A and B hired a pasture together for \$50. A put in 60 cows for 6 mo., and B put in 90 cows for 4 mo. What should each pay?

5. A, B, and C, enter into partnership. A puts in \$714 for 5 mo., B, \$742 for 7 mo., and C, \$308 for 11 mo., and they gain \$694.40. How much is each one's share?

6. Three contractors agree to dig a canal for \$1,010. A furnishes 30 men for 2 days; B, 20 men for 10 days; and C, 16 men for 9 days. Of the sum how much should A, B, and C receive?

7. A, B, and C agree to build an embankment for \$3,200. A is to furnish 14 men for 30 days; B, 10 men for 40 days; and C, 12 men for 32 days. How much should each receive after paying expenses of \$190?

1. Find the bank discount and proceeds: Face, \$1,500; date, Jan. 5; time, 60 days; day of discount, Feb. 1; rate of discount, 6%.

2. If goods are bought at 20% below list-price, with 5% off for cash, and sold at 14% above list-price, what per cent is gained?

3. On a note of \$800 at 6%, and dated March 4, 1903, are the following indorsements: April 12, 1903, \$75; July 9, 1903, \$150; Sept. 5, 1903, \$90; Dec. 8, 1903, \$200. What was due Jan. 1, 1904?

4. If a man receives \$318.75 as a dividend on \$5,000 of stock, what is the per cent of the dividend?

5. Including  $\frac{1}{8}$ % brokerage, what is the cost of 23 shares of stock bought at  $8\frac{1}{2}$ % discount?

6. A man invested \$7,570 in 5% bonds bought at  $5\frac{1}{4}$ % discount, brokerage  $\frac{1}{4}$ %. What is his annual income therefrom?

7. The assessed valuation of the property of a town is \$2,496,000. The estimate of expenses includes \$4,500 for schools, \$4,800 for streets, \$3,600 for salaries, and \$2,076 for contingent fund. What tax will be required of A, whose real estate is assessed at \$9,000 and personal property at \$650?

8. An insurance company asks \$120 as premium on property insured for \$16,000. At the same rate, what premium will they ask for insuring \$40,000?

9. A and B engaged in business Jan. 1, 1902. A invested \$5,000, and B \$8,000. On Aug. 1 they took in C as a third partner, who invested \$7,000. On Jan. 1, 1903, their net gain was \$7,640. What was the share of each partner?

10. How large an investment in Holyoke City 5's at 105 will give an income of \$1,500?

11. If a man should invest \$16,428 in 4% bonds at 74, what annual income would he receive?

1. What are the proceeds of a 90-days' note for \$789.96, discounted at 7%?
2. If the rate of discount at a bank is  $5\frac{1}{2}\%$ , what will be the proceeds of a 3-months' note of \$570, dated May 23, and discounted June 14?
3. Jan. 1, 1902, A and B engaged in business, each contributing \$6,000. April 1, 1902, C invested \$8,000; Oct. 1, 1902, D invested \$9,000. The gain was \$14,580. Find the share of each.
4. Which yields the greater percentage on the investment, 4% bonds at 80, or 6% bonds at 110?
5. A note for \$1,200 was given Jan. 1, 1900. On the 16th of May, 1900, \$360 was paid; and on the 1st of October, 1902, \$480 was paid. How much was due Dec. 1, 1903, interest at 7%?
6. If the net profits of a mill in 2 years are \$8,118, and the profits of the second year are 20% more than the first year, how much were the profits the first year?
7. 20% of a shipment of potatoes, originally 5,000 bu., were frozen. What per cent will be gained on the lot by selling the remainder at \$1 a bushel, if the cost was  $62\frac{1}{2}\%$  a bushel?
8. How many shares of railroad stock can be bought for \$33,293.75 at  $94\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ ?
9. How many cubic inches are there in a grindstone 5 ft. in diameter, 3 in. thick, and having a hole at the center 4 in. square?
10. Divide \$4.08 among 4 boys in the proportion of 3, 5, 7, and 9. How many cents will each boy receive?
11. A, B, and C formed a partnership. A put in \$2,000 for 10 mo.; B, \$1,800 for 8 mo.; and C, \$3,000 for 6 mo. If they gain \$2,620, what is each partner's share?
12. The sum of two numbers is 21 and their difference is 3. What are the numbers?

1. A horse, wagon, and harness cost altogether \$130. The horse cost twice as much as the wagon, and the wagon 4 times as much as the harness. Find the cost of each.

2. Mr. J. is worth \$6,550 more than S., and they are worth together \$35,978. How much is each worth?

3. I gained \$2,100 by selling  $\frac{3}{4}$  of my property for what  $\frac{7}{8}$  of it cost. At the same rate of profit, what ought I to receive for  $\frac{3}{4}$  of the remainder?

4. Mr. Jencks owns six U. S. 4% bonds of \$500 each. His brother has \$2,000 invested in a business that brings him in the same annual income. What rate per cent does the brother receive?

5. One side of a rhombus measures 16 ft., and the distance between its parallel sides is 10 ft. What is its area?

6. Find the proceeds of a note with the following data: Face, \$870; date, May 9; time, 60 days; date of discount, June 8; rate, 6%.

7. What is the interest on \$465.82 from May 15, 1901, to Jan. 6, 1903, at  $6\frac{3}{4}\%$ ?

8. If an insurance company takes a risk of \$12,000 at  $1\frac{1}{2}\%$ , and reinsures  $\frac{3}{4}$  of it in another company at  $1\frac{1}{8}\%$ , how much does the first company make, if no loss occurs?

9. What is the value of a stock of goods, if \$420 is paid for insurance on  $\frac{3}{4}$  of its value at  $1\frac{3}{4}\%$ ?

10. A merchant buys goods at discounts of 20%, 10%, and 5%. At what discount from the list-price must he sell to gain 25%?

11. A commission merchant sold 6,375 yd. of calico at  $4\frac{1}{4}\%$  a yard, 3,790 yd. of gingham at  $6\frac{1}{4}\%$  a yard, 3,780 yd. of gingham at  $7\frac{1}{4}\%$  a yard. Find his commission at  $2\frac{1}{2}\%$ .

12. A floor 14 ft. 6 in. by 12 ft. 8 in. is covered with carpet a yard wide, laid crosswise of the floor, at a cost of  $87\frac{1}{2}\%$  a yard. Find the cost.

1. One angle of a triangle is  $100^\circ$ . How many degrees in each of the other angles if they are equal?

2. Find the size of each angle of an isosceles triangle, when one of the equal angles is  $50^\circ$ ?

3. Draw a triangle. Measure two of the angles. What must the other measure?

4. Cut a paper isosceles triangle. Can you fold it so as to show that the perpendicular from the vertex divides an isosceles triangle into two equal right triangles?

Draw:

5. An acute isosceles triangle. An obtuse isosceles triangle.

6. A right isosceles triangle. An acute scalene triangle.

7. An obtuse scalene triangle. A right scalene triangle.

8. Define each of the above triangles.

9. Can you draw a triangle that contains two right angles? If not, why not?

10. Can you draw a triangle that contains a right and an obtuse angle? If not, why not?

11. Can you draw a triangle that contains a right and an acute angle? If not, why not?

12. Can you draw a triangle that contains three obtuse angles? If not, why not?

13. Can you draw a triangle that contains two obtuse angles? If not, why not?

14. Can you draw a triangle that contains one obtuse angle? Are the others acute?

15. Two angles of a triangle are: *a.*  $46^\circ$  and  $34^\circ$ . *b.*  $24^\circ 10'$  and  $32^\circ 15'$ . *c.*  $72^\circ$  and  $21^\circ$ . *d.*  $47^\circ 12' 20''$  and  $62^\circ 14' 21''$ . Find the third angle in each case.

16. Draw a triangle whose sides are 4 in., 6 in., and 8 in. What kind of a triangle is it? Measure its angles.

17. The sum of two angles of a triangle is  $122^\circ 45'$ . The third angle is how large?

1. A and B form a partnership with a capital of \$4,000, of which A puts in \$3,000, and B \$1,000. How shall they divide a gain of \$1,200?

2. Two boys, Charles and Henry, invest \$25 in a business, of which Charles invests \$15, and Henry \$10. How ought a gain of \$30 to be divided between them?

3. A, B, and C form a partnership. A puts in \$3,000; B, \$2,000; and C, \$4,000. If they gain \$3,600, how ought it to be divided?

4. How many feet in 10 boards, each 16 ft. long and 6 in. wide?

5. Find the interest of \$3,000 for 20 days at 8%.

6. List-price, \$675. Find the cost at  $33\frac{1}{3}\%$  off.

7. How many times is  $\frac{3}{4}$  in. contained in  $2\frac{1}{4}$  inches?

8. The divisor is  $\frac{3}{4}$ , the quotient is  $\frac{1}{2}$ . What is the dividend?

9. Mr. Brown purchased a horse, harness, and sleigh. The horse cost  $\frac{2}{3}$  and the harness  $\frac{1}{3}$  of the entire cost, and the price of the sleigh was \$30. Find the entire cost.

10. If 24 is  $\frac{3}{4}$  of a number, what is  $\frac{1}{4}$  of the number?

11. A man lost \$230. If he had  $\frac{1}{3}$  of his money left, how much had he at first?

12. Gain 54 cents, rate of gain 27%. Find the cost and selling-price.

13. Cost, 70 cents; rate of gain,  $7\frac{1}{2}\%$ . Find the gain and selling-price.

14. Selling-price, \$56; rate of loss, 20%. Find the cost and loss.

15.  $\frac{1}{3}$  of  $\frac{3}{4}$  of a dollar is 20% of what?

16. If Carrie's money is 25% less than her brother's, his money is what per cent more than hers?

17.  $\frac{1}{4}$  of a number exceeds  $\frac{1}{3}$  of it by 9. What is the number?

18. What is the ratio of 25% of a bushel to 50% of a peck?

1. What is the product of the two equal factors 4 and 4?
2. What is the product of the three equal factors, 4, 4, and 4?

3. A power is the product of equal factors. The product of two equal factors is a second power, or square. The product of three equal factors is a third power, or cube. The product of four equal factors is a fourth power, etc.

4. Each one of the equal factors used in producing the power is a root. If there are two equal factors, each is a second root, or square root. If there are three equal factors, each is a third root, or cube root, etc.

5.  $4^3$  is read "the third power of 4," or "4 to the third power," or "the cube of 4." The figure at the right and a little above the root (in this case 3) is the exponent. It always tells the power desired.

6. Find the powers as indicated:

$$54^3, 67^2, .14^3, 1.42^3, 37^5, 78^3, 45^4, 1.01^5, 36^3.$$

$$2.08^3, .004^2, 28^6, .01^3, 2.02^3, 1.05^4, .7^5.$$

7. A root is indicated by the sign  $\sqrt{\phantom{x}}$ , which is called the radical sign. If any other root than the square root is desired, a figure called the index is placed above the sign.

8. Find the roots indicated:

$$\sqrt{16}, \sqrt[3]{64}, \sqrt[3]{125}, \sqrt[4]{81}, \sqrt[6]{64}, \sqrt[3]{27},$$

$$\sqrt[3]{216}, \sqrt[4]{256}, \sqrt{900}, \sqrt[3]{8000}.$$

9. Square 36.

|                                                                                                    |                                        |                     |
|----------------------------------------------------------------------------------------------------|----------------------------------------|---------------------|
| $36$                                                                                               | $36 = 30 + 6$                          |                     |
| $36$                                                                                               | $30 + 6$                               |                     |
| $36$                                                                                               | $30^2$                                 | $= 900$             |
| $36$                                                                                               |                                        |                     |
| $36 \quad 6 \times 6 = (\text{units})^2$                                                           |                                        | $30 \times 6 = 180$ |
| $180 \quad 6 \times 30 = \text{units} \times \text{tens}.$                                         |                                        | $6 \times 30 = 180$ |
| $180 \quad 30 \times 6 = \text{tens} \times \text{units}.$                                         |                                        | $6^2 = 36$          |
| $900 \quad 30 \times 30 = \text{tens}^2.$                                                          |                                        |                     |
| $1,296 \quad 36 \times 36 = \text{tens}^2 + 2 (\text{tens} \times \text{units}) + \text{units}^2.$ | $30^2 + 2 (30 \times 6) + 6^2 = 1,296$ |                     |



1. In example 9, on page 211, is there any difference between  $30 \times 6$  and  $6 \times 30$ ? Is using them both the same as 2 times the product of the tens by the units?

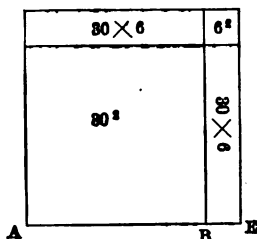
$$\begin{array}{r} 36 \\ 36 \\ \hline 900 = t^2 \\ 360 = 2tu \\ 36 = u^2 \\ \hline 1,296 = t^2 + 2tu + u^2. \end{array}$$

What is the square of the tens? What is two times the product of the tens by the units? What is the square of the units? In this process of squaring, how many partial products are obtained?

NOTE. — This is the algebraic method of squaring numbers.

2. Use this method in squaring the following: 25, 44, 52, 66, 75, 83, 91, 14, 28, 35, 42, 59, 67.

3. To square numbers geometrically.



Square 36. Let the line AB be 30 units long, and BE 6 units long. Construct on AB a square. What will be its area? Construct now on AE a square. How many additions must you make to your first square? Prove from the figure that the square of 36 will consist of  $30^2 + 2(30 \times 6) + 6^2$ .

4. Draw a figure, and square 18, 26, 32, 48, 63, 84, 73, 98.

5. Find the square root of 1,296.

$$\begin{array}{r} 36 \\ 12\ 96 \\ 9\ 00 = t^2 \\ 60 \overline{) 3\ 96} \quad 2tu + u^2 \\ \underline{3\ 60} \quad 2tu \\ 36 = u^2 \\ \underline{36} = u^2 \\ 3\ 6 \\ 12\ 96 \\ 9 \\ 60 \overline{) 39} \\ \underline{36} \\ 36 \\ \underline{36} \end{array}$$

Divide 1,296 in periods of two figures each. In our root we shall have as many figures as there are periods or fraction of a period. In example 1, what three parts did we have in the algebraic formula? What is the greatest square less than 12 in the tens period? What is its root? Place it above the tens period. Taking  $t^2$  away, what two parts remain? If  $t$  is 30, what is  $2t$ ? This is a trial divisor. How many times is it contained in the remainder? Place 6 above the units period. Find the value of  $2tu$ . Subtract it. What remains? Take away  $u^2$ . What then is the square root of 1,296?

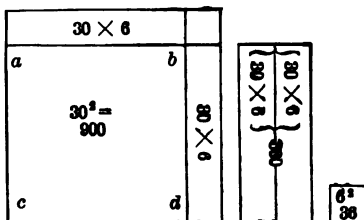
The short form is the same, only unnecessary ciphers are omitted, and only one figure brought down at a time.

6. Learn all the perfect powers between 1 and 100.

**NOTE.** — Page 212 should be thoroughly understood before attempting this page.

**1. Square Root illustrated geometrically:**

This diagram represents a square surface whose area is 1,296. What is the area of the square  $a b c d$ ? Taking this square away, how large an area is left? This remainder consists of how many equal rectangles, and how many squares? Think of these as placed end to end, forming a rectangle. What is its approximate length? If 396 is the area of a rectangle, and 60 is its approximate length, how wide is it? What is the area of each rectangle? Taking away the area of both rectangles, how large an area is left? How does the side of the little square compare with the width of the rectangles? Find the area of the little square, and take it away. What is left?



What is the area of each rectangle? Taking away the area of both rectangles, how large an area is left? How does the side of the little square compare with the width of the rectangles? Find the area of the little square, and take it away. What is left?

Find the square root of:

2. 3364, 3. 6241, 4. 5929, 5. 5329, 6. 8025, 7. 7569,  
8. 1369, 9. 2809, 10. 5184, 11. 4356, 12. 8836, 13. 4489.

**NOTE 1.** — If there are more than two figures in the root, double the root already found for a trial divisor, and proceed as at first.

**NOTE 2.** — When a cipher occurs in the root, place the cipher above its first period, and bring down the other figure of that period and the first figure of the next period.

**NOTE 3.** — When a number is not a perfect square, annex periods of ciphers and continue.

**NOTE 4.** — When a number contains a decimal, begin at the decimal point, and mark toward the left and right to form the periods.

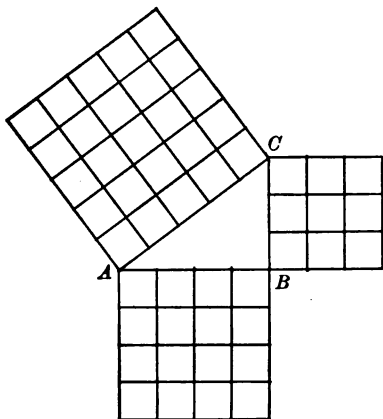
Find the square root of:

14. 12,544. 15. 15,625. 16. 87,025. 17. 93,025.  
18. 77,841. 19. 81,225. 20. 15,376. 21. 38,416.  
22. 27,225. 23. 29,241. 24. 617,796. 25. 334,084.  
26. 538,756. 27. 890,625. 28. 288,369. 29. 278,784.  
30. 214,369. 31. 948,676. 32. 143,641. 33. 823,69.  
34. 285.61. 35. 6.7081. 36. 32.7184. 37. 1866.24.

**NOTE.** — If a perfect square is resolved into its prime factors, the square root will consist of one-half of the number of each different factor.

$1,296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3. \therefore 2 \times 2 \times 3 \times 3 = 36. Ans.$

1. Draw a right triangle with a base 4 in. and a perpendicular of 3 in. On each side draw a square. Divide each square into square inches.



2. How many square inches in each square?

3. Add the square inches in the square of the base and perpendicular. How does the sum compare with the square inches in the square of the hypotenuse?

4. If there are 25 sq. in. in the square of the hypotenuse, what is the length of the hypotenuse?

5. Subtract the square of the base from the square of the hypotenuse. How many

square inches? How does it compare with the square of the perpendicular?

6. If there are 16 sq. in. in the square of the perpendicular, what is the length of the perpendicular?

7. Subtract the square of the perpendicular from the square of the hypotenuse. Extract the square root of the remainder. How does the result compare with the length of the base?

8. How many sides of a right triangle must be known in order to find the remainder? Formulate a rule for finding each side.

9. The base of a right triangle is 12 and the perpendicular

16. What is the hypotenuse?

10. A ladder is 39 ft. in length. How far from the base of a building must the foot of the ladder be placed, in order that the top may reach a window 36 ft. above the base?

1. If the area of a square is 196 sq. ft., what is the length of one side?
2. How much will it cost at \$1.75 a rod to build a fence round a square field whose area is 10 acres?
3. A rectangular lot contains 1,875 sq. yd. If it is 3 times as long as it is wide, what are its dimensions?

NOTE. — Into how many equal squares can this rectangle be divided? How many square yards are there in each square?

4. How much more will it cost to fence a rectangular field 312 rd. long and 78 rd. wide, than a square field of the same area, at \$2.50 a rod?
5. Find the dimensions of a cubical box, if the area of its faces is 12,696 square inches.
6. When 2,255 men were arranged in the form of the largest square possible, 46 men were left over. How many men were in rank and file?
7. A park in the form of a rectangle is 80 rd. long and 72 rd. wide. What is the length in rods of a walk between the opposite corners?
8. Two vessels sail from the same port. One sails due north 8 miles an hour, and the other due west 6 miles an hour. What is their distance from each other at the end of 5 hours?
9. What is the length of a field twice as long as wide, containing 70 A. 50 sq. rd.?
10. A field contains 63 A. 60 sq. rd. Its length is to its breadth as 5 to 3. What will it cost to fence it at 72¢ a rod?
11. A field four times as long as it is wide contains 15½ acres. Find the dimensions.
12. Find how long the rafters must be for a house 32 ft. wide, if the ridgepole is 12 ft. above the attic floor, and the eaves project 2 ft. beyond the walls.
13. The product of what two equal numbers is 5184?

1. A room measures 18 ft. by 24 ft., and is 12 ft. high. Find the distance from one corner of the floor to the corner of the ceiling diagonally opposite.

NOTE. — Find the diagonal of the floor. Use this diagonal as the base, and the height as the perpendicular. See if you can discover a shorter way.

2. Find the side of a square that shall contain as many square feet as an oblong measuring 210 ft. by  $52\frac{1}{2}$  ft.

3. Find the square root of 65.1249.

4. Find the square root of  $1\frac{5}{8}\frac{1}{2}\frac{1}{5}$ .

NOTE. — Take the square root of each term separately when possible. If not possible, change to a decimal.

5. Find the square root of  $\frac{1}{3}\frac{1}{4}$ .

6. How many square yards of canvas will make a conical tent 10 ft. in diameter and 15 ft. high? First find the slant height.

7. What is the largest square that can be cut from a circular cardboard 32 in. in diameter?

8. At  $87\frac{1}{2}$ ¢ a rod, how much more will it cost to fence 25 A. 96 sq. rd. in the form of a rectangle whose length is to its width as 16 to 25, than to fence the same area in the form of a square?

9. A square field, containing 10 A., has round the outside a driveway. If the driveway contains  $\frac{2}{5}$  of the whole square, how wide is it?

10. The foot of a ladder, 52 ft. long, is 20 ft. from the base of a building. How high a window can the top of the ladder reach?

11. What was the height of a tree, standing 25 ft. from a building, the top of which in falling struck the building 30 ft. from the ground?

12. Hypotenuse, 25; base, 15; find the perpendicular.

13. The length of an oblong is three times its width. It contains 768 square feet. How wide is it?

1. Extract the square root of 5,669,161.
2. A block of granite is 38 ft. long, and  $9\frac{1}{2}$  ft. square at the ends. How many cubic feet must be cut away to leave a perfectly cylindrical pillar?
3. A farmer fed  $\frac{1}{3}$  of the corn he raised to his horses,  $\frac{1}{4}$  he sold,  $\frac{1}{8}$  he saved for seed, and the remainder, 520 bu., he has in his granary. How many bushels did he raise?
4. What sum must I invest in Massachusetts 5's, purchased at  $97\frac{1}{2}$ , to get an annual income of \$1200?
5. Find the annual income from investing \$14,229 in New York 6's at  $104\frac{1}{2}$ , brokerage  $\frac{1}{8}\%$ .
6. A 4-months' note for \$564.50 was dated April 9, and bore interest at 6%. If it was discounted May 6, what were the proceeds?
7. On May 24, 1903, Mr. B. borrowed \$1,200. If this sum remained on interest at  $7\frac{1}{2}\%$  until Oct. 7, 1904, what amount would Mr. B. then owe?
8. A lawyer collected  $87\frac{1}{2}\%$  of a bill of \$2,400, and charged 6%. How much did he remit to his client?
9. A merchant sold goods to the amount of \$31,378 this year. If this was  $8\frac{1}{2}\%$  more than he sold last, what was the amount of his sales last year?
10. A man owned 600 acres of woodland. He sold 25% of it to one man, and  $33\frac{1}{3}\%$  of the remainder to another. What part of the land remained unsold? and what is it worth at \$75 an acre?
11. A lot containing 2 A. 120 sq. rd. 186 sq. ft. was sold for \$80. At the same rate, how much land can be bought for \$1,400?
12. Find the inner and outer circumference of a walk  $6\frac{1}{2}$  ft. wide, running round a circular grass plot that measures 90 ft. in diameter.
13. Find the area of the walk in example 12.

1. How deep must a bin 12 ft. square be made to hold 864 bu.? Approximate measurement.
2. Find the area of a right triangle, whose base is 25 ft., and hypotenuse 60 ft.
3. If the circumference of the base of a cone is multiplied by  $\frac{1}{2}$  its slant height, what is obtained?
4. The slant height of a square pyramid is 15 inches, and one side of the base 24 inches. Find its contents.
5. Two poles are 40 ft. apart. One is 50 ft. high and the other 80 ft. How long a line will connect their tops?
6. A house is 24 ft. wide. The ridge-pole is 9 ft. higher than the plate. How long are the rafters if they project 1 ft.?
7.  $\sqrt{2033.1081}$ .  $\sqrt{3444736}$ .
8. A's share of the gain is  $\frac{1}{3}$  of the whole gain. B's capital is \$8,500. What is A's capital?
9. Divide 256 hundredths by 16 hundred-thousandths.
10. Find two important facts from the following data: Amount retained by an agent for purchasing wheat, \$72; rate of commission, 2%; cost of wheat a bushel, 90 cents.
11. What will be the rate of income of a 4% bond, bought at 114 $\frac{1}{2}$ , and  $\frac{1}{4}$ % brokerage?
12. The capital stock of a company is \$200,000. There is a debt this year of \$10,000. If I own 40 shares, how much must I pay of the assessment levied?
13. I bought some railroad stock at 60% premium, paying \$19,200. How many shares did I get?
14. A 60-days' note for \$429 was dated Feb. 21, and discounted Mar. 11, at 4 $\frac{1}{2}$ %. Find the proceeds.
15. If your father places \$600 in the Savings Bank, when the rate of interest is 4% per annum payable semi-annually, how much can he withdraw at the end of 1 yr. 6 mo.?
16. I received from my agent \$7,720, the net proceeds of a sale of flour at 3 $\frac{1}{2}$ % commission. Find the gross proceeds.

1. A square field contains 22 A. 80 sq. rd. At the rate of a mile in 8 min., how long will it take a boy to ride his bicycle round the boundary of the field?

2. At \$1.10 a square yard, it will cost \$495 to carpet a room whose length is double its breadth. Find the length.

3. A man obtains an income of \$60 from an investment of \$1,560 in 5's. What was the market price of the bonds?

4. After spending  $\frac{1}{3}$  of his income, then  $\frac{1}{4}$  of the remainder,  $\frac{1}{5}$  of the second remainder, and  $\frac{1}{6}$  of the third remainder, a man had \$1,200 left. What was his income?

5. A man invested \$76,800 in  $2\frac{3}{4}\%$  bonds at  $95\frac{1}{8}\%$ . How much stock at  $109\frac{1}{8}$  can he buy with his first semi-annual interest; brokerage,  $\frac{1}{4}\%$  in each transaction?

6. The nearest of the fixed stars is estimated to be twenty trillion miles distant. If light travels 190,000 miles a second, how long does it take the light of the star to reach the earth?

7. Find the trade discount on a bill of goods for \$2,920, with 15% and 7% off.

8. What is the cost of concreting the bottom of a circular fountain 70 ft. in diameter at \$1.75 a square yard?

9. A farmer sold some sheep for \$950, and took in payment a 3-mo. interest-bearing note dated Jan. 6, rate  $5\frac{1}{2}\%$ . On March 1 he had the note discounted at 5%. How much did he receive for his sheep?

10. A man, after deducting \$2,000 from his income, pays \$85 income tax on the remainder. If the \$2,000 had not been deducted, the tax would have been \$125. Find his income.

11. The outer diameter of a spherical iron shell is 10 in., and the inner diameter is 6 in. Find the cubic inches of iron in the shell.

12. The breadth of a room is twice its height, and the length is 3 times the height. It cost \$86.40 to paint the walls at 6¢ a square foot. Find the dimensions of the room.



1. A man's assets are \$3,000, and his liabilities \$4,000. How much can he pay on the dollar?

2. A man's resources are \$2,400, and his liabilities \$3,600. How much can he pay on the dollar?

3. A merchant bought a bankrupt stock at 40 cents on the dollar, and sold it at 20% below the original price. How much per cent did he gain?

4. Find the cost of insuring a cargo of goods for \$16,000 at  $\frac{1}{4}$ %.

5. What will it cost to insure a house for \$4,200 at  $2\frac{1}{4}$ %?

6. How much stock will \$6,400 buy at 80?

7. How much stock will \$37,500 buy at 75?

8. If I buy oranges at the rate of 60¢ a score, at how much a dozen must I sell to gain  $33\frac{1}{3}$ %?

9. If by selling an article for \$9.50 I lose 5%, for how much should I sell it so as to gain 5%?

10. 10% of a number is what per cent of  $\frac{1}{4}$  of the number?

11. What fraction of 96 is 12? What per cent?

12. The rent of a house is \$360, which is 12% of its value. What is its value?

13. Find 10% of \$428.

14. Find  $16\frac{2}{3}$ % of \$792.

15. Find  $33\frac{1}{3}$ % of \$624.

16. A horse traveled  $5\frac{1}{2}$  miles in 33 minutes. What was his average time a mile?

17. Make six different numbers with the figures 6, 4, and 8.

18. What number must be subtracted from  $\frac{1}{4}$  to leave .15?

19. Find the number of board feet in a board 12 ft. long, 10 in. wide, and 1 inch thick.

20. Bought a typewriter for \$120, and sold it for \$100. What per cent did I lose? What per cent should I have gained if I had sold it for \$150?

21. 250 is what per cent of 5000?

Find the value of  $x$ :

1.  $7(x+3) - 4(3x-16) = 45$ .    2.  $3x - 20 = -(x+4)$ .
3.  $4x+12 = 2x - (x-21)$ .    4.  $9(x+1) = 12(x-2)$ .
5.  $2(x-6) + 3(2x+5) = 3(3x-2) - 1$ .
6.  $2(x-1) - 2(2x-19) = 3(x-3)$ .
7.  $x - \frac{2x+1}{5} = 2 + \frac{x+3}{3}$ .    8.  $\frac{6x-1}{4} - \frac{3-4x}{6} = \frac{4}{3} - \frac{x}{8}$ .
9.  $\frac{x+3}{4} + \frac{7x-2}{5} = \frac{5x-1}{4} + \frac{5x+4}{9}$ .
10.  $\frac{3x-1}{4} = \frac{2x+1}{3}$ .    11.  $\frac{60-x}{14} - \frac{3x-5}{7} = \frac{3x}{4}$ .
12.  $x+3 + \frac{x-1}{5} = 7 + \frac{x}{2}$ .    13.  $\frac{11-6x}{5} - \frac{9-7x}{2} = \frac{5x-5}{6}$ .
14.  $\frac{x}{6} + \frac{x-8}{4} = 1 + \frac{x-6}{3}$ .    15.  $\frac{x+8}{3} = 2 + \frac{x-6}{7}$ .
16.  $\frac{3x-1}{10} - \frac{x-1}{4} = \frac{2x-31}{3}$ .    17.  $\frac{x-6}{4} + 7 - \frac{2x-3}{3} = -6$ .
18.  $21 + \frac{2x-2}{16} = \frac{5x-5}{8} + \frac{97-7x}{2}$ .
19.  $\frac{3x}{4} - \frac{x-1}{2} = 6x - \frac{20x+13}{4}$ .
20.  $\frac{x-3}{2} + \frac{x}{3} = 20 - \frac{x+19}{2}$ .    21.  $2x - \frac{x+3}{3} + 15 = \frac{12x+26}{5}$ .
22.  $\frac{x+1}{2} + \frac{x+2}{3} = 16 - \frac{x+3}{4}$ .    23.  $\frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} = -2$ .
24.  $\frac{7x-8}{9} + x - 5 = \frac{4x+4}{6} - \frac{x-2}{3}$ .
25.  $\frac{x}{5} - \frac{x-2}{3} = -\frac{x}{2} + \frac{13}{3}$ .    26.  $x + \frac{5x}{7} + \frac{5x}{14} = 116$ .
27.  $\frac{x+5}{2} + \frac{5(x+5)}{6} = 3(x+5) - 20$ .
28.  $3(x-1) - 2(x-3) + (x-2) - 5 = 0$ .

1. A number is as much larger than 10 as 10 is larger than  $\frac{1}{2}$  of the number. What is the number?
2. A certain number and two-thirds of the number equals 15. What is the number?
3. What number is  $5b$  larger than  $10b$ ?  $6x$  larger than  $-3x$ ? 12 larger than  $-10$ ?
4. A drover bought the same number of sheep and cows. For the sheep he paid \$4 each, and for the cows \$32. If he paid \$288 for all, how many did he buy of each?
5. 36 divided by a certain number gives 2 less than 48 divided by the same number. What is the number?
6. Divide 36 into three parts so that the first will equal  $\frac{1}{2}$  the third, and the second will equal the sum of the first and third.
7. To what number can 3 be added making  $\frac{1}{2}$  of the sum equal to  $\frac{1}{2}$  of the number?
8.  $\frac{1}{4}$  of a given number added to 2 equals  $\frac{3}{4}$  of itself plus 1. Find the number.
9. If from  $\frac{1}{2}$  of a number you take 6, the remainder will equal 2 less than  $\frac{1}{4}$  of the number. Find the number.
10. From  $\frac{3}{4}$  of a number take one-sixth of it, and the remainder will equal 6. Find the number.
11.  $\frac{1}{4}$  of a number added to 5 gives the same result as 8 plus  $\frac{3}{4}$  of the same number. Find the number.
12. To  $\frac{3}{4}$  of a certain number 8 was added, and the sum was 12 more than  $\frac{1}{2}$  of the number. Find the number.
13. If from  $\frac{1}{2}$  of a number you take 1, and to  $\frac{1}{4}$  of the number add 2, the results will be equal. Find the number.
14. George has 12 more than  $\frac{1}{2}$  as many cents as his sister Mary. If together they have \$1.74, how many cents has each?
15. The sum of two numbers is 25, and the larger is 3 less than three times the smaller. What are the numbers?

1. A farm is taxed for \$81.60. The rate of taxation is \$13.60 on a thousand, and the assessed valuation is  $\frac{3}{4}$  of the real value. Find the real value of the farm.

2. An insolvent debtor has liabilities of \$40,000, and assets worth \$15,000. How much will a creditor obtain to whom he owes \$4,280?

3. If 3% more be gained by selling a horse for \$133.20 than by selling for \$129.60, what was the original cost of the horse?

4. What per cent of 3 h. 45 m. are 5 m.?

5. A lady paid \$27 for a cloak.  $\frac{2}{3}$  of the cost of the cloak was 90% of the sum paid for other clothing. How much did she pay out in all?

6. At a forced sale, a bankrupt sold a suit of clothes for \$8, which was 20% less than its real value. If the suit had been sold for \$12, what per cent above its real value would it have brought?

7. A man paid \$5,000 for a farm, and then spent a sum equal to 80% of this amount for a new house. He then sold the whole for \$12,000. What per cent did he make?

8. A carriage was sold for \$185, at an advance of 15% on its cost. What would have been the gain per cent if it had been sold for \$222?

9. Ten sheep were sold for \$69, at a gain of 15%. For how much a head on the average should they have been sold to gain 10%?

10. A commission merchant received \$450 as his commission at  $2\frac{1}{2}$ % for purchasing 3,600 bbl. of flour. What was the price paid a barrel?

11. In a city of 3,000 polls, each paying \$1.50, the sum of \$166,500 is to be raised by taxation. The property is assessed at \$13,500,000. What is the tax of a man who pays for one poll and tax on property assessed at \$16,470?

1. One side of a square field of  $22\frac{1}{2}$  A. abuts on a road. This side is divided into building-lots 110 ft. deep, having a frontage along the road of 90 ft. each. The building-lots are sold at \$60 each, and the rest of the field at \$75 an acre. What is the total received?

2. What is the least amount of carpet  $\frac{3}{4}$  yd. wide that is required for a floor 24 ft. by 21 ft.?

3. If a cubic foot of ice weighs  $62\frac{1}{2}$  lb., how many tons of ice can be stored in an ice-house which is 175 ft. long, 36 ft. high, and 18 ft. wide?

4. The hot-air register in our schoolroom is 2 ft. 4 in. by 1 ft. 8 in., and  $\frac{1}{3}$  of the area is taken up by the grating. How much air a minute must pass through each square foot of the opening of this register into the room to supply each of 48 pupils with 4 cu. ft. of fresh air every minute?

5. How many pounds of lead will be required to line an open cistern, whose dimensions are  $5\frac{1}{2}$  ft. long,  $3\frac{1}{4}$  ft. wide, and  $2\frac{1}{4}$  ft. deep, if the lead weighs  $3\frac{1}{4}$  lb. to the square foot?

6. If each person on an average breathes 28 cu. ft. of air in an hour, how many hours will the air in a room 15 ft. long, 12 ft. wide, and 8 ft. high, last 9 persons, supposing the air can be breathed only once?

7. If a man takes 110 steps a minute, and the average length of his step is 30 in., how far can he walk in 2 hours?

8. How high must wood be piled on a car, which is 28 ft. long and 8 ft. wide, to contain 14 cords?

9. Find the cost of papering a room 36 ft. long, 24 ft. wide, 14 ft. high, with paper 18 inches wide. The paper costs 42¢ a roll, and 64 sq. yd. are deducted for openings.

10. A room is 32 ft. long, 20 ft. wide, and 14 ft. high. Allowing for 3 doors, each 8 ft. by 4 ft., 4 windows, each 7 ft. by  $3\frac{1}{2}$  ft., and a wainscot 26 in. high, find the cost of plastering the room at  $12\frac{1}{2}$  cents a square yard.

1. A contractor found that it would take 8 men 12 weeks to do a piece of work. He wished to complete the work in 8 days. How many men must he employ?

2. How many loads of gravel will it require to cover to a depth of 3 in. a path 150 yd. long and 4 ft. wide?

3. Divide .00625 by 250 ; and 62.5 by .025.

4. If a merchant sells  $\frac{3}{4}$  of an article for  $\frac{1}{4}$  of its cost, what per cent does he gain?

5. A started to walk a distance of 80 miles at the rate of 5 miles an hour, stopping to rest 20 min. at the end of every two hours. Two hours and a half after A started, B started to drive the same distance at the rate of 8 miles an hour. If B stopped  $3\frac{1}{2}$  hours to rest his horse, how many hours ahead of A would he reach his destination?

6. From a piece of cloth measuring  $36\frac{1}{2}$  yd., a merchant sold  $8\frac{1}{2}$  yd. at \$1.25 a yard ;  $12\frac{1}{2}$  yd. at \$1.12 $\frac{1}{2}$  ; and the remainder at 95¢ a yard, throwing in  $\frac{1}{4}$  of a yard that was damaged. How much did he make on the whole, if it cost him  $87\frac{1}{2}$ ¢ a yard?

7. A cistern has 3 pipes ; the first will fill it in 2 hr., the second in 3 hr., and the third in 4 hr. In what time will they together fill the cistern? Suppose the water flows out of the first pipe, and in through the second and third, in what time will the cistern be filled?

8. If a bird can fly  $8\frac{1}{4}$  miles in 20 min., how far can it fly in 2 hr. 30 min.?

9. A can do a piece of work in 12 days ; A and C can do it in 9 days ; A and B can do it in 8 days. In how many days can B and C together do it?

10. A and B can finish a piece of work in 25 days. They work together for 15 days, and then A finishes it by himself in 20 days. How long will it take A and B, working separately, to do it?

1. Turn to a map of the hemispheres in your geographies, or, better, study the globe. What are the lines called that extend from the North to the South Pole?

2. Meridian means the line of midday; i.e., all places situated on the same line have midday or noon at the same time.

3. We usually call the meridian that passes through Greenwich the first or prime meridian.

4. Longitude is distance east or west from this meridian.

5. In which direction does the earth revolve? In which direction, then, does the sun *appear* to move?

6. Do places east or west of us see the sun rise first?

7. If the places east of us begin the day earlier, will they have earlier or later time than we have?

8. Into how many degrees is any circle divided? Draw a circle with a 4-inch radius, and divide it into degrees.

9. How many hours are there in one day? Draw a circle with a  $3\frac{1}{2}$  in. radius, and divide it into 24 equal parts.

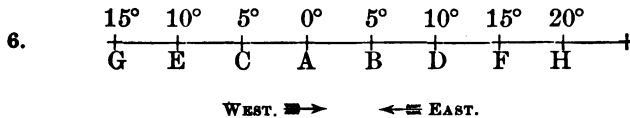
10. Fasten these two circles by an eyelet at the center, so that one can revolve upon the other. You have now a mechanical contrivance for solving all examples in Longitude and Time.

11. Since 360 degrees of the earth's surface passes under the sun in 24 hours, we say 24 hours of time corresponds to 360 degrees of longitude. One hour of time corresponds to how many degrees of longitude?

12. If 1 hour of time corresponds to 15 degrees of longitude, what does 1 minute of time correspond to? One second of time?

13. What is the difference in longitude between two places, if the difference in time is 1 hour? 3 hours?  $4\frac{1}{2}$  hours? 30 minutes? 4 minutes? 2 hours, 15 minutes?

1. Formulate a rule for finding the difference in longitude when the difference of time is known.
2. What is the difference in time of two places, if the difference in longitude is 15 degrees? 60°? 5°? 15'? 45'? 30° 30'?
3. Formulate a rule for finding the difference in time when the difference in longitude is known.
4. When it is noon here, what time is it at a place 15 degrees east of here? 30° west? 120° west? 90° east? 45° 15' west? 15° 30'' east? 45° 15'' west?
5. It is 0° on the meridian of Greenwich. In what direction, and how many degrees distant, is a place whose time is 2 hours earlier? 1 hr. 10 min. earlier? 3 hr. 20 min. later? 2 hr. 15 min. later?



How far apart are A and D? How found? B and F? How found? A and C? How found? C and G? How found? B and C? How found? D and G? How found?

7. From the illustration above, formulate a rule for finding the difference in longitude between two places when both are in east longitude? When both are in west longitude? When one is east and the other west longitude?

8. Table of longitude:

New York, 74° 0' 3'' W.  
 San Francisco, 122° 26' 48'' W.  
 Paris, 2° 20' 22'' E.  
 Constantinople, 28° 59' E.  
 Rome, 12° 27' 14'' E.  
 Washington, 77° 0' 15'' W.

Boston, 71° 3' 30'' W.  
 Canton, 113° 14' 0'' E.  
 Calcutta, 88° 19' 2'' E.  
 St. Louis, 90° 15' 15'' W.  
 Chicago, 87° 35' 0'' W.  
 New Orleans, 90° 3' 28'' W.



1. Find the difference in longitude between Boston and each of the other places given in the table on page 227.

2. When it is 2 P.M. at A,  $60^\circ$  west longitude, what time is it at B,  $90^\circ$  east longitude?

$$60^\circ + 90^\circ = 150^\circ$$

$150^\circ$  corresponds to 10 hr.

$$2 \text{ P.M.} + 10 \text{ hr.} = 12 \text{ midnight}$$

How do you find the difference in longitude between two places? How do you change difference in longitude to difference in time? Is B east or west of A? In going east will you find earlier or later time? If you wish to find later time, do you add the difference to the given time or subtract it?

3. Directions for using the chart: Turn the hour dial until 2, the hour at A, coincides with  $60^\circ$  W., the longitude of A. Find  $90^\circ$  E., the longitude of B, and on the corresponding dial read its time.

4. When it is 9 A.M. at C,  $120^\circ$  W., it is 12 M. at D. Find the longitude of D.

$$12 \text{ hr.} - 9 \text{ hr.} = 3 \text{ hr.}$$

3 hr. corresponds to  $45^\circ$

$$120^\circ - 45^\circ = 75^\circ \text{ Ans.}$$

How do you find the difference in time between two places? How do you change difference in time to difference in longitude? Does D have earlier or later time? If later time, is it east or west of C? If C is  $120^\circ$  W., and D  $45^\circ$  east of it, how do you find the longitude of D?

5. Directions for using the chart: Turn the hour dial until 9 A.M. coincides with  $120^\circ$ . Find 12, and read the longitude of the corresponding dial.

6. When it is 11 A.M. at Boston, what time is it at Paris?

7. When it is 3 P.M. at New York, what time is it at New Orleans?

8. When it is noon at Canton, what time is it at Washington?

9. When it is 6 A.M. at St. Louis, what time is it at Rome?

10. When it is 4 P.M. at Chicago, what time is it at Boston? Find the missing term in the following:

11. A. Longitude  $20^\circ 15' 20''$  E. Time 6.30 A.M.

B. Longitude  $15^\circ 10' 20''$  W. Time?

1. This difference in time caused such inconvenience to the railroads and persons traveling that in November, 1883, the principal cities and railroads in the United States adopted what is called "*Standard Time*."

2. Four meridians were chosen, 15 degrees apart, as central meridians; these are the 75th, 90th, 105th, 120th. All places  $7\frac{1}{2}$  degrees on either side of each of these meridians form a belt. Thus from  $7\frac{1}{2}$  degrees east of the 75th meridian to  $7\frac{1}{2}$  degrees west of it is called the Eastern Belt, and the solar time of the 75th meridian is the standard time of all places in that belt. The other belts are called the Central Belt, the Mountain Belt, the Pacific Belt.

3. Trace these Central Meridians on the map. Locate each belt. Learn approximately the boundary of each, so as to tell what States lie mostly in each belt.

4. Since these central meridians are just 15 degrees apart, what is the difference in time between the Eastern and Central Belt? The Eastern and Pacific Belt?

5. When it is 2 P.M. at Philadelphia, what time is it at Detroit?

6. When it is 10 A.M. at Denver, what time is it at New Haven?

7. How should a watch be changed in going from Connecticut to Minnesota? From California to Ohio?

8. What is the difference between the true time and the standard time at a place whose longitude is  $85^{\circ}$  W.?  $125^{\circ}$  W.?

Give the difference in standard time, and tell which place has earlier time: —

9. Boston and St. Louis. New York and St. Louis.

10. Washington and New Orleans. Portland, Me., and Seattle.

11. Baltimore and Omaha. Philadelphia and Chicago.

12. How should a watch be changed in going from Boston to St. Louis? From Chicago to Washington?

1. A piece of cloth containing 12 yd. was sold for \$60, at a gain of 25%. What was the gain on each yard?
2. By selling 6 yd. of cloth for \$20 a merchant gained  $\frac{1}{3}$  of what the cloth cost. What did it cost a yard?
3. What will 5 gal. of molasses cost, if 6 pt. cost 45 cents?
4. What will 1 quart of seed cost, if 2 pk. cost \$3.20?
5.  $5\frac{1}{2}$  times 5 is  $\frac{1}{2}$  of what number?
6. If 9 lb. of oatmeal cost \$.54, what will 27 lb. cost?
7. If a pole 8 ft. long cast a shadow  $4\frac{1}{2}$  ft., what will be the length at the same time of day of the shadow of a pole which is 15 ft. long?
8.  $1\frac{1}{2}$  are how many times  $\frac{3}{8}$ ?
9. How many times is  $2\frac{1}{2}$  contained in  $8\frac{3}{4}$ ?
10. A man after spending  $\frac{3}{4}$  of his money found that  $\frac{3}{8}$  of what remained equaled \$12. How much money had he?
11. What part of 2 is  $\frac{3}{4}$  of  $\frac{1}{2}$ ?
12. What part of 3 is  $\frac{1}{2}$  of  $\frac{1}{3}$ ?
13.  $\frac{3}{4}$  of 16 is  $\frac{1}{2}$  of how many times  $\frac{1}{3}$  of 21?
14. If 15 cords of wood are worth \$45, how much are  $3\frac{3}{4}$  cd. worth?
15. Divide the number 45 into two parts which shall be to each other as 7 to 2.
16. Divide the number 14 into two parts which shall be to each other as 4 to 3?
17. Two men hired a pasture for \$40. One put in 5 horses and the other 3 horses. What ought each to pay?
18. Two boys have 41 marbles. If one has 5 marbles more than the other, how many marbles has each?
19. If 9 times a certain number is 24 more than 6 times the same number, what is the number?
20. What number is that, to which if  $\frac{1}{3}$  of itself be added, the sum will be 21?
21.  $12x - 7x = 25$ . Find  $x$ .

1. If it cost \$270 to inclose a rectangular field 50 rd. long and 40 rd. wide, how much will it cost to inclose a square field of the same area with the same kind of a fence?

2. The longitude of Boston is  $71^{\circ} 3' 30''$  west, and that of San Francisco  $122^{\circ} 26' 15''$  west. When it is 4 A.M. at Boston, what is the time at San Francisco?

3. The longitude of Cincinnati is  $84^{\circ} 26'$  west, and that of Berlin  $13^{\circ} 23' 45''$  east. When it is 15 min. 20 sec. past 10 A.M. at Cincinnati, what is the time at Berlin?

4. When it is 10 o'clock A.M. at Philadelphia it is 10 min. past 3 P.M. at Paris. What is the longitude of Paris, if that of Philadelphia is  $75^{\circ} 10'$  west?

5. The perimeter of a square and the circumference of a circle are each 16 rd. Which has the greater area? and how much greater?

6. The parallel sides of a trapezoid are 62 yd. and 48 yd. and the altitude 68 ft. What is its area?

7. At 66¢ a square yard, it cost \$9.90 to pave a triangular space the base of which was 18 ft. What was the altitude?

8. A ladder 60 ft. long stands 15 ft. from a building, and the upper end rests against the building 4 ft. from the top. How high is the building?

9. If I buy 5% bank stock on which there is a semi-annual dividend of \$400, what will it cost at \$125 a share?

10. A board fence 5 ft. high is built round a square piece of land measuring 48 ft. on a side. At \$16.50 per M., find the cost of the boards. At 15¢ a running yard, find the cost of building.

11. Find one of the two equal factors of 811,801.

12.  $1\frac{1}{2}$  is one of 4 equal factors of what number?

13. Find the square root of 2.5 raised to the fourth power.

14. How many tiles, 4 in. by  $2\frac{1}{2}$  in., are required to make a hearth 6 ft. 4 in. long, 3 ft. 9 in. wide.

1. Posts are placed 8 ft. apart, round a square field of 40 acres, and round a rectangular field of 50 acres whose width is  $\frac{1}{2}$  of its length. How many posts are needed for both fields?

2. A lot 88 ft. wide contains  $1\frac{1}{2}$  acres. Find the length of the field.

3. As a wholesale grain-dealer you sell, March 1, to James Merrick, 38 bbl. flour @ \$5.80, 136 bu. corn @ 35¢, 220 bu. wheat @ 66¢, 410 bu. oats @ 35¢, with 5% discount for cash. Make out his bill and note in proper form, supposing he pays one-half cash, and gives his note for 3 mo. for the balance.

4. The difference between two numbers is 144.  $\frac{1}{3}$  of the greater number equals  $\frac{2}{3}$  of the smaller number. Find the two numbers.

5. A grocer bought 150 boxes of oranges at \$2.50 a box. He found 16% spoiled, but sold the rest at \$3 a box. Did he gain or lose? and how much?

6. The base of a triangular lot is 24 ft., the altitude is 45 ft. Find the area.

7. A room is 18 ft. by 20 ft. How many yards of carpeting 27 in. wide are required, if we allow a waste of 9 in. on each breadth for matching figures?

8. A hall measures 40 ft. by 25 ft. by 14 ft. At 36¢ a square yard, find the cost of plastering the room, allowing  $\frac{1}{2}$  for doors and windows and 1 ft. all round for the base-board. There are 10 windows 5 ft. by 10 ft., and 5 doors 5 ft. by 8 ft.

9. Find the cost of 45 bu. 3 pk. 6 qt. of wheat at 75¢ a bushel.

10. Find the cost of a load of lumber consisting of 40 planks, 16 ft. long, 8 in. wide, and  $2\frac{1}{2}$  in. thick, at \$16 per M.

11. From 7 times a certain number subtract 5 times itself and 10 more. The remainder is 6. What is the number?

12. A horse and wagon cost \$115. The horse cost \$5 less than twice the cost of the wagon. Find the cost of each.

1. D commenced business with \$1,800 stock; 2 months later he took in E with a capital of \$1,500, and a month later F with a capital of \$2,400. At the end of the year the firm had gained \$1,164. Find the share of each.

2. A, B, and C are partners with \$4,040 capital. A's gain for the year is \$492, B's \$450, and C's \$270. What capital did each furnish?

3. A man spent  $\frac{1}{3}$  of his money, and then  $\frac{2}{3}$  of the remainder. If he spent \$91 more than he had left, how much had he at first?

4. At \$3 $\frac{1}{2}$  a rod it costs \$420 to fence a field in the form of a square. What will it cost to fence a rectangular field of equal area whose sides are to each other as 2 to 4 $\frac{1}{2}$ ?

5. Bought a horse for \$120, which is  $\frac{2}{3}$  of 5 $\frac{1}{3}$  times the cost of a buggy. Find the cost of the buggy.

6. A and B sailed together from New York. A kept his watch by New York time, and B set his by the sun every day. In a few days the watches were 2 h. 15 min. apart. Whose watch was the faster? In what longitude were they?

7. In grading a lot 162 ft. long and 40 ft. wide, it was found necessary to raise it 15 inches. How many loads (1 cu. yd.) of earth were needed?

8. How many square feet of tin in 140 feet of furnace pipes 8 in. in diameter?

9. A circular park is 60 rd. in diameter. At \$1.85 a rod, what will it cost to fence the park?

10. A lot 6 times as long as it is wide contains 18,481 $\frac{1}{2}$  square feet. What are its dimensions?

11.  $\sqrt{11.56} + \sqrt{2.56} = ?$

12. The square of a number divided by the number equals

34. What is the number?

13. The cube of the fifth power is what power?

14. Cube the square of 9.

(For tables, see page 305.)

1. Examine carefully a meter stick. Into how many equal parts is it divided?

2. Call each part a decimeter; i.e., a tenth of a meter.

3. Into how many parts is each decimeter divided? Call each part a centimeter; i.e., a hundredth of a meter.

4. Into how many parts is each centimeter divided? Call each part a millimeter; i.e., a thousandth of a meter.

NOTE. — The prefixes “*deci*,” “*centi*,” and “*milli*” come from Latin words, and mean a tenth, a hundredth, and a thousandth.

5. A millimeter is what part of a centimeter?

6. A centimeter is what part of a decimeter?

7. A millimeter is what part of a decimeter?

8. Draw several lines of different lengths. Estimate their length. Test your estimate.

9. The multiples of the meter are designated by the Greek prefixes “*deka*” (10), “*hecto*” (100), “*kilo*” (1,000), and “*myria*” (10,000).

10. Fill in the blanks, and learn the abbreviations in the following :

|                         |                      |
|-------------------------|----------------------|
| ..... millimeters (mm.) | = 1 centimeter (cm.) |
| ..... centimeters       | = 1 decimeter (dm.)  |
| ..... decimeters        | = 1 meter (m.)       |
| ..... meters            | = 1 dekameter (Dm.)  |
| ..... dekameters        | = 1 hectometer (Hm.) |
| ..... hectometers       | = 1 kilometer (Km.)  |
| ..... kilometers        | = 1 myriameter (Mm.) |

NOTE. — The word “*meter*” means *measure*. The standard meter is a bar of platinum carefully preserved in Paris. It was intended that the meter should be one ten-millionth of the distance from the equator to the poles. It is 39.37 in. in length.

11. Measure off 10 meters on a string with knots to indicate the meters. Find the length and breadth of the school-yard in dekameters.

1. Draw a square decimeter. Divide the sides of this square decimeter into centimeters. Connect the points. Into how many little squares is the square decimeter divided?

2. How many square centimeters, then, in a square decimeter?

3. Take the upper right-hand square centimeter, and divide its sides into millimeters. Connect the points. Into how many parts is the square centimeter divided?

4. How many square millimeters in a square decimeter?

5. Write the table for Square Measure as follows:

— square millimeters (sq. mm.) 1 square centimeter (sq. cm.).

6. How many square centimeters in a square meter?

7. Find the area of a square whose side is: 72 m. 4.25 m. 545 dm.

8. When the area of a square is as follows, find its side: 625 sq. m. 515.29 sq. m. 63.2025 sq. m. 54.76 sq. Dk.

9. A square park contains 81 sq. Hm. How many trees 20 m. apart can be set out round it?

10. Find the area of a rectangle when the sides measure: 40 m.  $\times$  36 m. 7.5 Km.  $\times$  60 m. 45 Dm.  $\times$  14.24 Dm.

11. Find the other dimension of a rectangle when the area and one side are as follows: 72 sq. m. and 30 m. 11.2 a. and 16 m. 100 ha. and  $\frac{5}{8}$  Km. 7.2 ha. and 0.25 Km.

NOTE. — The square dekameter is usually called an Ar (a.), and the square hectometer a Hectar (ha.), and the square meter a Centar (ca.). They are employed chiefly in measuring land.

12. How many ars in a field 64.5 m. by 42.8 m.?

13. The perimeter of a rectangle is 36 m. If the length is 13.8 m., find the breadth and the area.

14. If the area of a rectangle is 1,500 sq. m., and its length 48m., what is its breadth?

15. The area of a rectangle is 288 sq. m., and the breadth 8 m. Find the length and the perimeter.



1. The base of a triangle is .6 Dm., and the altitude 40 dm. Find the base of a triangle with the same altitude and area twice as large.
2. Two triangles have the same altitude, and their bases are 800 cm. and 2.4 Dm. Find the ratio of their areas.
3. The perimeter of a rectangle is 120 m. If the length is twice the breadth, find the length, breadth, and area.
4. The roof of a tower is in the form of a pyramid with a square base. If a side of the base is 4.6 m., and its slant height 6.2 m., how many square meters of tin are required to cover the roof?
5. Find the area of a trapezoid, if the parallel sides are 4 m. and 300 cm., and the altitude 1.8 m.
6. The area of a trapezoid is 369 sq. m. If the parallel sides are 1,600 cm., and 2.5 Dm., find the altitude.
7. The area of a trapezoid is 12,480 sq. dm., the altitude .64 Dm., and one of the parallel sides 128 dm. Find the other side.
8. One man has a garden in the form of a trapezoid. The parallel sides 6 Dm. and 32 m., and their distance apart 124 dm. His neighbor has a square garden of equal area. Find the side.
9. The diagonal of a trapezium is 32 m. Find the area of the trapezium if the altitudes of the two triangles made by the diagonal are 1.8 Dm. and 200 dm.
10. The sides of a rectangle are 28 m. and 63 m. Find the side of a square equivalent to the rectangle.
11. A square and a rectangle have the same perimeter, 90 m. The length of the rectangle is twice its width. Compare the area of the rectangle with the area of the square.
12. Compare the area of a rectangular field whose length is four times its width, and perimeter 1,000 m., with the area of a square field whose perimeter is 800 m.

1. Learn : 1,000 cubic millimeters = 1 cubic centimeter.  
1,000 cubic centimeters = 1 cubic decimeter.  
1,000 cubic decimeters = 1 cubic meter.
  2. Write the following metric quantities as cubic meters and decimals :  
4 cu. m. 7 cu. dm. 8 cu. cm.    43 cu. m. 19 cu. dm. 8 cu. cm.  
64,532 cu. cm. 34 cu. mm.    48,675 cu. dm. 46 cu. cm.  
6,537 cu. m. 7 cu. cm.
  3. Reduce 13.46 cu. m. to cubic decimeters ; 42,300 cu. dm. to cubic meters ; 12 cu. m. to cubic centimeters ; 412 cu. cm. to cubic decimeters ; to cubic meters.
  4. How many cubic meters in a box 1.40 m. long, 1.10 m. wide, and 0.6 m. deep ?
  5. How many cubic decimeters in a wall 45 m. long, 26 dm. high, and 246 mm. thick ?
  6. How many bricks 20 cm.  $\times$  10 cm.  $\times$  5 cm. will it take to build a solid wall 60 m. long, 2.5 m. high, and 65 cm. thick ?
  7. A hollow cannon-ball measures 5 dm. in diameter. If the diameter of the hollow part is 4 dm., find the volume of iron in the ball.
  8. Compare the surface and volume of a cube whose edge is 3 m. with the surface and volume of a cube whose edge is 1 m.
  9. A box measures 4 dm. by 3 dm. by 2 dm. Compare its volume with the volume of a box if one dimension is doubled. Compare it with another if two dimensions are doubled. Compare it with a third if all three dimensions are doubled.
- NOTE. — Is it necessary, to find the volume, to perform the work in Ex. 9 ?
10. How many sters of wood in a cubical pile, one edge of which is 8 m. ?
- NOTE. — In measuring wood, a cubic meter is called a ster (st.).
11. What is the cost of digging a cellar 12.5 m. long, 6.4 m. wide, and 262 cm. deep, at the rate of 75¢ a cubic meter ?

(For table, see page 305.)

1. A cubic decimeter of water is a liter (l.).
2. The prefixes are the same as in the table for determining length. Write the table of liquid measure.
3. What name will you give to the tenth of a liter? The hundredth of a liter? The thousandth of a liter? What will you call ten liters? A hundred liters? A thousand liters?
4. How many cubic centimeters are there in a liter? In a deciliter?
5. How many cubic millimeters are there in a milliliter?
6. Write 65 l. as dekaliters; as deciliters; as centiliters; at hektoliters?
7. Write 2345 cl. as deciliters; as liters; as dekaliters.
8. Find the price of a liter at \$5 a hektoliter.
9. Find the price of a liter at 3¢ a centiliter.
10. How many hektoliters in a bin 4 m. long, 3 m. wide, and 2 m. deep?
11. A bin is 12 m. long, and 8 m. wide. How deep must it be to hold 1,440 hektoliters of grain?
12. How many liters in a tank 5.6 m. long, 3.25 m. wide, and 1.4 m. deep?
13. A rectangular tank 4.8 dm. long, and 25 cm. wide, contains 56 liters of water. What is the depth of the water?
14. 416 hektoliters of potatoes are put into a bin, 16 m. long, and 5.2 m. wide. What is the height of the bin?
15. Find the volume of a rectangular prism when the length is 45 dm., breadth, 7 m., and height, 340 cm.
16. How many cubes, each with an edge of 1 dm., can be put into a box 16 dm. by .8 m. by 6 dm., inside measurement?
17. Find the volume of a square pyramid, if the height is 4.5 cm. and a side of the base .8 dm.
18. The total surface of a cube is 2,400 square meters. Find its volume.

(For table, see page 305.)

1. The metric unit of weight is called a gram. It is the weight of a cubic centimeter of pure ice-water.

2. The prefixes used with the terms meter and liter are used with the gram. Give the name of the tenth of a gram; of ten grams; of the hundredth of a gram; of a hundred grams; of the thousandth of a gram; of a thousand grams.

3. Write the table of metric weights.

4. Examine carefully the set of weights. Of what are these weights made?

5. Weights representing the fractions of a gram are usually made of thin sheets of aluminum or platinum. They are graduated in the same way as the larger weights.

6. What is the weight of a liter of ice-water? Of a cubic millimeter of ice-water?

7. Change 6.4872 mg. to centigrams.

8. Change 2345 Dg. to decigrams.

9. Change 45789 cg. to kilograms.

10. How many centiliters of water will weigh 146 dg.?

11. How many dekaliters of water will weigh 14.64 Kg.?

12. Add 44 dg.; 4.638 Dg.; and 2.45189 Hg.

13. From 16.4895 Dg. take 244.68 dg.

14. Multiply 2.48 dg. by 2.42, and express the result in dekagrams.

15. Divide 148680 g. by 6.3, and express the result in kilograms.

16. Divide 63.258 Dg. by 39 mg.

17. What will 474.6 Hg. of beef cost at 28¢ a kilogram?

NOTE. — A kilogram is  $2\frac{1}{2}$  lb.

18. At \$6.50 a ton, what will the coal cost to keep a fire a week if 30 kilos (kilograms) are burned each day?

19. Find the weight of water that may be contained in a cistern 4 m. deep, 1.5 m. long, and 1.2 m. wide.

1. A room is 10 ft. by 20 ft., and 10 ft. high. How many square feet on the four walls?
2. A room is 9 ft. square and 10 ft. high. What will it cost to plaster it at 25¢ a square yard?
3. Jennie has a piece of ribbon 15 inches long which contains 45 square inches. How wide is it?
4. Four square feet are what part of 4 feet square?
5. A square contains 144 square feet. How long and how wide is it?
6. How many feet long is a wall 11 ft. high, which contains 264 square feet, not including the top?
7. 360 square feet are  $\frac{1}{6}$  of the square feet in the floor of a room 30 ft. long. How wide is the room?
8. How many acres in a lot 20 rd. by 16 rd.?
9. How many acres in a lot 28 rd. by 32 rd.?
10. How many board feet in 8 boards, each 5 in. wide and 12 ft. long?
11. How many board feet in 40 boards, each 6 in. wide and 10 ft. long?
12. A 2-inch cube is made of inch cubes. One of the inch cubes is what per cent of one row of the 2-inch cube? What per cent of one layer? What per cent of the whole cube?
13. Two inch cubes are what per cent of one row? Of one layer? Of the whole 2-inch cube?
14. Three inch cubes are what per cent of the 2-inch cube? Four inch cubes are what per cent? Six? Seven? Eight?
15. How many  $\frac{1}{8}$ -inch cubes will build a 2-inch cube?
16. How many cords of wood in a pile 16 ft. long, 4 ft. wide, and 8 ft. high? In a pile 24 ft. long, 8 ft. wide, and 4 ft. high? In a pile 40 ft. long, 4 ft. wide, and 16 ft. high? In a pile 32 ft. long, 4 ft. wide, and 8 ft. high?

NOTE. — Picture the cord as a unit, and see how many units there are. Do not change to cubic feet.

In Algebra we have two kinds of quantities, positive and negative. The sign  $+$  is prefixed to positive quantities and the sign  $-$  to negative quantities.

1. If we call gain in business positive, what shall we call the loss?

2. If  $15^\circ$  below zero is negative, what should  $15^\circ$  above zero be called?

3. If direction to the east is called positive, what should direction to the west be called?

NOTE. — In Algebra negative quantities are as real as positive quantities, and compared with positive quantities mean opposite in direction or effect.

4. Change the following expressions, using positive terms for negative: (a)  $-4$  south latitude; (b) I traveled  $-5$  miles to the east; (c) I lost  $-\$2$ ; (d) Subtract  $-6$ ; (e) Add  $-4$ .

$$- \leftarrow \boxed{-} \quad \boxed{+} \rightarrow +$$

5.  $-7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8$ .

1.  $+4 + (+2) = +6$ .

2.  $-4 + (-2) = -6$ .

3.  $+4 + (-2) = +2$ .

4.  $-4 + (+2) = -2$ .

(a) To illustrate (1): place your pencil on  $+4$ , and move in the positive direction two points. Where are you? What is the algebraic sum of two positive quantities?

(b) To illustrate (2): place your pencil on  $-4$ , and move in the negative direction two points. Where are you? What is the algebraic sum of two negative quantities?

(c) To illustrate (3): place your pencil on  $+4$ , and move in the negative direction two points. Where are you? What is the algebraic sum of a positive and a negative quantity, when the positive quantity is the larger?

(d) To illustrate (4): place your pencil on  $-4$ , and move in the positive direction two points. Where are you? What is the algebraic sum of a positive and negative quantity, when the negative quantity is the larger?

6. Add  $-a, -4a, -7a$ , and  $-2a$ .

7. What is the sum of  $8b$  and  $-4b$ ?

Unite the terms in each of the following algebraic expressions:

8.  $4x + 2x - 3x$ .

9.  $4bc + 3bc + 2bc$ .

10.  $4x + 4x - 5x$ .

11.  $2xy - 5xy + 4xy$ .

12.  $3a - 2a + 6a$ .

13.  $8b + 2b - 6b$ .

1. Add  $14a$ ,  $-3a$ ,  $8a$ ,  $-2a$ ,  $-4a$ ,  $7a$ ,  $-a$ ,  $18a$ .

NOTE. — Combine the positive quantities, and then the negative, before finding the algebraic sum.

2. Find the sum of  $-16ab$ ,  $ab$ ,  $7ab$ ,  $-ab$ ,  $11ab$ ,  $-2ab$ .

3. Find the sum of  $15a^2$ ,  $-6a^2$ ,  $5a^2$ ,  $-8a^2$ .

Add the following examples:

$$4. \quad 2ac + 4xy$$

$$5ac - 2xy$$

$$3ac + 2xy$$

$$-2ac + 3xy$$

$$5. \quad 3a - 2b + 6c$$

$$2a + 3b - 5c$$

$$4a - b + 2c$$

$$-5a - 4b - 3c$$

$$6. \quad 4b + 3c, -3b, -5c - 2b, -5c, -6b + c.$$

$$7. \quad 6a - 5c + b, -5b - 4x + 4c - 3.$$

$$8. \quad 5ab - 8a^2b + x^2y + xy^2, 4a^2b - 7ab^2 - 4x^2y + 5xy^2, 3ab^2 + 4a^2b - 3x^2y + 4xy^2.$$

$$9. \quad a^3 + a^2 + a, 2a^3 + 3a^2 - 2a, 3a^3 - 4a^2 + a.$$

$$10. \quad -3x + 2y + z, x - 3y + 2z, 2x + 3y - z.$$

$$11. \quad 5ab + 6bc - 7ac, 3ab - 9bc + 4ac, 3bc + 6ac.$$

$$12. \quad b^3 + 4b^2c - 3bc^2, -4b^2c - 5bc^2 - c^3, 3b^2c + 4bc^2.$$

$$13. \quad x^3 + cx^2 + ax + 5, 3x^3 - 4cx^2 - 6ax + 5, 3x^3 - 3cx^2 - 7ax - 11.$$

$$14. \quad 2a - 3b + c, 15a - 21b - 8c, 3a + 24b + 7c.$$

$$15. \quad 5a - 3c + d, b - 2a + 3d, 4c - 2a - 3d.$$

$$16. \quad 4a - 2ab + 8ab + 15b, -2a + 4ab - 2ab - 12b, b - 2ab + 4ab - a, 2ab - 2a - 4b - ab.$$

$$17. \quad 2a - 5b + 2c, 2b - 5c + 2a, 2c - 5a + 2b.$$

$$18. \quad 6a + 3b + 7, -5a - 5b - 11, -7a - 12b + 6, 3a + 4b - 9.$$

$$19. \quad 4ab + bc - 3x, 3ab + 7bc - 3x, 3bc + 7x + 4, 4x - 2ab + 7, ab + 2x + bc - 3.$$

$$20. \quad 7x + 3y + 8z - 4, 5z - 7 + 3x - 8y, 3y - 5x + 6, -2x, 2 - 4x + 3y - 2z.$$

$$21. \quad 6a - 5b + 2d, 4a - 2b + 3d, 5a - 2b - 4d.$$

Be sure that the pupils understand positive and negative quantities, page 241.

1. Subtraction is the process of finding the difference between two quantities.



-7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7.

Subtract :

|      | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|------|---|----|----|----|----|----|----|----|
| Min. | 4 | -4 | 4  | -4 | 2  | -2 | 2  | -2 |
| Sub. | 2 | -2 | -2 | 2  | 4  | -4 | -4 | 4  |
|      | 2 | -2 | 6  | -6 | -2 | 2  | 6  | -6 |

To illustrate (1), place your pencil on the subtrahend, +2, and move it to the minuend, +4. In what direction did you move? Over how many spaces? What is the algebraic difference?

To illustrate (3), place your pencil on the subtrahend, -2, and move it to the minuend, +4. In what direction did you move? and over how many spaces? What is the algebraic difference?

To illustrate (5), place your pencil on the subtrahend, +4, and move it to the minuend, +2. In what direction did you move, and over how many spaces? What is the algebraic difference?

Illustrate 2, 4, 6, 7, and 8 in the same way.

2. In (1) which is the greater, the minuend or subtrahend? Is the difference positive or negative? In (2) which is the greater, the minuend or subtrahend? Is the difference positive or negative? Test the others in the same way.

3. When you subtract a larger number from a smaller, the difference is always ——. Which of the eight examples above illustrate this?

4. When you subtract a smaller number from a larger, the difference is always ——. Which of the eight examples above illustrate this?

5. From a study of these eight examples, when is the algebraic difference found by adding the minuend and subtrahend? when by subtracting them?



NOTE. — The pupils at first should be required to tell in each case whether they are subtracting a less from a greater or a greater from a less. This will be less confusing than to give them the usual rule for subtraction.

$$\begin{array}{r} 1. \quad 7a \\ \underline{4a} \end{array} \quad \begin{array}{r} 2. \quad 8x \\ \underline{-5x} \end{array} \quad \begin{array}{r} 3. \quad -9y \\ \underline{-4y} \end{array} \quad \begin{array}{r} 4. \quad -7b \\ \underline{2b} \end{array} \quad \begin{array}{r} 5. \quad 12ab \\ \underline{8ab} \end{array}$$

$$\begin{array}{r} 6. \quad -11z \\ \underline{-7z} \end{array} \quad \begin{array}{r} 7. \quad 2x \\ \underline{11x} \end{array} \quad \begin{array}{r} 8. \quad -3y \\ \underline{9y} \end{array} \quad \begin{array}{r} 9. \quad -7abc \\ \underline{8abc} \end{array} \quad \begin{array}{r} 10. \quad -6xy \\ \underline{-9xy} \end{array}$$

$$\begin{array}{r} 11. \quad 14a \\ \underline{-7a} \end{array} \quad \begin{array}{r} 12. \quad -2b \\ \underline{8b} \end{array} \quad \begin{array}{r} 13. \quad 2c \\ \underline{9c} \end{array} \quad \begin{array}{r} 14. \quad 2d \\ \underline{-7d} \end{array} \quad \begin{array}{r} 15. \quad -11xy \\ \underline{-2xy} \end{array}$$

$$\begin{array}{r} 16. \quad 7a - 4b - 6c \\ \quad 3a + 2b - 4c \\ \hline 4a - 6b - 2c \end{array} \quad \begin{array}{r} 17. \quad 4x - 2y - 5z \\ \quad 6x \quad \quad + 5z - a \\ \hline -2x - 2y - 10z + a \end{array}$$

18. From  $6a - 2b + 5c$  take  $a - b + 2c$ .

19. From  $6x - 2y - 3z$  take  $3x - 4y + 7z$ .

20. From  $4ab - 2ac + 3bc + 2bd$  take  $4ab + 3ac - 2bc + 4bd$ .

21. From  $8b^3 - 6abc - 5c^3$  take  $6b^3 + 5c^3 - 11abc$ .

22. From

$2b^4 - 3a^3b^3 + 6ab^2 - 2a^2b$  take  $a^3b - 3ab^2 + 4a^3b^3 - 3b^4$ .

23. From  $9a - 5b + c$  take  $2a + 5b - 2c + 11$ .

24. From  $7a - 5x + 4$  take  $8a - 9x + y^2$ .

25. From  $4abc - 7x + 3y - 24$  take  $7abc + 3x - 4y + 38$ .

26. From  $a + b$  take  $a - b$ .

27. From  $x - y$  take  $x + y$ .

28. From  $7x + 5y - 3a$  take  $x - 7y + 5a$ .

29. From  $4x - c$  take  $-3ab + c$ .

Reduce to its simplest form:

30.  $6a + 2x - [2a + 6x - (4a - 3x)]$ .

31.  $7a - [3a + 7x - (3a - 3x - 5a - 4x)]$ .

32.  $8x - \{6y - [4z - (4x + 2z)]\}$ .

33.  $(a + b - c) - (a - b + c) + (b - a - c) - (c - a - b)$ .

1. Multiply  $a^3$  by  $a^2$ .

$$a^3 = a \times a \times a. \quad a^2 = a \times a. \quad \therefore a^3 \times a^2 = (a \times a \times a) \times (a \times a) \\ = a \times a \times a \times a \times a = a^5.$$

2.  $x^4 \times x^3 = x^7$        $c^4 \times c^3 = c^7$ .

NOTE. — In each of these examples you notice that the exponent of the product is found by adding together the exponents of the factors. This is a general law.

Multiply:

- |                           |                          |                            |
|---------------------------|--------------------------|----------------------------|
| 3. $x^7$ by $x^3$ .       | 4. $a^3c$ by $ac^2$ .    | 5. $b^2x^2$ by $bx$ .      |
| 6. $a^4$ by $a^2$ .       | 7. $a^2x$ by $ax^3$ .    | 8. $c^2x$ by $cx^2$ .      |
| 9. $a^2b$ by $ab^2$ .     | 10. $x^4y$ by $x^2y^3$ . | 11. $xyz^2$ by $x^2yz^3$ . |
| 12. $ab^2c^3$ by $a^2c$ . | 13. $y^3z$ by $xyz$ .    | 14. $c^3d^2$ by $ab^2d$ .  |

Multiply:

- |                                                       |                          |                              |
|-------------------------------------------------------|--------------------------|------------------------------|
| 15. $2a^2$ by $3a^2$ .                                | 16. $6b$ by $4b^4$ .     | 17. $2ab$ by $3a^2b^2$ .     |
| 18. $6xy$ by $5x^3y^2$ .                              | 19. $3b^2c$ by $3bc^2$ . | 20. $3a^2x^2$ by $4a^3x^4$ . |
| 21. If a man is \$2 in debt, how many dollars has he? |                          |                              |

We say he has — \$2. If another man has 2 times as much, would he be out of debt, or would he be deeper in debt?

22.  $-2 \times 2 = -4$ .      A negative quantity multiplied by a positive gives a negative product.

23. Multiply:

|                |                |               |               |                 |                  |                  |
|----------------|----------------|---------------|---------------|-----------------|------------------|------------------|
| $x$            | $x$            | $x$           | $x$           | $x$             | $x$              | $x$              |
| $\frac{3}{3x}$ | $\frac{2}{2x}$ | $\frac{1}{x}$ | $\frac{0}{0}$ | $\frac{-1}{-x}$ | $\frac{-2}{-2x}$ | $\frac{-3}{-3x}$ |

In these examples the multiplicand remains the same, and the multiplier is one less each time. Notice that the products are also one  $x$  less each time. One  $x$  less than  $x$  is 0. What is one  $x$  less than 0? What is one  $x$  less than  $-x$ ? Here we see that a positive quantity multiplied by a negative gives a negative result.

|     |                        |     |                        |     |                       |     |                       |
|-----|------------------------|-----|------------------------|-----|-----------------------|-----|-----------------------|
| 24. | $4x$                   | 25. | $2abx$                 | 26. | $-7c^2d^4$            | 27. | $-8a^2x^3$            |
|     | <u><math>-3</math></u> |     | <u><math>-6</math></u> |     | <u><math>3</math></u> |     | <u><math>4</math></u> |

25. State three laws of multiplication learned from this page.

1. On page 245, Ex. 23, make the multiplicand  $-x$ . The products will be  $-3x$ ,  $-2x$ ,  $-x$ ,  $0$ ,  $x$ ,  $2x$ ,  $3x$ . The multiplier is one less in each case. Do the products grow smaller or larger? Two negative quantities multiplied together give a positive product.

|                                           |                                        |                                            |                                              |
|-------------------------------------------|----------------------------------------|--------------------------------------------|----------------------------------------------|
| 2. $-4a^2b^2$<br><u>      -7      </u>    | 3. $-2x^2y^3$<br><u>      -7      </u> | 4. $-3a^2x$<br><u>      -7      </u>       | 5. $-12cd^2$<br><u>      -7      </u>        |
| 6. $-5ab^2c$<br><u>      -7bc^2      </u> | 7. $-8xyz$<br><u>      -2axz      </u> | 8. $-9a^2d^2$<br><u>      -3a^3d      </u> | 9. $-7x^2y^3$<br><u>      -4x^3y^4      </u> |

10. Multiply  $x^2 + 2xy + 2yz + y^2$  by  $4xy$ .
11. Multiply  $a^b - 3a^3b + 4a^2b^2 + b^4$  by  $2a^2b^2$ .
12. Multiply  $x^4 - 4x^3y + 4x^2y^2 - 2xy + y^2$  by  $3xy^2$ .
13. Multiply  $a^3 - 2a^2c - 3ac^2 + 2a^3c^3$  by  $5a^2b$ .
14. Multiply  $a^2 + b^2 + c^2 - ab - bc - ac$  by  $abc$ .
15. Multiply  $3x^2y^2 + 2xy^3 - 5x^3y - 2xy^3$  by  $5x^2y^2$ .
16. Multiply  $4a^4 - 2a^3b - 3a^2b^2 - 2a^2$  by  $4a^2b^3$ .
17. Multiply  $2a^2b - 3ab^2 - 4a^2b^2 + 2ab$  by  $3ab^2$ .
18. Multiply  $a^3 - 6ay + 2y^2 - 2a^2$  by  $6a^3y^3$ .
19. Multiply  $a^4 + 6a^2b^2 + b^4 - 4a^3b - 4ab^3$  by  $2ab^3$ .

|                                                                                                                                    |                                                                                                                      |
|------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| 20. $a^2 + ay + y^2$<br><u>      a - y      </u><br>$a^3 + a^2y + ay^2$<br><u>      - a^2y - ay^2 - y^3      </u><br>$a^3$ - $y^3$ | 21. $2x - 4y$<br><u>      x - 2y      </u><br>$2x^2 - 4xy$<br><u>      - 4xy + 8y^2      </u><br>$2x^2 - 8xy + 8y^2$ |
|------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|

22. Multiply  $x^2 + 4x + 5$  by  $x + 3$ .
23. Multiply  $a^4 - 2a^3b + 3a^2b^2 - 5ab^3 + 7b^4$  by  $a + 4b$ .
24. Multiply  $x^4 - x^3 + x^2 - x + 1$  by  $x + 1$ .
25. Multiply  $1 + 3x - 7x^2$  by  $1 - 5x + 4x^2$ .
26. Multiply  $x^2 + x - 2$  by  $x^2 + x - 4$ .
27. Multiply  $a^3 - 3a^2 + 3a - 1$  by  $a^2 + 3a + 1$ .
28. Multiply  $x^2 - 2xy + y^2$  by  $x - y$ .

NOTE.—Since division is the inverse of multiplication, it follows that the sign of the quotient must be + when the divisor and dividend have like signs and — when the divisor and dividend have unlike signs.

If  $a^3 \times a^2 = a^{3+2} = a^5$ , it follows that  $a^5 \div a^3 = a^{5-3} = a^2$ .

1. Divide  $15 a^2 b^3 c^4$  by  $3 abc^2$ .

$$\frac{15 a^2 b^3 c^4}{3 abc^2} = 5 ab^2 c^2.$$

2.  $\frac{96 x^3 y^4 z^2}{-12 x^2 y^2 z} = -8 xy^2 z.$

3.  $a^4 b^3 c^2 \div a^2 bc = ?$

4.  $25 a^3 b^3 c^4 \div -5 ab^3 = ?$

5.  $-18 a^2 b^2 c \div 2 ac = ?$

6.  $3 x^2 y^2 z^3 \div xy^2 z^2 = ?$

7.  $21 axy^3 \div -3 ay = ?$

8.  $-28 a^2 bc \div -7 ab = ?$

9.  $-18 a^3 x \div -6 ax = ?$

10.  $15 axy^2 \div -3 ay = ?$

11.  $3 a^2 \overline{) 12 a^5 - 9 a^3 b + 6 a^2 c.}$   
 $4 a^3 - 3 ab + 2 c$

12.  $-3 yz \overline{) 3 xyz + 12 bxyz - 9 y^2 z}$   
 $-x - 4 bx + 3 y$

13.  $-12 x^2 yz + 9 xyz - 6 xy^2 z \div -3 xy = ?$

14.  $25 a^2 bc - 15 a^2 bc + 5 a^3 bcx^2 \div -5 a^2 c = ?$

15.  $6 a^4 y^2 - 8 a^3 y^3 + 12 a^2 y^4 \div 2 ay = ?$

16.  $-36 a^3 x + 54 a^2 x - 18 ax + 18 x = ?$

17.  $3 x^4 - 12 x^3 + 15 x^2 \div -3 x^2 = ?$

18.  $15 a^2 bc - 10 a^3 b^4 c^5 y^3 + 5 a^2 b^3 c^2 \div -5 abc = ?$

19.  $4 x^3 + 36 ax^2 - 16 x \div -4 x = ?$

20.  $3 a^3 - 9 a^2 b - 6 ab^2 \div -3 a = ?$

21. Divide  $6 a^2 b - 3 a^3 b + 6 a^2 b^2 - 9 ab$  by  $3 ab$ .

22. Divide  $8 xy^3 + 4 x^2 y^2 - 4 xy^3 + 12 x^3 y^2$  by  $4 xy^2$ .

23. Divide  $2 a^3 c - 4 a^2 c^2 + 6 a^2 c^3 + 8 a^3 c$  by  $2 a^2 c$ .

24. Divide  $4 a^4 x^2 - 2 a^3 x^3 + 4 a^2 x^3 - 6 a^3 x^4$  by  $2 a^2 x^2$ .

25. Divide  $6 x^5 y^3 + 12 x^4 y^5 - 18 x^7 y^5 - 24 x^5 y^8$  by  $6 x^4 y^3$ .

26. Divide  $18 x^{11} y^2 + 27 x^{13} y^6 - 45 x^9 y^8$  by  $-9 x^7 y$ .

27. Divide  $14 a^5 b^3 + 28 a^7 b^4 - 7 a^8 b^5 - 21 a^6 b^6$  by  $7 a^4 b^2$ .

28. Divide  $15 a^6 y^6 - 12 a^7 y^4 + 18 a^3 y^3 + 21 a^9 y^2$  by  $-3 a^6 y^2$ .

29. Divide  $8 b^4 x^3 + 24 b^5 x^7 + 16 b^6 x^8 - 40 b^7 x^5$  by  $8 b^4 x^5$ .

30. Divide  $21 a^4 b^3 c + 14 a^3 b^2 c^2 - 7 a^2 b c^3$  by  $7 abc$ .

1.

$$\begin{array}{r|l}
 x^3 - 3x^2y + 3xy^2 - y^3 & x^2 - 2xy + y^2 \\
 x^3 - 2x^2y + xy^2 & x - y \\
 \hline
 -x^2y + 2xy^2 - y^3 & \\
 -x^2y + 2xy^2 - y^3 & \\
 \hline
 & 
 \end{array}$$

Divide the first term of the dividend by the first term of the divisor. What is the first term of the quotient? Multiply the whole divisor by this term. What

is the product? Subtract it from the dividend. What is the remainder? Consider the remainder as a new dividend, and proceed as at first.

2.

$$\begin{array}{r|l}
 x^4 - y^4 & x - y \\
 x - x^3y & x^3 + x^2y + xy^2 + y^3 \\
 \hline
 x^3y - y^4 & \\
 x^3y - x^2y^2 & \\
 \hline
 x^2y^2 - y^4 & \\
 x^2y^2 - xy^3 & \\
 \hline
 xy^3 - y^4 & \\
 xy^3 - y^4 & \\
 \hline
 & 
 \end{array}$$

3.

$$\begin{array}{r|l}
 a^3 - 3abc + b^3 + c^3 & a + b + c \\
 a^3 + a^2b + a^2c & a^2 - ab - ac + b^3 - bc + c^2 \\
 \hline
 -a^2b - a^2c - 3abc + b^3 + c^3 & \\
 -a^2b - ab^2 - abc & \\
 \hline
 -a^2c + ab^2 - 2abc + b^3 + c^3 & \\
 -a^2c & -abc - ac^2 \\
 \hline
 ab^2 - abc + ac^2 + b^3 + c^3 & \\
 ab^2 & +b^3 + b^2c \\
 \hline
 -abc + ac^2 - b^2c + c^3 & \\
 -abc & -b^2c - bc^2 \\
 \hline
 ac^2 + bc^2 + c^3 & \\
 ac^2 + bc^2 + c^3 & 
 \end{array}$$

4.  $a^3 - y^3 \div a - y = ?$  5.  $x^6 - y^6 \div x^2 + x^2y^2 + y^4 = ?$

6.  $x^4 - 9ax^3 + 12a^2x^2 + 35a^3x + 15a^4 \div x^2 - 4ax - 3a^2 = ?$

7.  $6x^4 + 21x^3y + 31x^2y^2 + 27xy^3 - 5y^4 \div 3x^2 + 6xy - y^2 = ?$

8.  $a^3 - 8a - 3 \div a - 3 = ?$

9.  $x^2 - 2xy + y^2 - z^2 \div x - y - z = ?$

10.  $x^3 + y^3 + z^3 - 3xyz \div x + y + z = ?$

11.  $x^5 + y^5 \div x + y = ?$

12. Divide  $8a^3 + 8a^2b + 4ab^2 + b^3$  by  $2a + b$ .

13. Divide  $x^2 + 11x + 30$  by  $x + 5$ .

14. Divide  $9x^2 - 3x - 2$  by  $3x - 2$ .

15. Divide  $9x^3 - 18x^2 + 26x - 24$  by  $3x - 4$ .

16. Divide  $a^4 - b^4$  by  $a - b$ .

17. Divide  $a^3 - 3a^2b + 3ab^2 - b^3$  by  $a^2 - 2ab + b^2$ .

1. *Mathematics* is the science that treats of measuring of quantities to ascertain their properties and relations.

2. This measuring demands a unit of measurement, as 1 ft., 1 oz., 1 two-dollar bill, 1 tenth, 1 doz., etc.

3. Measuring quantity by a unit of measurement demands number.

4. *Number* answers the question, How many?, or shows the ratio of the quantity measured to the unit of measurement.

5. *Arithmetic* is the science of numbers and the art of using them.

6. *Algebra* is that branch of mathematics which reasons about quantity by the use of letters.

7. *Geometry* is that branch of mathematics which treats of space and its relations, and the measurement of lines, angles, surfaces, and solids.

#### NOTATION AND NUMERATION.

8. *Notation* is the art of expressing numbers by symbols or characters.

9. *Numeration* is the art of reading numbers that are expressed by figures.

10. There are two systems of notation, the Arabic and the Roman.

11. The Arabic is the system in general use, and is so called because it is supposed that it was introduced into Europe by the Arabs.

12. The Arabic system employs ten characters, called *figures*, to represent numbers, thus, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

13. The first of these characters is called naught, cipher, zero. The other characters are called digits. By means of these characters any number can be written.

1. The position occupied by any figure gives it its value. A figure standing alone or immediately at the left of the decimal point represents single units, or units of the first order.

2. Ten units of any order make one unit of the next higher order, and the orders increase in value from right to left.

3. A figure at the left of the single-units figure represents tens of the unit, or units of the second order. A figure at the right of the single-units figure represents tenths of the unit, or units of the first decimal order.

4. For convenience three orders form a period.

5. From right to left the names of the periods are, *units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quattuordecillions, quindecillions, sexdecillions, septendecillions, octodecillions, novemdecillions, vigintillions.*

6. At the right of the decimal point, and reading from left to right, the names of the decimal orders are *tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ten-millionths, hundred-millionths, billionths, ten-billionths, etc.*

7. Use the word *and* in reading numbers in place of the decimal point only.

Write in figures :

8. Forty thousand, six and five tenths.

9. Twenty thousand, four hundred and four hundredths.

10. Three hundred and three hundred-thousandths.

11. Three hundred three hundred-thousandths.

12. Twenty-eight million, two thousand eighty-five and sixteen millionths.

13. Six quadrillions, two hundred trillions, three hundred six millions, forty-four, and five hundred-billionths.

14. One hundred sixteen trillion, two thousand and one million, four hundred six hundred-millionths.

NOTE. — See Part II, pages 162, 163.

Read the following numbers :

1. 406.00608. 40,040,040,040. 4,060,001,604.00004.
2. 1.0106487. 111,010,001,070. 28,014,643,005,004.72.
3. 100001.01. 200,200,002,002. 58,076,104,074.01016.
4. 36,071,026,108,000,010,042. 5000.0000004648.
5. 1,010,100,000,600,010,002,074. 16004.0001017484.
6. 100.002. .102. 600.16. .616. 300.025. .325.
7. 216.00112. 100.008008. 106.000000106.
8. 1000.001. 711,468.143464. 1,111,111,111.000111.
9. 216.000216. 114,106.00048. 2,002,002,020.0202.
10. 400,000,010.01. 7,000,000,007.7. 4,400,040,004.004.

Express the following numbers in figures :

11. Five million, three, and five millionths.
12. Four billion, two million, sixty-four thousand, nine.
13. Six million, six thousand, six hundred six and six hundredths.
14. Sixteen billion, fourteen million, twelve thousand, twenty-two, and twelve hundred-thousandths.
15. Forty-seven thousand twenty-four ten-billionths.
16. One thousand six and twelve thousand four hundred-millionths.
17. Eighty-one million, five hundred eight thousand, one hundred seventy-eight and six thousand thirty-two ten-thousandths.
18. Two hundred million, seventy-two thousand, seven hundred fifty-four and two ten-billionths.
19. One billion, twenty million, three hundred four thousand, fifty and one hundred five thousandths.
20. Six billion, eight hundred seventy million, twenty-eight thousand two hundred six and one hundredth.
21. One hundred twenty-five thousand ten and sixty-seven thousandths.



1. In this system seven letters are used to express numbers :

|         |   |   |    |    |     |     |      |
|---------|---|---|----|----|-----|-----|------|
| Letters | I | V | X  | L  | C   | D   | M    |
| Value   | 1 | 5 | 10 | 50 | 100 | 500 | 1000 |

2. All other numbers are expressed by writing two or more of these letters side by side.

3. The following four principles must be learned :

(a) When a letter is followed by another of equal or less value the number expressed is equal to the *sum* of the simple values, thus, III = 3, XV = 15, CC = 200, LXX = 70.

(b) When a letter is followed by one of greater value than itself, the number denoted by the expression is the difference of their values, thus, XC = 90, IX = 9, CD = 400.

(c) When a letter is placed between two letters, each of greater value, its value is subtracted from the sum of the other letters to find the value of the expression, thus, XIV = 14, DXL = 540, XIX = 19.

(d) A dash or bar placed over a letter multiplies its value by 1000, thus,  $\overline{V}$  = 5000,  $\overline{XIV}$  = 14,000,  $\overline{CLX}$  = 100,060.

Express in Arabic notation :

4. XLV, CCCV, MMM, DXC, MMD.
5. LXV, XLVI, XCIV, XCIX, LXXXIX.
6. CIX, DLIV, MDCL, MXCV, MDCCCXIX.
7. XCI,  $\overline{CM}$ VI, MCDX,  $\overline{CC}$ XC,  $\overline{CD}$ CCXLIX.
8. DCXC, DCCX, CLXXV, XXIX, CCCCLXVIII.

Express in Roman notation :

9. 16, 24, 38, 52, 65, 78, 89, 91.
10. 156, 204, 560, 478, 892, 999, 312, 463.
11. 1186, 1776, 1890, 1896, 1900, 1902, 1492, 1886.
12. 1209, 1680, 1756, 1876, 1879, 1905, 1910, 5648.

1. *Addition* is the process of uniting two or more numbers into one number. Each of the quantities added must have the same measuring unit; that is, they must be of the same kind.

2. The numbers to be united are called *addends*.

3. The *sum* is the number obtained by adding.

4. The *sign of addition* is +, and is read plus.

5. The *sign of equality* is =, and is read equals. Thus  $5 + 4 = 9$  is read, 5 plus 4 equals 9.

6. There are two principles to be memorized:

(a) Only like numbers can be added.

(b) The sum is the same denomination as the addends.

7. Drill on the following forty-five combinations till accuracy and rapidity are secured.

2,  $\frac{1}{1}$ ; 3,  $\frac{2}{1}$ ; 4,  $\frac{2}{2}, \frac{3}{1}$ ; 5,  $\frac{3}{2}, \frac{4}{1}$ ; 6,  $\frac{3}{3}, \frac{4}{2}, \frac{5}{1}$ ; 7,  $\frac{4}{3}, \frac{5}{2}, \frac{6}{1}$ ; 8,  $\frac{4}{4}, \frac{5}{3}, \frac{6}{2}, \frac{7}{1}$ ;

9,  $\frac{5}{4}, \frac{6}{3}, \frac{7}{2}, \frac{8}{1}$ ; 10,  $\frac{5}{5}, \frac{6}{4}, \frac{7}{3}, \frac{8}{2}, \frac{9}{1}$ ; 11,  $\frac{6}{5}, \frac{7}{4}, \frac{8}{3}, \frac{9}{2}$ ; 12,  $\frac{6}{6}, \frac{7}{5}, \frac{8}{4}, \frac{9}{3}$ ;

13,  $\frac{7}{6}, \frac{8}{5}, \frac{9}{4}$ ; 14,  $\frac{7}{7}, \frac{8}{6}, \frac{9}{5}$ ; 15,  $\frac{8}{7}, \frac{9}{6}$ ; 16,  $\frac{8}{8}, \frac{9}{7}$ ; 17,  $\frac{9}{8}$ ; 18,  $\frac{9}{9}$ .

8. All problems in addition are simply repetitions of these problems, though the numbers may not always occur in the same order. Thus the numbers may be  $\frac{7}{8}$  or  $\frac{8}{7}$ , the sum remaining the same.

9. Enlarge each of these combinations thus:—

(a) Place a tens figure before one of the addends, thus,

$\begin{array}{r} 12 \\ 2 \end{array}$   $\begin{array}{r} 22 \\ 2 \end{array}$   $\begin{array}{r} 32 \\ 2 \end{array}$  etc.

(b) Place a tens figure before each of the addends, thus,

$\begin{array}{r} 25 \\ 24 \end{array}$   $\begin{array}{r} 35 \\ 44 \end{array}$   $\begin{array}{r} 65 \\ 84 \end{array}$  etc.

| 1.          | 2.           | 3.            | 4.             |
|-------------|--------------|---------------|----------------|
| 346         | 7965         | 23756         | 868963         |
| 465         | 9654         | 12762         | 527878         |
| 654         | 6543         | 66431         | 859667         |
| 542         | 5432         | 96246         | 787238         |
| 426         | 3219         | 53569         | 984794         |
| 268         | 2190         | 86725         | 589468         |
| 683         | 9083         | 92358         | 948973         |
| 832         | 6835         | 58649         | 494747         |
| 327         | 4776         | 25735         | 636836         |
| <u>796</u>  | <u>7792</u>  | <u>76482</u>  | <u>966775</u>  |
| 5.          | 6.           | 7.            | 8.             |
| 7.32        | 81.078       | 427.36        | 4.84           |
| 28.397      | 16.004       | 61.037        | 132.468        |
| 11.016      | 8.74         | 44.074        | 230.067        |
| 248.318     | 164.8        | 8.74          | 57.8           |
| .428        | 82.063       | 236.7         | 9.732          |
| <u>1.08</u> | <u>8.165</u> | <u>85.798</u> | <u>162.875</u> |

9. Add six hundred twenty million, two hundred six thousand, four hundred eight; nine million, three thousand, four; twenty-three million, fifteen thousand, five hundred four; seven million, thirty-two thousand, seventy-five; three million, four thousand, forty-eight.

10. Add thirty-six and twenty-eight thousandths; twenty and eight hundred five ten-thousandths; forty-one thousand two hundred eight hundred-thousandths; two million, three thousand, one hundred ten and three thousand sixty-two ten-thousandths; five tenths; twelve hundredths.

11. Add 5.4307; 48.6512; 7564.02; 314.065; 864.23; 1026.0087; 2346.002004; 86.24; 1209.00643; 3109.02041; 3287.0074; 6.007; 704.0049; 1010.00101; 4.07; 16.00109.

1. *Subtraction* is the process of taking one number from another, or it is the process of finding what part of a given number remains when a part has been taken away.

2. *Minuend* is the number from which another number is taken, or it is the given number from which the part is taken.

3. *Subtrahend* is the number taken away from another number, or it is the given part which is taken from the minuend.

4. The result of an example in subtraction is called *difference* or *remainder*.

5. The sign of subtraction is  $-$ . It is read *minus*. Thus  $9-5$  is read 9 minus 5, and indicates that 5 is to be subtracted from 9.

6. The eighty-one primary facts of subtraction should have been learned while learning the facts of addition. If not, each pupil should be made perfectly familiar with them now. See Page 253.

7. Enlarge each combination as suggested in addition on Page 253.

8. From 763 take 486.

763 We cannot take 6 units from 3 units, so we take 1 ten from the 486 6 tens, leaving 5 tens. We add this ten which equals 10 units to the 277 3 units, making 13 units. 6 units from 13 units leaves 7 units, which we write in units' column. 8 tens from 5 tens we cannot take, so we take 1 hundred, leaving 6 hundreds. We add this one hundred, or 10 tens, to 5 tens, making 15 tens. 8 tens from 15 tens leaves 7 tens, which we write in tens' column. 4 hundreds from 6 hundreds leaves 2 hundreds, which we write in hundreds' column. The remainder is 2 hundreds, 7 tens, and 7 units, or 277.

| 9.          | 10.         | 11.         | 12.         | 13.         | 14.         |
|-------------|-------------|-------------|-------------|-------------|-------------|
| 342         | 604         | 738         | 820         | 701         | 812         |
| <u>163</u>  | <u>467</u>  | <u>419</u>  | <u>644</u>  | <u>418</u>  | <u>619</u>  |
| 15.         | 16.         | 17.         | 18.         | 19.         | 20.         |
| 8052        | 5409        | 8025        | 6317        | 3020        | 4004        |
| <u>5148</u> | <u>3645</u> | <u>7184</u> | <u>3189</u> | <u>2185</u> | <u>3074</u> |

Subtract:

1.  
2030405060  
1468194782

2.  
7988362  
4726845

3.  
4421618  
2567487

4.  
20040060070  
16417381246

5.  
75103040  
57068172

6.  
800206080  
434184165

7.  
60407030809  
41625818714

8.  
40000000  
23456789

9.  
84000605045  
42613417632

10. To what number must 472587 be added to make 604032?

11. How much less than 3002020004 is 1436817486?

12. What number must be subtracted from 2001004000 that the remainder may be 981607106?

13. What number must be added to 647583 to make 1047102?

14. From two million, two thousand four, take sixty-eight thousand seventy-one.

15. How much greater is one billion than two hundred thirty-two million, forty-five thousand, seven hundred thirty-two?

16. Find the difference between two hundred and two thousandths, and two hundred two thousandths.

17. From thirteen thousandths take forty-one millionths.

18.  $479.0128 - (114.016 + 96.0074) = ?$

19. If the whole quantity is 45602, and one of the parts is 26715, find the other part.

20. The subtrahend is 1054608; the minuend is 4610072. What is the difference?

1. *Multiplication* is the process of repeating a quantity a certain number of times, or it is the process of finding the sum of several equal numbers.

2. The *multiplicand* is the number repeated.

3. The *multiplier* shows how many times the quantity is to be repeated.

4. The *product* is the result of the process of multiplication.

5. The sign of multiplication is  $\times$ , and is read *times* or *multiplied by*; thus,  $4 \times \$3$  is read 4 times \$3 and  $\$3 \times 4$  is read \$3 multiplied by 4. In both cases 4 is the multiplier, and shows the *ratio* of the product to the multiplicand.

6. Learn the following principles of multiplication: —

(a) The multiplier denotes ratio, and is always an abstract number.

(b) The product is always of the same denomination as the multiplicand.

7. The sixty-four facts of multiplication, as found in multiplication tables, should be thoroughly memorized.

8. The multiplicand and multiplier are sometimes called *factors*.

9. Multiply 684 by 476.

$$\begin{array}{r} 684 \\ 476 \\ \hline 4104 = 684 \times 6 \text{ units} \\ 47880 = 684 \times 7 \text{ tens, or } 70 \text{ units} \\ 273600 = 684 \times 4 \text{ hundreds, or } 400 \\ \hline 325584 \end{array}$$

units

10. Multiply 5600 by 130.

$$\begin{array}{r} 5600 \\ 130 \\ \hline 168 \\ 56 \\ \hline 728000 \end{array}$$

Omit ciphers in multiplying, and annex to the product as many ciphers as have been omitted.

11. To prove an example in multiplication: —

(a) Use the multiplicand as the multiplier.

(b) Divide the product by either factor. The quotient will be the other factor.

Multiply: —

|                      |                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| 1. 593<br><u>566</u> | 2. 672<br><u>748</u> | 3. 878<br><u>896</u> | 4. 839<br><u>456</u> | 5. 789<br><u>658</u> |
|----------------------|----------------------|----------------------|----------------------|----------------------|

|                       |                       |                       |                       |                        |
|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|
| 6. 2876<br><u>186</u> | 7. 6542<br><u>347</u> | 8. 9734<br><u>296</u> | 9. 6542<br><u>347</u> | 10. 9678<br><u>978</u> |
|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|

|                         |                         |                         |                         |                         |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| 11. 92646<br><u>675</u> | 12. 37595<br><u>793</u> | 13. 45674<br><u>962</u> | 14. 82457<br><u>406</u> | 15. 83729<br><u>596</u> |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|

|                           |                           |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 16. 695736<br><u>3928</u> | 17. 843934<br><u>7926</u> | 18. 836719<br><u>5849</u> | 19. 547386<br><u>9657</u> | 20. 840673<br><u>6489</u> |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|

|                           |                           |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 21. 874007<br><u>9047</u> | 22. 900608<br><u>4705</u> | 23. 960087<br><u>7008</u> | 24. 930807<br><u>9060</u> | 25. 670509<br><u>7060</u> |
|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|

26. What will 943 barrels of flour cost at \$6 a barrel?

|               |               |
|---------------|---------------|
| (a) \$ 6      | (b) 943       |
| 943           | \$ 6          |
| <u>18</u>     | <u>\$5658</u> |
| 24            |               |
| <u>54</u>     |               |
| <u>\$5658</u> |               |

Method (a) is correct and needs no explanation. Pupils should be allowed to use the other form for brevity. It is important, however, that they should keep clearly in mind that \$6 is the number to be repeated in each case.

27. How many bushels of potatoes can be raised on 678 acres at the rate of 87 bu. an acre?

28. Multiply seven thousand sixty-four by nine thousand six hundred five.

29. Multiply seven thousand ninety-six by five thousand seventy.

30. Multiply seven hundred sixty-five thousand, six hundred seventy-seven by eight thousand sixty-eight.

31. What is the value of 8679 acres of land at \$467 an acre?

32. Multiply the sum of 96142 and 87310 by three times their difference.

1. *Division* is the process of finding how many times one number is contained in another, or it is the process of separating a number into equal parts.

NOTE. — At 3¢ each how many peaches can be bought for 15 cents?  $15¢ \div 3¢ = 5$ . This illustrates the first definition, or division. I paid 15¢ for 5 peaches. What did each cost?  $15¢ \div 5 = 3¢$ . This illustrates the second definition, or as it is sometimes called, Partition.

2. The *dividend* is the number that is to be divided or separated into parts.

3. The *divisor* is the number by which we divide.

4. The *quotient* is the result obtained by division.

5. The *remainder* is the part of the dividend remaining, when the divisor is not contained an exact number of times.

6. The *sign of division* is  $\div$ , and is read *divided by*. It indicates that the number before the sign is to be divided by the number after it.

7. Division is expressed in four ways: —

(a) Fractional method, thus  $\frac{1}{3} = 3$ .

(b) Using sign,  $\div$ , thus,  $12 \div 4 = 3$ .

(c) Using sign,  $:$ , thus,  $12 : 4 = 3$ . Ratio method.

(d) Working method, thus  $4 \overline{)12}$ .

8. Division is the converse of multiplication. Multiplication is thus a proof of division.

9. Divide 6048 by 36.

|         |                                                                     |
|---------|---------------------------------------------------------------------|
| 168     | 36 is not contained in 6 thousands. 36 is contained in 60           |
| 36)6048 | hundreds, 1 hundred times. Write the 1 over hundreds' figure        |
| 3600    | of the dividend. 36 times 1 hundred are 36 hundred. 36 hundred      |
| 2440    | from 60 hundred leaves 24 hundred, or 240 tens. 240 tens            |
| 2160    | and 4 tens are 244 tens. 36 is contained in 244 tens, 6 tens times. |
| 288     | 36 times 6 tens are 216 tens. 216 tens from 244 tens leaves 28      |
| 288     | tens, or 280 units. 280 units and 8 units are 288 units. 36 is      |
| —       | contained in 288 units, 8 units times. 36 times 8 units are 288     |
|         | units. There is no remainder.                                       |

NOTE. — Teachers should explain that ciphers marked  $\emptyset$  in the example are not usually written.



Divide : —

1. 2862 by 27.
2. 6680 by 34.
3. 11997 by 43.
4. 34894 by 58.
5. 83927 by 89.
6. 109604 by 94.
7. 79692 by 916.
8. 61074 by 783.
9. 13,746,232 by 386.
10. 56,882,034 by 594.
11. 441,239,442 by 4863.
12. 489,877,092 by 3721.
13. 25,446,182,662 by 62593.
14. 26,015,867,442 by 75407.
15. 4,580,423,662 by 27418.
16. 1,047,293,812 by 86414.
17. The dividend is 531288 and the divisor 628. What is the quotient?
18. How many times can 326 be subtracted from 1,056,566?
19. By what must 4006 be multiplied to make 356,534?
20. The dividend is 17,590,800 and the quotient 5136. What is the divisor?
21. The product of two factors is 403326, and one of the factors is 126. What is the other factor?
22. The multiplier is 245 and the product 562520. What is the multiplicand?
23. What number will divide 866,775 exactly 195 times?
24. What must be taken from 232,696 that the remainder may be exactly divisible by 512?
25. What is the nearest number to 632186 that will contain 321 without a remainder?
26. The sum of 365 equal numbers is 3, 113, 815. What is each number?
27. The product is 10,365,051 and the multiplicand is 3021. What is the multiplier?
28. If 13 horses and 15 cows cost \$2320, and the average price of a horse is \$130, what is the average price of a cow?
29. A rectangle contains 4824 sq. ft. If it is 72 ft. long, how wide is it?

1. Numbers are *integral, fractional, or mixed*.
  2. Integral numbers are divided into two classes, *even* and *odd*.
  3. An *even* number is a number that is **exactly** divisible by 2, as, 4, 8, 32, etc.
  4. An *odd* number is a number that is *not* exactly divisible by 2, as, 5, 9, 13, 41, etc.
  5. Integral numbers are also classified as *prime* or *composite*.
  6. *Factors* of a number are the numbers that multiplied together will produce the number, as 4 and 3 are the factors of 12; 2, 3, and 5 are the factors of 30.
  7. A *prime number* is a number that has no integral factors except itself and 1, as 5, 11, 29, etc.
  8. A *composite* number is a number that has integral factors, as, 9, 25, 36, etc.
  9. *Factoring* is the process of finding the factors of a number.
  10. *Prime factors* of a number are the prime numbers which **exactly** divide the given number.
  11. **Learn:** To resolve a number into its prime factors, divide the number by any prime factor, and do the same with each quotient until the quotient becomes a prime number. The several divisors and the last quotient are the prime factors required.
- Find the prime factors of : —
- |         |         |          |          |
|---------|---------|----------|----------|
| 12. 105 | 13. 429 | 14. 3735 | 15. 3224 |
|---------|---------|----------|----------|
16. A *divisor* of a number is a number that exactly divides it.
  17. Name all the divisors of 45, 51, 96, 81, 32.
  18. A *common* divisor of two or more numbers is a divisor of each of them.
  19. What divisors are common to 9 and 12? 24 and 36? 45 and 60? 6, 9, and 12? 14, 21, 28, and 35?

1. The *Greatest Common Divisor* of two or more numbers is the largest number that will exactly divide each of them.

2. Find the greatest common divisor of 45, 60, 75.

3. Find the greatest common divisor of 92 and 115.

$$\begin{array}{r} 3 \overline{)45, 60, 75} \\ 5 \overline{)15, 20, 25} \\ \hline 3, 4, 5 \end{array}$$

Divide by any common prime factors of all the numbers. Do the same with the quotients, till the quotients have no common factor. The product of all the divisors will be the greatest common divisor.

$$\begin{array}{r|l} 92 & 115 \\ 92 & 92 \\ \hline & 23 \end{array} \begin{array}{l} 1 \\ 4 \end{array}$$

Divide the greater number by the less, the divisor by the remainder, and thus continue until there is no remainder. The last divisor will be the greatest common divisor.

Find the greatest common divisor of :

4. 18, 27, and 45

5. 42, 56, and 84

6. 909 and 1414

7. 1917 and 2556

8. 350 and 475

9. 759 and 1155

#### LEAST COMMON MULTIPLE.

10. The *Least Common Multiple* of two or more numbers is the least number that is exactly divisible by each of them. It contains all the prime factors found in each of the numbers, and no other factors.

11. Find the least common multiple of 8, 12, and 20.

$$2 \overline{)8-12-20}$$

$$2 \overline{)4-6-10}$$

$$2-3-5$$

$$2 \times 2 \times 2 \times 3 \times 5 = 120$$

Divide the numbers by any prime factor of two or more of them; write the quotients and undivided numbers beneath, divide as at first, and so continue until the numbers in the last line are prime to each other. The product of the divisors and the numbers in the last line will be the least common multiple.

Find the least common multiple of :

12. 28, 42, 63, 108

13. 171, 592, 703

14. 65, 78, 104, 130

15. 115, 161

NOTE. — When the numbers are not readily factored, find the greatest common divisor by the second method. Divide one number by the greatest common divisor and multiply the quotient by the other number.

1. A *fractional unit* is one of the equal parts of a unit.
2. A *fractional number* is a collection of fractional units.
3. A *fraction* is one or more of the equal parts of a unit.
4. The *denominator* of the fraction shows the number of equal parts into which the integral unit has been divided. It therefore gives the *name* to the fraction.
5. The *numerator* of the fraction shows the number of the parts taken to form the fraction.
6. The numerator and denominator are the *terms* of the fraction.
7. Fractions are classified with respect to their *value* into proper and improper fractions.
8. A *proper fraction* is one whose numerator is less than its denominator, i.e., its value is less than a unit, thus,  $\frac{3}{4}$ ,  $\frac{1}{2}$ , etc.
9. An *improper fraction* is one whose numerator is equal to or less than its denominator, i.e., its value is equal to or greater than a unit, thus,  $\frac{5}{4}$ ,  $\frac{3}{2}$ .
10. Fractions are classified with respect to their *form* into simple and complex fractions.
11. A *simple fraction* is one whose terms are integers, as  $\frac{3}{4}$ .
12. A *complex fraction* is one which contains a fraction in one or both of its terms, as  $\frac{2\frac{1}{2}}{4}$ .
13. A *mixed number* is a number consisting of an integer and a fraction, as  $4\frac{1}{2}$ .
14. Write two proper and two improper fractions.
15. Name the numerator and denominator of each.
16. Write a complex fraction having a simple fraction in both terms. Write a complex fraction having a mixed number in both terms.
17. Reduction of fractions is the process of changing their form without changing their value.

*To change a mixed number to an improper fraction:—*

1. Change  $9\frac{5}{6}$  to an improper fraction.

$9\frac{5}{6} = \frac{49}{6}$  In one unit there are 6 sixths. In 9 units there are 9 times 6 sixths or 54 sixths. 54 sixths and 5 sixths are 59 sixths.

Learn: Change the integer to an equivalent fraction having the denominator of the fraction, and then unite the two fractional parts.

2. Change the following to improper fractions:—

$9\frac{3}{4}$ ;  $15\frac{3}{4}$ ;  $46\frac{1}{2}$ ;  $38\frac{1}{2}$ ;  $16\frac{3}{8}$ ;  $96\frac{3}{8}$ ;  $41\frac{1}{8}$ ;  $29\frac{7}{8}$ ;  $87\frac{7}{8}$ ;  $16\frac{7}{8}$ .

*To change an improper fraction to a whole or mixed number:—*

3. Change  $\frac{27}{5}$  to a mixed number.

$\frac{27}{5} = 5\frac{2}{5}$  Since there are 5 fifths in 1 unit, in  $\frac{27}{5}$  there must be as many units as five is contained times in 27, or  $5\frac{2}{5}$ .

Learn: Since a fraction is an expression of division, the rule follows: Divide the numerator by the denominator.

4. Change to whole or mixed numbers:— $\frac{2^6}{7}$ ;  $\frac{2^6}{8}$ ;  $\frac{8}{3}$ ;  $\frac{7}{4}$ ;  $\frac{257}{2}$ ;  $\frac{893}{17}$ ;  $\frac{458}{23}$ ;  $\frac{883}{37}$ ;  $\frac{324}{17}$ ;  $\frac{237}{18}$ ;  $\frac{345}{24}$ ;  $\frac{796}{31}$ .

*To change a fraction to its lowest terms:—*

5. Change  $\frac{30}{5}$  to its lowest terms.

$5 \mid \frac{30}{5} = \frac{6}{1}$  Since dividing both terms of the fraction by the same number does not change its value, we divide both terms by the common factor 5 and our result is  $\frac{6}{1}$ .

Learn: Divide both terms by a common factor till no common factor can be found. When no common factor can be found by inspection, find the greatest common divisor of the terms, and divide both terms by it.

6. Change to lowest terms:— $\frac{1}{2}$ ;  $\frac{2}{4}$ ;  $\frac{3}{12}$ ;  $\frac{11}{10}$ ;  $\frac{21}{10}$ ;  $\frac{11}{11}$ ;  $\frac{625}{525}$ ;  $\frac{213}{324}$ ;  $\frac{1123}{1124}$ ;  $\frac{11}{13}$ ;  $\frac{783}{888}$ ;  $\frac{209}{338}$ ;  $\frac{735}{1280}$ .

To change fractions to equivalent fractions having the same denominator:—

1. Change  $\frac{1}{2}$  and  $\frac{3}{4}$  to 12ths.

$\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$      $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$     Since multiplying both numerator and denominator by the same number does not change the value of the fraction, we multiply both terms by such a number as will change the denominator to the required denominator.

2. Change to the same denominator.

|                                                   |                                                                 |
|---------------------------------------------------|-----------------------------------------------------------------|
| $\frac{2}{4}, \frac{3}{4}, \frac{5}{8}$ to 12ths. | $\frac{1}{4}, \frac{1}{3}, \frac{7}{8}, \frac{7}{12}$ to 48ths. |
| $\frac{3}{4}, \frac{1}{2}$ to 15ths.              | $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ to 36ths.  |
| $\frac{2}{4}, \frac{5}{8}, \frac{7}{8}$ to 24ths. | $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{5}{12}$ to 60ths. |

Fractions having the same denominator are called like fractions or similar fractions.

#### ADDITION OF FRACTIONS.

3. Add  $\frac{2}{3}$  and  $\frac{3}{5}$ .

$\frac{2}{3} = \frac{4}{6}$      $\frac{3}{5} = \frac{6}{10}$     Since only like or similar fractions can be added, find the least common multiple of the denominators for the new denominator and make the fractions similar.

Add:—

- |                                                             |                                                                |
|-------------------------------------------------------------|----------------------------------------------------------------|
| 4. $\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{7}{12}$ .  | 5. $\frac{3}{5}, \frac{1}{15}, \frac{7}{10}, \frac{5}{6}$ .    |
| 6. $\frac{5}{6}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}$ .  | 7. $\frac{1}{3}, \frac{7}{9}, \frac{1}{6}, \frac{5}{6}$ .      |
| 8. $\frac{3}{8}, \frac{1}{16}, \frac{9}{10}, \frac{2}{5}$ . | 9. $\frac{5}{8}, \frac{5}{6}, \frac{1}{24}, \frac{1}{12}$ .    |
| 10. $\frac{1}{4}, \frac{1}{8}, \frac{7}{12}, \frac{5}{6}$ . | 11. $\frac{7}{12}, \frac{8}{15}, \frac{2}{20}, \frac{7}{10}$ . |

In the following examples add the integers and fractions separately.

- |                                                                           |                                                                           |
|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| 12. $61\frac{3}{4} + 112\frac{5}{8} + 78\frac{5}{8} + 176\frac{3}{4}$ .   | 13. $216\frac{3}{8} + 141\frac{1}{8} + 85\frac{1}{2} + 415\frac{1}{2}$ .  |
| 14. $412\frac{1}{4} + 371\frac{1}{2} + 211\frac{3}{4} + 911\frac{1}{2}$ . | 15. $86\frac{3}{4} + 116\frac{9}{10} + 28\frac{7}{10} + 196\frac{3}{4}$ . |
| 16. $74\frac{5}{8} + 83\frac{5}{8} + 136\frac{3}{4} + 672\frac{3}{4}$ .   | 17. $59\frac{8}{15} + 64\frac{1}{10} + 75\frac{7}{12} + 118\frac{1}{4}$ . |
| 18. $96\frac{1}{12} + 71\frac{3}{4} + 461\frac{3}{4} + 164\frac{3}{12}$ . | 19. $64\frac{3}{4} + 96\frac{3}{4} + 84\frac{5}{8} + 178\frac{7}{12}$ .   |
| 20. $46\frac{1}{4} + 56\frac{8}{15} + 435\frac{1}{6} + 126\frac{5}{8}$ .  | 21. $45\frac{7}{9} + 81\frac{1}{6} + 66\frac{1}{2} + 148\frac{7}{12}$ .   |
| 22. $18\frac{1}{8} + 94\frac{3}{4} + 397\frac{7}{12} + 326\frac{1}{8}$ .  | 23. $65\frac{5}{12} + 42\frac{1}{3} + 18\frac{1}{6} + 211\frac{1}{4}$ .   |

1. From  $\frac{5}{8}$  take  $\frac{3}{8}$ .

$$\frac{5}{8} = \frac{32}{32} \cdot \frac{3}{8} = \frac{9}{32} \quad \text{Change both fractions to similar fractions having the least common denominator, then find the difference of the numerators, and write it over the common denominator.}$$

$$\frac{32}{32} - \frac{9}{32} = \frac{23}{32}$$

2. From  $11\frac{1}{2}$  take  $7\frac{1}{8}$ .

$$11\frac{1}{2} = 11\frac{4}{8} = 10\frac{12}{8}$$

$$7\frac{1}{8} = 7\frac{1}{8} = \frac{57}{8}$$

$$\frac{10\frac{12}{8}}{\frac{57}{8}} = \frac{7\frac{5}{8}}{3\frac{1}{8}} \quad \text{This process is identical with that of subtraction of integers.}$$

- |                                                                      |                                                                       |
|----------------------------------------------------------------------|-----------------------------------------------------------------------|
| 3. $45\frac{3}{4} - 18\frac{5}{8} = ?$                               | 4. $76\frac{3}{10} - 34\frac{2}{5} = ?$                               |
| 5. $121\frac{7}{8} - 96\frac{3}{8} = ?$                              | 6. $111\frac{7}{8} - 74\frac{5}{8} = ?$                               |
| 7. $127\frac{1}{2} - 48\frac{1}{8} = ?$                              | 8. $79\frac{1}{4} - 46\frac{5}{8} = ?$                                |
| 9. $219\frac{1}{8} - 111\frac{1}{4} = ?$                             | 10. $116\frac{3}{4} - 87\frac{1}{8} = ?$                              |
| 11. $18\frac{1}{2} + 7\frac{3}{4} - 3\frac{1}{2} + 4\frac{1}{8} = ?$ | 12. $48\frac{1}{8} - 6\frac{1}{4} - 2\frac{1}{2} - 15\frac{3}{8} = ?$ |

#### MULTIPLICATION OF FRACTIONS.

*To multiply a fraction by an integer, or an integer by a fraction:—*

1. (a) Multiply  $\frac{3}{4}$  by 5.

$$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4} \quad \text{Since the numerator expresses the number of parts, the fraction is multiplied by multiplying its numerator.}$$

(b) Multiply  $\frac{7}{10}$  by 5.

$$\frac{7}{10} \times 5 = \frac{7}{2} = 3\frac{1}{2} \quad \text{Dividing the denominator of the fraction by the integer multiplies the fraction, since it increases the size of the parts without increasing their number.}$$

Multiply the following:—

- |                            |                             |                             |                            |
|----------------------------|-----------------------------|-----------------------------|----------------------------|
| 2. $\frac{5}{8}$ by 8.     | 3. $\frac{9}{10}$ by 5.     | 4. $\frac{7}{8}$ by 8.      | 5. $1\frac{1}{8}$ by 9.    |
| 6. $7\frac{1}{4}$ by 29.   | 7. $3\frac{1}{2}$ by 26.    | 8. $3\frac{1}{8}$ by 34.    | 9. $3\frac{1}{4}$ by 17.   |
| 10. 75 by $1\frac{1}{7}$ . | 11. 625 by $2\frac{1}{5}$ . | 12. 407 by $1\frac{1}{3}$ . | 13. 27 by $\frac{2}{3}$ .  |
| 14. 32 by $1\frac{1}{8}$ . | 15. 22 by $3\frac{1}{3}$ .  | 16. 36 by $1\frac{1}{2}$ .  | 17. 24 by $1\frac{1}{2}$ . |
| 18. $8\frac{1}{8}$ by 33.  | 19. $\frac{5}{8}$ by 927.   | 20. 54 by $1\frac{1}{7}$ .  | 21. $1\frac{1}{8}$ by 575. |

To multiply a fraction by a fraction :—

1. Multiply  $\frac{5}{8}$  by  $\frac{2}{3}$ .

$$\frac{1}{8} \text{ of } \frac{5}{8} = \frac{1}{8}$$

$$\frac{5}{8} = 2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

or

$$\frac{5}{8} \times \frac{2}{3} = \frac{1}{3}$$

This means find  $\frac{2}{3}$  of  $\frac{5}{8}$ . First find  $\frac{1}{3}$  of  $\frac{5}{8}$  or  $\frac{1}{8}$ .  $\frac{2}{3}$  is 2 times  $\frac{1}{3}$  or  $\frac{2}{8} = \frac{1}{4}$ . This work may be much shortened by using cancellation, hence cancel all factors common to the numerators and denominators, and multiply the remaining factors of the numerator for a new numerator and the remaining factors of the denominators for a new denominator.

Multiply :

- |                                      |                                      |                                      |                                      |
|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| 2. $\frac{3}{8}$ by $\frac{1}{4}$ .  | 3. $\frac{1}{2}$ by $\frac{3}{8}$ .  | 4. $\frac{3}{8}$ by $\frac{1}{2}$ .  | 5. $\frac{3}{8}$ by $\frac{3}{4}$ .  |
| 6. $\frac{4}{8}$ by $\frac{1}{4}$ .  | 7. $\frac{3}{4}$ by $\frac{1}{8}$ .  | 8. $\frac{2}{8}$ by $\frac{3}{4}$ .  | 9. $\frac{3}{8}$ by $\frac{1}{4}$ .  |
| 10. $\frac{1}{8}$ by $\frac{3}{4}$ . | 11. $\frac{3}{4}$ by $\frac{3}{4}$ . | 12. $\frac{1}{8}$ by $\frac{3}{8}$ . | 13. $\frac{3}{8}$ by $\frac{1}{8}$ . |

14. Multiply  $15\frac{3}{4}$  by  $12\frac{1}{2}$ .

$$15\frac{3}{4} = \frac{63}{4} \quad 12\frac{1}{2} = \frac{25}{2} \quad \frac{63}{4} \times \frac{25}{2} = \frac{1575}{8} = 196\frac{7}{8}$$

or

|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                 |
|------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 15\frac{3}{4} \\ 12\frac{1}{2} \\ \hline 180 \\ 187\frac{1}{2} \\ \hline 196\frac{7}{8} \end{array}$ | <p>Change mixed numbers to improper fractions, and multiply as in the first illustration, or as in second illustration without changing. <math>\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}</math>. <math>\frac{1}{4} \times 15 = 3\frac{3}{4}</math>. <math>12 \times \frac{3}{4} = 9</math>. <math>12 \times 15 = 180</math>. Use this method when the numbers are small.</p> |
|------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Multiply :

- |                                          |                                          |                                           |
|------------------------------------------|------------------------------------------|-------------------------------------------|
| 15. $85\frac{3}{4}$ by $24\frac{1}{2}$ . | 16. $25\frac{1}{2}$ by $2\frac{1}{2}$ .  | 17. $16\frac{3}{4}$ by $6\frac{3}{4}$ .   |
| 18. $15\frac{1}{2}$ by $9\frac{3}{4}$ .  | 19. $24\frac{1}{2}$ by $12\frac{1}{2}$ . | 20. $27\frac{1}{10}$ by $20\frac{3}{4}$ . |
| 21. $25\frac{3}{4}$ by $32\frac{3}{4}$ . | 22. $27\frac{1}{2}$ by $16\frac{3}{4}$ . | 23. $23\frac{3}{4}$ by $49\frac{3}{4}$ .  |
| 24. $69\frac{1}{2}$ by $72\frac{3}{4}$ . | 25. $75\frac{1}{2}$ by $6\frac{1}{2}$ .  | 26. $86\frac{3}{4}$ by $27\frac{3}{4}$ .  |
- 
- |                                                                                                                      |                                                                                                      |
|----------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| 27. $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{8} \times \frac{3}{4} = ?$                                       | 28. $12\frac{3}{4} \times 8\frac{1}{2} \times 4\frac{1}{10} \times 7\frac{3}{8} = ?$                 |
| 29. $5\frac{1}{2} \times 1\frac{1}{4} \times 1\frac{1}{2} \times 1\frac{1}{8} = ?$                                   | 30. $\frac{1}{2} \times 2\frac{1}{4} \times 3\frac{3}{4} \times 2\frac{1}{4} \times \frac{1}{8} = ?$ |
| 31. $3\frac{3}{4} \times 4\frac{1}{2} \times 6\frac{3}{4} \times \frac{3}{8} = ?$                                    | 32. $3\frac{1}{2} \times 7\frac{1}{8} \times 2\frac{1}{4} \times \frac{5}{8} = ?$                    |
| 33. $2\frac{1}{2} \times \frac{7}{8} \times \frac{3}{4} \times 3\frac{3}{8} \times 5\frac{1}{2} = ?$                 | 34. $6 \times 7\frac{1}{2} \times 1\frac{1}{2} \times 3\frac{1}{4} = ?$                              |
| 35. $\frac{3}{4} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{8} = ?$ | 36. $12\frac{3}{4} \times 18\frac{1}{2} \times 2\frac{1}{4} = ?$                                     |



*To divide a fraction or a mixed number by an integer : —*

1. (a) Divide  $\frac{2}{3}$  by 3.

$\frac{1}{3}$  of  $\frac{2}{3} = \frac{2}{9}$       This means find  $\frac{1}{3}$  of  $\frac{2}{3}$ . This is the same as multiplication, page 267, or divide the numerator of the fraction by the integer.

(b) Divide  $\frac{2}{3}$  by 3.

$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$ .      In this case we cannot divide the numerator, so we multiply the denominator, since multiplying the denominator divides the fraction.

(c) Divide  $25\frac{2}{3}$  by 6.

1)  $6 \overline{)25\frac{2}{3}}$       1) Divide as in whole numbers, 6 is contained in 25, four times and 1 remainder.  $1 = \frac{3}{3}$ .  $\frac{3}{3} + \frac{2}{3} = \frac{5}{3}$ .  $\frac{5}{3} \div 6 = \frac{5}{18}$ , or,  
 $4\frac{1}{3}$       2) change the mixed number to an improper fraction.  
 2)  $25\frac{2}{3} = \frac{157}{3}$        $25\frac{2}{3} = \frac{157}{3}$ .  $\frac{157}{3} \div 6 = \frac{157}{18} = 8\frac{13}{18}$ .

Divide :

- |                            |                            |                           |                           |
|----------------------------|----------------------------|---------------------------|---------------------------|
| 2. $4\frac{1}{2}$ by 12.   | 3. $1\frac{1}{2}$ by 25.   | 4. $\frac{7}{12}$ by 3.   | 5. $3\frac{1}{2}$ by 5.   |
| 6. $3\frac{1}{2}$ by 7.    | 7. $25\frac{1}{2}$ by 5.   | 8. $32\frac{1}{2}$ by 4.  | 9. $40\frac{1}{2}$ by 12. |
| 10. $37\frac{1}{2}$ by 15. | 11. $62\frac{1}{2}$ by 25. | 12. $1\frac{1}{3}$ by 12. | 13. $1\frac{1}{2}$ by 11. |

*To divide an integer by a fraction : —*

1. Divide 6 by  $\frac{2}{3}$ .

$6 = \frac{18}{3}$ .  $\frac{18}{3} \div \frac{2}{3} = 18 \div 2 = 9$       Change the integer to the same denomination as the fraction, and divide the numerators.

or

Or

$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = 9$       Multiply the integer by the fraction inverted, because multiplying by the reciprocal of a number is the same as dividing by that number.

Divide :

- |                              |                              |                             |
|------------------------------|------------------------------|-----------------------------|
| 2. 12 by $\frac{2}{3}$ .     | 3. 8 by $\frac{2}{3}$ .      | 4. 18 by $\frac{1}{10}$ .   |
| 5. 45 by $\frac{1}{10}$ .    | 6. 16 by $\frac{2}{3}$ .     | 7. 52 by $\frac{1}{3}$ .    |
| 8. 87 by $1\frac{1}{2}$ .    | 9. 231 by $2\frac{1}{2}$ .   | 10. 330 by $4\frac{1}{2}$ . |
| 11. 74 by $7\frac{1}{2}$ .   | 12. 308 by $8\frac{1}{2}$ .  | 13. 264 by 9.               |
| 14. 166 by $10\frac{1}{2}$ . | 15. 288 by $19\frac{1}{2}$ . | 16. 176 by $5\frac{1}{2}$ . |
| 17. 60 by $1\frac{1}{2}$ .   | 18. 54 by $\frac{2}{3}$ .    | 19. 126 by $\frac{2}{3}$ .  |

To divide a fraction by a fraction:—

1. (a) Divide  $\frac{10}{9}$  by  $\frac{5}{3}$ .

$$\frac{10}{9} \div \frac{5}{3} = \frac{10 \div 5}{9 \div 3} = \frac{2}{3}$$

Divide the numerators for a new numerator and the denominators for a new denominator.

(b) Divide  $\frac{5}{3}$  by  $\frac{3}{4}$ .

$$\frac{5}{3} = \frac{20}{12} \quad \frac{3}{4} = \frac{9}{12}$$

$$\frac{20}{12} \div \frac{9}{12} = 20 \div 9 = 2\frac{2}{9}$$

Change the fractions to equivalent fractions having their least common denominator, then divide the numerators.

(c) Divide  $\frac{3}{4}$  by  $\frac{2}{3}$ .

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$$

On the principle that dividing by any number is the same as multiplying by the reciprocal of the number, we obtain the rule: Multiply by the inverted divisor.

Divide:

- |                                         |                                         |                                         |                                       |
|-----------------------------------------|-----------------------------------------|-----------------------------------------|---------------------------------------|
| 2. $\frac{3}{4}$ by $\frac{2}{3}$ .     | 3. $\frac{5}{8}$ by $\frac{3}{4}$ .     | 4. $\frac{11}{12}$ by $\frac{1}{12}$ .  | 5. $\frac{3}{8}$ by $2\frac{1}{4}$ .  |
| 6. $4\frac{1}{2}$ by $2\frac{1}{2}$ .   | 7. $8\frac{1}{2}$ by $1\frac{1}{2}$ .   | 8. $4\frac{1}{2}$ by $2\frac{1}{3}$ .   | 9. $5\frac{1}{2}$ by $1\frac{1}{2}$ . |
| 10. $2\frac{1}{2}$ by $1\frac{1}{2}$ .  | 11. $7\frac{1}{2}$ by $1\frac{1}{4}$ .  | 12. $2\frac{3}{4}$ by $\frac{5}{8}$ .   | 13. $\frac{3}{8}$ by $\frac{7}{8}$ .  |
| 14. $69\frac{1}{2}$ by $4\frac{1}{8}$ . | 15. $42\frac{1}{2}$ by $2\frac{5}{8}$ . | 16. $12\frac{1}{2}$ by $1\frac{1}{2}$ . | 17. $6\frac{3}{4}$ by $\frac{3}{4}$ . |

#### COMPLEX FRACTIONS.

Reduce to simple fractions:

- |                                                                                                                                   |                                                                                                                                                              |                                                                                            |                                                                                         |
|-----------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1. $\frac{5\frac{3}{4}}{6\frac{2}{3}}$ .                                                                                          | 2. $\frac{11\frac{3}{4}}{5\frac{2}{3}}$ .                                                                                                                    | 3. $\frac{2\frac{1}{3}}{5\frac{1}{4}}$ .                                                   | 4. $\frac{3\frac{1}{2}}{9\frac{3}{8}}$ .                                                |
| 5. $\frac{\frac{3}{4} \times \frac{3}{4} \times \frac{7}{8}}{\frac{1}{2} \times \frac{2}{3} \times \frac{4}{5}}$ .                | 6. $\frac{\frac{1}{2} + \frac{3}{4}}{\frac{3}{4} + \frac{5}{8}}$ .                                                                                           | 7. $\frac{\frac{3}{4} \times \frac{5}{8}}{\frac{3}{4} \times \frac{1}{11}}$ .              |                                                                                         |
| 8. $\frac{\frac{8}{12} - \frac{5}{6}}{\frac{1}{12} - \frac{3}{4}}$ .                                                              | 9. $\frac{3\frac{1}{2} + 6\frac{3}{8} - 7\frac{5}{8} + 5\frac{1}{2}}{4\frac{1}{15} - 2\frac{2}{3} + 6\frac{3}{4} - 1\frac{5}{6}}$ .                          | 10. $\frac{4\frac{1}{2} + 3\frac{5}{8} - 2\frac{1}{3}}{7\frac{1}{3} - 3\frac{3}{4} + 2}$ . |                                                                                         |
| 11. $\frac{1\frac{5}{8} + 2\frac{1}{2}}{2\frac{1}{4} - 1\frac{1}{3}}$ .                                                           | 12. $\frac{\frac{3}{8} \times \frac{3}{4} \times \frac{7}{8} \times \frac{1}{11}}{\frac{8}{9} \times \frac{3}{4} \times \frac{1}{11} \times \frac{7}{12}}$ . | 13. $\frac{\frac{3}{4} \times 9\frac{3}{8}}{9\frac{3}{8} \div \frac{7}{8}}$ .              |                                                                                         |
| 14. $\frac{\frac{3}{4} \div 1\frac{3}{8}}{\frac{1}{25}} \times \frac{2\frac{1}{2} + 1\frac{1}{4}}{4\frac{1}{2} - 3\frac{1}{8}}$ . | 15. $\frac{\frac{1}{2} \times \frac{5}{7} \times 7\frac{3}{8}}{12\frac{2}{11}}$ .                                                                            | 16. $\frac{\frac{3}{4} \times 13\frac{1}{4}}{\frac{7}{8} \times 7\frac{7}{8}}$ .           |                                                                                         |
| 17. $\frac{3\frac{1}{4}}{4\frac{1}{2}}$ .                                                                                         | 18. $\frac{7\frac{3}{10}}{\frac{3}{4}}$ .                                                                                                                    | 19. $\frac{6\frac{2}{3}}{8\frac{2}{3}}$ .                                                  | 20. $\frac{7\frac{5}{8}}{4\frac{3}{8}}$ .                                               |
| 21. $\frac{402\frac{1}{2}}{27\frac{1}{3}}$ .                                                                                      | 22. $\frac{14\frac{5}{8}}{7\frac{3}{8}}$ .                                                                                                                   | 23. $\frac{40\frac{5}{8}}{6\frac{3}{8}}$ .                                                 | 24. $\frac{\frac{3}{4} \text{ of } \frac{3}{8}}{\frac{7}{8} \text{ of } \frac{1}{3}}$ . |

1. In a certain school  $\frac{1}{10}$  of the pupils are under 8;  $\frac{1}{5}$  between 8 and 12;  $\frac{1}{4}$  between 12 and 14;  $\frac{3}{8}$  between 14 and 16; and 27 are over 16. What is the whole number of pupils?

2. If two men can do a piece of work in  $6\frac{3}{4}$  days, what part of it can one man do in  $3\frac{1}{2}$  days?

3. Three-sevenths of a certain number exceeds  $\frac{1}{4}$  of the same number by 25. What is the number?

4. If a man can build  $\frac{1}{4}$  of a wall in  $5\frac{1}{2}$  days, how long will it take him to finish it?

5. If  $\frac{1}{11}$  acres of land cost \$18 $\frac{7}{11}$ , how much must be paid for  $78\frac{5}{11}$  A.?

6. If  $1\frac{1}{2}$  A. of land cost \$76, what will  $1\frac{3}{8}$  of an acre cost?

7. If a man owns  $\frac{5}{11}$  of a mill, and sells  $\frac{1}{4}$  of his share for \$10,000, what is the value of the entire mill?

8. Find the cost of  $57\frac{1}{2}$  yd. of cloth  $\frac{3}{4}$  yd. wide, if  $39\frac{1}{2}$  yd. of the same cloth  $\frac{3}{4}$  wide cost \$118.50.

9. If a man owns  $\frac{2}{3}$  of a mill, and sells  $\frac{1}{3}$  of his interest for \$4,000, what is the value of the mill?

10. A farmer raises  $4\frac{1}{2}$  tons of hay on  $2\frac{3}{4}$  acres of land. How many tons can he raise on  $12\frac{3}{4}$  acres?

11. How many square inches of tin will be required to make a box 9 in. long,  $3\frac{3}{4}$  in. wide, and  $3\frac{1}{2}$  in. deep?

12. If  $\frac{2}{3}$  of A's money is increased by  $\frac{1}{3}$  of  $\frac{1}{3}$  of his money, the sum will equal \$198. How much money has he?

13. A boy's money diminished by  $\frac{1}{4}$  and  $\frac{1}{5}$  of itself, equals \$1.32. How much has he?

14. A owned  $\frac{2}{3}$  of a factory, and sold  $\frac{1}{3}$  of his share to B, who sold  $\frac{1}{3}$  of his share to C, who sold  $\frac{1}{3}$  of what he bought to D. What part of the factory did each then own?

15. A owns  $79\frac{5}{8}$  acres of land, B  $9\frac{7}{8}$  acres less, while C owns  $26\frac{1}{4}$  acres less than A and B together. How many acres have B and C?

16. If  $5\frac{1}{2}$  bu. of wheat cost \$6.60, how much will \$121 $\frac{1}{2}$  buy?

1. The word *decimal* comes from the Latin word *decem*, which means ten.

2. A *decimal fraction*, usually called a *decimal*, is a fraction therefore whose denominator is some power of ten.

3. The denominator is not usually written, but is shown by the position of the decimal point.

4. In reading decimals, read as if the decimal were an integral number and add the name of the lowest decimal place.

5. Read the following: 1.5; 2.06; 3.007; 4.0016; 5.00025; 9.000164; 200.02; 20.002; 300.003; .303.

6. Write the following decimally: three tenths; eleven hundredths; one hundred twelve hundred-thousandths; six hundred four thousandths; six hundred and four thousandths; eighteen and fifteen hundred-thousandths; two and one half tenths; forty-five hundred and forty-five hundredths.

7. Notice the similarity of sound and difference of value in the following:—101000; 100.001; .101.

*To change a decimal to a common fraction.*

8. Change .25 to a common fraction.

$.25 = \frac{25}{100} = \frac{1}{4}$  Express the decimal as a common fraction, and reduce it to its lowest terms.

Change the following decimals to common fractions:—

- |                       |                        |                        |                        |                       |
|-----------------------|------------------------|------------------------|------------------------|-----------------------|
| 9. .75                | 10. .64                | 11. .032               | 12. .12 $\frac{1}{2}$  | 13. .87 $\frac{1}{2}$ |
| 14. .3 $\frac{1}{3}$  | 15. .00125             | 16. .0005              | 17. .024               | 18. .1625             |
| 19. 1.6 $\frac{2}{3}$ | 20. .16 $\frac{2}{3}$  | 21. .008 $\frac{1}{3}$ | 22. .081 $\frac{1}{4}$ | 23. .00 $\frac{1}{3}$ |
| 24. .0725             | 25. 8.66 $\frac{2}{3}$ | 26. 22.0 $\frac{1}{3}$ | 27. 2.03 $\frac{1}{3}$ | 28. 75.25             |

*To change a common fraction to a decimal.*

1. Change  $\frac{3}{8}$  to a decimal.

8)3.000  $\frac{3}{8}$  means  $3 \div 8$ . Perform the indicated operation by annexing ciphers to the numerator and dividing by the denominator.

Change to decimals:

- |                     |                       |                         |                                |                                |                         |                                |
|---------------------|-----------------------|-------------------------|--------------------------------|--------------------------------|-------------------------|--------------------------------|
| 2. $\frac{3}{8}$ .  | 3. $\frac{7}{8}$ .    | 4. $\frac{3}{16}$ .     | 5. $\frac{4}{25}$ .            | 6. $\frac{1^2}{25}$ .          | 7. $\frac{5}{16}$ .     | 8. $16\frac{1}{4}$ .           |
| 9. $6\frac{1}{8}$ . | 10. $7\frac{9}{25}$ . | 11. $\frac{1^9}{250}$ . | 12. $\frac{1}{3}\frac{1}{2}$ . | 13. $\frac{1}{2}\frac{8}{8}$ . | 14. $\frac{1^2}{400}$ . | 15. $\frac{1}{8}\frac{1}{4}$ . |

## 272 ADDITION AND SUBTRACTION OF DECIMALS.

1. Add 2.514; 6.7; 18.1205; 8.24.

$$\begin{array}{r} 2.514 \\ 6.7 \\ 18.1205 \\ 8.24 \\ \hline 35.5745 \end{array}$$

Write the units of the same order in the same column, and add as in whole or integral numbers.

2. Add 4.615; 7.1; .4589; 7.46819; 8.01.

3. Add .612; 5.0; 9.2178; 5.00623; .2002.

4. Add 10.017; 265; 8.001; 3.1205; 0.45; 3.07.

5. Add eighty-two and three hundred sixteen thousandths; one and two hundredths; four and one hundred two ten-thousandths; six thousandths; fourteen.

6. Add four tenths; twenty-four hundredths; eighteen thousandths; one hundred nine hundred-thousandths; one thousand two hundred one millionths; seventeen hundredths; seventy-five ten-millionths.

7. Add  $2\frac{1}{2}$  tenths;  $16\frac{1}{2}$  thousandths;  $19\frac{1}{2}$  hundredths;  $2\frac{1}{2}$  thousandths;  $46\frac{1}{2}$  thousandths.

8. From 2.79 take 1.07.

$$\begin{array}{r} 2.79 \\ 1.07 \\ \hline 1.72 \end{array}$$

Write units of the same order in the same column, and subtract as in integral numbers.

9. From 6.6 take .49.

10. From 2.106 take .0004.

11. From 4.01 take 2.004.

12. From 3.04 take 1.906.

13. From 3.05 take 1.075.

14. From 6.4 take 4.806.

15. From 75.01 take 61.964.

16. From 0.716 take 0.6418.

17. From .025 take .000487.

18. From 400 take .004.

19. From 1.046 take .00687.

20. From 100 take .0001.

21. From  $6\frac{1}{2}$  hundredths take  $4\frac{1}{2}$  thousandths.

22. From two take two hundredths.

23. From two hundred take two hundredths.

24. From twelve hundredths take twelve millionths.

25. From three hundred and three thousandths take three hundred three thousandths.

1. Multiply 3.5 by .5.

- (a)  $\begin{array}{r} .3 \\ \times .5 \\ \hline 1.75 \end{array}$  (b) 3.5  $\begin{array}{r} .5 \\ \times 3.5 \\ \hline 1.75 \end{array}$  (a) This means find  $\frac{5}{10}$  of 3.5.  $\frac{5}{10}$  of 3.5 is .35, then  $\frac{5}{10}$  are 5 times .35 or 1.75.  
(b) 35 multiplied by 5 are 175, and tenths multiplied by tenths gives hundredths, hence the result is 175 hundredths, or 1.75.

Multiply as in whole numbers, and make the decimal places in the product equal the sum of those in the multiplicand and multiplier.

Multiply: —

- |                                                               |                                                |                  |
|---------------------------------------------------------------|------------------------------------------------|------------------|
| 2. .7 by .465                                                 | 3. .5 by .064                                  | 4. .6 by .049    |
| 5. .15 by .628                                                | 6. .07 by .085                                 | 7. .24 by .184   |
| 8. .09 by .007                                                | 9. .08 by .009                                 | 10. .06 by .007  |
| 11. .462 by .005                                              | 12. .074 by .641                               | 13. .075 by .028 |
| 14. 1.007 by 2.005                                            | 15. .0046 by .00098                            |                  |
| 16. 2078 by .0047                                             | 17. 40.079 by .046                             |                  |
| 18. 62.174 by 2.16                                            | 19. 7.216 by .463                              |                  |
| 20. 5.002 by 5.06                                             | 21. 10.005 by .105                             |                  |
| 22. 16 by 1.0705                                              | 23. 148 by .00148                              |                  |
| 24. 5.028 $\frac{1}{2}$ by .064                               | 25. 48.072 $\frac{3}{4}$ by .081 $\frac{1}{2}$ |                  |
| 26. 99.9 $\frac{1}{2}$ by .666 $\frac{2}{3}$                  | 27. 1000 by .055 $\frac{1}{2}$                 |                  |
| 28. Multiply 48062 thousandths by 4078 hundredths.            |                                                |                  |
| 29. Multiply 508 millionths by 32 ten-thousandths.            |                                                |                  |
| 30. Multiply 15 hundred-thousandths by 76 ten-millionths.     |                                                |                  |
| 31. Multiply 25 hundreds by 25 hundredths.                    |                                                |                  |
| 32. Multiply 16 thousands by 16 thousandths.                  |                                                |                  |
| 33. What is the product of one tenth and one tenth?           |                                                |                  |
| 34. What is the product of one tenth and one thousandth?      |                                                |                  |
| 35. What is the product of one thousandth and one thousandth? |                                                |                  |
| 36. What is the product of one thousandth and one millionth?  |                                                |                  |
| 37. What is the product of one thousand and one millionth?    |                                                |                  |
| 38. What is the product of one million and one thousandth?    |                                                |                  |

## 1. Divide 12.4 by 4.

3.1 Notice that in dividing a decimal by an integer the decimal point must be placed in the quotient when the point in the dividend is reached. Also notice that in short division it is better to place the quotient above the dividend.

4) 12.4

## 2. a. Divide 125 by .5 b. Divide .125 by .05.

(a)  $\begin{array}{r} 250. \\ 5 \overline{)125} \end{array}$  (b)  $\begin{array}{r} 2.5 \\ .05 \overline{)12.5} \end{array}$  Since multiplying the divisor and dividend by the same number does not change the quotient, we multiply both terms by that number which will make the divisor an integer, then the rule for the decimal point is the same as in the first illustration.

3. Learn: Move the decimal point in both divisor and dividend to the right as many places as is necessary to make the divisor a whole number, then divide as in whole numbers and place the decimal point in the quotient when the point in the dividend is reached.

Divide: —

- |                     |                      |
|---------------------|----------------------|
| 4. 91.512 by 3.72   | 21. 1.46475 by 3.15  |
| 5. 177.66 by 31.5   | 22. 1501 by 31.6     |
| 6. 151.411 by 6.13  | 23. 1.3792 by 8.62   |
| 7. 44.591 by 73.1   | 24. 46.224 by 96.3   |
| 8. 683.76 by 8.4    | 25. 3.7284 by 4.78   |
| 9. 20.88 by 8.7     | 26. .78387 by .087   |
| 10. 52.26 by 7.8    | 27. .82848 by .096   |
| 11. 61.41 by 6.9    | 28. 199.525 by 57.5  |
| 12. 5.175 by .9     | 29. 3604.68 by 52.7  |
| 13. 471.42 by 9.7   | 30. 299.052 by 639   |
| 14. 872.64 by 86.4  | 31. 7.20252 by 7.41  |
| 15. 197.316 by 20.3 | 32. 229059 by .279   |
| 16. 24.412 by 71.8  | 33. .0721512 by .911 |
| 17. .20976 by .46   | 34. 303107 by 8.17   |
| 18. 32.7 by 65.4    | 35. 35.926 by 781    |
| 19. 469.56 by 8.6   | 36. 29.632 by 64     |
| 20. 1.1439 by .31   | 37. 67.643 by 1.73   |

1. Find the cost of  $4\frac{1}{2}$  cords of wood at \$3.75 a cord;  $6\frac{1}{2}$  tons of hay at \$12.50 a ton and  $47\frac{3}{4}$  bu. of potatoes at 55¢ a bushel.

2. How many times will a wheel 4 ft. in diameter revolve in going  $2\frac{1}{2}$  miles?

3. If a road rises 3.75 ft. in every 50 ft., how much does it rise in a mile?

4. A man sold a mill for \$14,500, which was .04 more than he paid for it. How much did he pay for it?

5. A bushel even measure contains 2150.42 cubic inches. If this is .783 of a heaped bushel, how many cubic inches are there in a heaped bushel?

6. My gas meter registered Nov. 1, 59,500 feet, and on Oct. 1, 56,400 feet. What is the amount of my gas bill for the month of October at \$1.10 a thousand feet?

7. A meter is 39.375 inches. How many yards are there in 18 meters?

8. If a man travel 29.6 miles a day, in how many days will he travel 1016.168 miles?

9. How many rods of fence will inclose a rectangular field that is 75.08 rd. long and 46.48 rd. wide?

10. How much pure iron in 64,148 lb. of iron ore, if .75 of it is pure iron?

11. If 40.5 yd. of cloth are bought for \$253 $\frac{1}{2}$ , what will 18.75 yards cost?

12. Twenty-five hundredths of a farm cost \$1200. What will seven-tenths of it cost?

13. For the roof of a building 9000 tiles are to be used. What will they cost at \$7.62 $\frac{1}{2}$  a thousand?

14. Two men bought 2160 acres of western land, and divided it so that one man received  $.37\frac{1}{2}$  of it, and the other man received the remainder. How many acres did each man receive?



1. *Mensuration* is the process of computing the lengths of lines, the areas of surfaces, and the volumes of solids.

2. A *solid* is a portion of space bounded by a surface or surfaces. It has three dimensions, length, breadth, and thickness.

3. A *surface* is the boundary or limit of a solid. It has two dimensions, length and breadth.

4. A *line* is the boundary or limit of a surface. It has only one dimension, length.

5. A *straight line* is a line that does not change its direction.

6. A *curved line* is a line that changes its direction at every point.

7. *Parallel lines* are lines which have the same direction.

8. An *angle* is the difference in direction of two lines.

9. When one straight line meets another, two angles are formed. If the angles are equal, they are *right angles* and the lines are perpendicular to each other.

10. An *acute angle* is smaller than a right angle.

11. An *obtuse angle* is larger than a right angle.

12. A *polygon* is a plane figure bounded by straight lines.

13. Polygons are named from the number or relations of their sides or from their angles.

14. A *quadrilateral* is a polygon of four sides.

15. A *parallelogram* is a quadrilateral whose opposite sides are parallel.

16. A *rectangle* is a right-angled parallelogram.

17. A *square* is an equilateral rectangle.

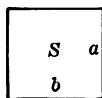
18. A *rhomboid* is an oblique-angled parallelogram.

19. A *rhombus* is an equilateral rhomboid.

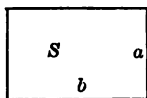
20. A *trapezoid* is a quadrilateral only two of whose sides are parallel.

21. A *trapezium* is a quadrilateral having no parallel sides.

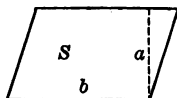
To find the area of parallelograms.



Square.



Rectangle.



Rhomboid.

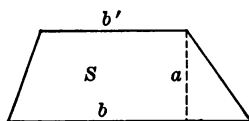
Let  $S$  equal the surface or area,  $b$  the base, and  $a$  the altitude. Any two of these being given, the other may be found.

General formulas:  $S = ab.$   $a = \frac{S}{b}.$   $b = \frac{S}{a}.$

Write these formulas as rules, thus: The area of a rectangle is found by multiplying the base by the altitude. Divide the area of a rectangle by the base to find the altitude, etc.

Using formula, find the missing term: —

1. Base 20 ft., altitude 12 ft., area  $x$ .
2. Altitude 9 ft., area 720 sq. ft., base  $x$ .
3. Base 17 ft., area 85 sq. ft., altitude  $x$ .
4. Base 15 ft., altitude 24 ft., area  $x$ .
5. Base 12 ft., area 96 sq. ft., altitude  $x$ .
6. Altitude 15 ft., area 180 sq. ft., base  $x$ .



Trapezoid.

Let  $S$  = area,  $a$ , altitude,  $b$  and  $b'$ , the bases.

General formulas:  $S = \frac{a(b + b')}{2}.$

$$a = 2 \frac{S}{(b + b')} \cdot b = \frac{S}{\frac{1}{2}a} - b'.$$

Write these formulas as rules.

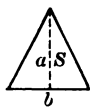
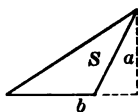
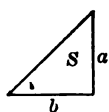
Using formula, find the missing term: —

7. Bases 20 ft. and 30 ft., altitude 10 ft., area  $x$ .
8. Bases 15 ft. and 35 ft., area 500 sq. ft., altitude  $x$ .
9. Base ( $b$ ) 110 ft., altitude 50 ft., area 4625 sq. ft. Find  $b'$ .
10. There is a house-lot with four straight sides, two of which are parallel, 80 ft. apart, and measuring 120 and 132 ft. What is the value of the lot at 25¢ a square foot.

## 278 SUMMARY OF MEASUREMENTS—TRIANGLES.

1. A *triangle* is a three-sided polygon.
2. A *right triangle* is a triangle having a right angle.
3. An *equilateral triangle* is a triangle having three equal sides.
4. An *isosceles triangle* is a triangle, two of whose sides are equal.
5. A *scalene triangle* is a triangle no two of whose sides are equal.
6. Any side upon which a triangle rests is the *base*.
7. The angle opposite the base is the *vertex*.
8. The altitude of a triangle is the perpendicular drawn from the vertex to the base or the base extended.

General formulas:



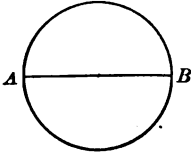
$$S = \frac{1}{2} ab. \quad a = 2 \frac{S}{b}.$$

$$b = 2 \frac{S}{a}.$$

Write these formulas as rules.

Using the formula, find the missing term:—

9. Base 40 ft., altitude 25 ft., area  $x$ .
10. Base 28 ft., altitude 20 ft., area  $x$ .
11. Altitude 45 ft., area 900 sq. ft., base  $x$ .
12. Base 30 ft., area 540 sq. ft., altitude  $x$ .
13. What is the area of a piece of ground in the form of a triangle, when the length of one side is 80 rd. and the perpendicular distance to the vertex of the opposite angle is 60 rd.?
14. A triangular piece of land contains  $2\frac{1}{4}$  A. If the altitude is 36 rd., what is the base?
15. At \$85 an acre find the value of a triangular piece of land whose base is 24 rods and altitude 21 rods.
16. The area of a triangle is 1080 sq. ft. and the altitude is 54 ft. What is the base?



1. A *circle* is a plane figure bounded by a curved line, every point of which is equally distant from the center, or a circle is a regular polygon of an infinite number of sides.

2. The *circumference* of the circle is its boundary line.

3. The *diameter*,  $AB$ , is a straight line passing through the center of the circle and touching the circumference at both ends.

4. The *radius* is a straight line joining the center and the circumference of a circle. It is one-half of the diameter.

5. The ratio of the circumference of a circle to its diameter is 3.1416.

*To find the circumference or diameter of a circle :—*

Let  $c$  = circumference,  $R$ , the radius,  $2R$ , the diameter, and  $\pi = 3.1416$ . General formulas ;  $c = 2\pi R$ .  $2R = \frac{c}{\pi}$ .

Using the formula, find the missing term and state the rule :—

6. Diameter 5 ft., circumference  $x$ .
7. Diameter 9 ft., circumference  $x$ .
8. Circumference 21.9912 ft., diameter  $x$ .
9. Circumference 25.1328 ft., diameter  $x$ .
10. What is the distance one-half round a circular piece of ground, which measures 180 ft. across the middle?
11. How far has a carriage gone when one of its wheels, measuring  $3\frac{1}{2}$  ft. in diameter, has made 1200 revolutions?
12. There are two circles drawn from the same center. The circumferences measure 196 ft. and 264 ft. respectively. Find the width of the ring formed by these two circles.
13. What is the circumference of a circle whose radius is 28 ft.? Of one whose diameter is 14 rd.?

*To find the area of circles : —*

General formulas :

$$S = 2 \pi R \times \frac{R}{2} = \pi R^2.$$

$$R = \sqrt{\frac{S}{\pi}}. \qquad 2 R = \sqrt{\frac{S \times 4}{\pi}}.$$

Write these formulas as rules.

1. Find the area of a circle whose diameter is 30 ft.
2. Find the area of a circle whose circumference is 314.16 ft.
3. Find the diameter of a circle whose area is 392.7 sq. rd.
4. Find the radius of a circle whose area is 28.2744 sq. ft.
5. Find the circumference of a circle whose area is 78.54 sq. rd.
6. Find the area of a circle whose radius is 12 ft.
7. Find the area of a circle whose radius is 20 ft.
8. What is the circumference of a circular fountain whose area is 872½ sq. yd.?
9. Find the diameter of a circle whose area is 144 sq. ft. Of another circle whose area is 36 sq. ft. How do these two diameters compare?
10. A horse is tied to a stake by a rope 20 ft. long. Over how many square feet can he graze?
11. The perimeter of a square and the circumference of a circle are each 15.708 ft. Find the difference in area.
12. What is the area of a semicircle, if the diameter of the circle is 72 ft.?
13. How many yards are there in the radius of a circle whose area is 706.86 sq. yds.?
14. If the area of a circle is 475.24 sq. ft., what is the diameter of the circle?
15. The areas of two circles are to each other as 4 to 16. Find the diameter of the smaller when the diameter of the greater is 60 ft.

To find the surface of a sphere:—



1. A *sphere* is a solid bounded by a curved surface every point of which is equally distant from its center.

Let  $S$  = surface,  $R$  = radius,  $2R$  = diameter,  $C$  = circumference, and  $\pi = 3.1416$ .

General formulas:

$$S = 2RC. \quad S = 4\pi R^2. \quad 2R = \sqrt{\frac{S}{\pi}}.$$

The surface of a sphere is equal to four times the square of the radius multiplied by 3.1416, or it is the product of the square of the diameter and  $\pi$ .

2. Find the surface of a sphere whose radius is 6 ft.
3. Find the surface of a sphere whose diameter is 10 ft.
4. Assuming the earth to be a sphere 7960 miles in diameter, how many square miles are therein its surface?
5. Find the diameter of a sphere whose surface contains 1000 square inches.
6. Find the radius of a sphere whose surface is 314.16 sq. ft.
7. Find the circumference of a sphere whose surface is 804.2496 sq. ft.

*Volume of a sphere:—*

A sphere may be regarded as composed of pyramids whose bases taken together form the surface of the sphere, whose tops are at the center, and whose height is the radius. Hence the rule, multiply the surface by one-third of the radius.

Formula: 
$$v = 4\pi R^2 \times \frac{R}{3} = \frac{4\pi R^3}{3}.$$

1. Find the volume of a sphere whose radius is 2 ft.
2. Find the volume of a sphere whose diameter is 8 ft.
3. Find the volume of a sphere whose circumference is 31.4156 ft.

*Surface of a cylinder : —*

General formulas : Let  $l$  = length, or altitude, then  $2\pi R^2$  = area of bases, and  $2\pi Rl$  = convex surface.

Find the entire surface of the following cylinders : —

1. Radius of base 5 in., length 20 in.
2. Diameter of base 12 in., length 24 in.
3. Diameter of base 5 ft., length 40 ft.

*Surface of a cone : —*

General formulas: Let  $h$  = slant height, then  $h\pi R$  = convex surface, and  $h\pi R + \pi R^2$  = the entire surface.

1. Find the surface of a cone, when the radius of the base is 4 in. and slant height 8 in.
2. A tent in the form of a cone has a slant height of 20 ft. and a diameter of 30 ft. How many square yards of cloth are required to make it?
3. The circumference of the base of a cone is 75.3984 ft., and the slant height is 50 ft. Find its entire surface.

*Surface of a right pyramid : —*

The convex surface of a right pyramid is equal to the perimeter of the base multiplied by half the slant height. To this must be added the area of the base to find the entire surface.

1. Find the entire surface of a square pyramid whose slant height is 45 ft. and each side of the base 18 ft.
2. Find the convex surface of a pyramid whose slant height is 36 ft. and the base a hexagon whose sides are each 15 ft.

*Surface of prisms : —*

Multiply the perimeter by the height to find the surface of the sides. To this add the area of the top and bottom to find the entire surface.

Find the entire surface of the following rectangular prisms : —

1. Length  $16\frac{1}{2}$  ft., width 12 ft., height 11 ft.
2. Length  $20\frac{1}{2}$  ft., width  $18\frac{3}{4}$  ft., height 14 ft.

*Volume of a cylinder or prism:—*

To find the volume of a cylinder or prism multiply the area of the base by the height. Formula for the volume of a cylinder is  $v = l\pi R^2$ .  $l$  = length.

1. Find the volume of a cylinder when the radius of the base is 8 in. and length 21 in.
2. Find the volume of a cylindrical iron vat  $4\frac{1}{2}$  ft. in diameter and 10 ft. deep.
3. Find the volume of a rectangular prism whose altitude is 40 ft. and the sides of the base 5 ft. and 9 ft. respectively.
4. The volume of a cylinder is 144 cu. ft. The diameter is 4 ft. Find the entire surface.
5. Find the capacity in cubic feet of a cylindrical cistern 6 ft. in diameter and 9 ft. deep.

*Volume of a pyramid or cone:—*

To find the volume of a pyramid or cone multiply the area of the base by one third of the altitude. Formula for the volume of a cone,

$$v = \frac{a\pi R^2}{3}. \quad a = \text{altitude.}$$

Find the volume of the following:—

1. A pyramid whose base contains  $9\frac{1}{2}$  sq. ft. and whose height is 12 ft.
2. A cone the radius of whose base is 9 in. and its altitude 20 in.
3. A cone the diameter of whose base is 16 in. and its altitude 70 in.
4. A pyramid whose base is 6 in. by 9 in. and altitude 15 in.
5. Find the volume of a cone the circumference of whose base is 36 in., and whose altitude is 5 ft.



1. Similar plane figures are those having the same shape, but not necessarily the same size.

2. The *areas* of similar plane figures are proportional to the *squares* of their corresponding or like lines; i.e., the area of a 2-in. square: the area of a 3-in. square = 4:9.

3. The diameters of 2 circles are 3 ft. and 4 ft. What is the ratio of their areas?

4. If the area of a hexagon is 20 sq. ft., what is the area of a similar hexagon each of whose sides is three times as long?

5. A square whose side is 3 ft. is what part of a square whose side is 6 ft.?

6. A circle whose diameter is 2 ft. is what part of a circle whose radius is 2 ft.?

7. A circle whose diameter is 4 ft. is how many times a circle whose diameter is 6 in.?

8. A horse is tied to a stake so that he can graze over 250 square feet of land. Another horse is tied by a rope 3 times as long. Over how much land can the second horse graze?

#### VOLUMES OF SIMILAR SOLIDS.

The volumes of similar solids are proportional to the cubes of their corresponding line.

1. How many 2-in. cubes are equal in volume to an 8-in. cube?

2. A sphere whose diameter is 3 in. is what part of a sphere whose diameter is 6 in.?

3. If a cannon ball weighs 42 lb., what will one weigh whose diameter is 3 times as great?

4. If a cubical block of wood one foot long weighs 4 lb., find the weight of a cubical block 5 ft. long.

5. The weight of a cube of metal each edge of which measures 4 in. is  $18\frac{1}{2}$  lb. What is the weight of a cube of copper each edge of which measures 5 in.?

1. Percentage is a system of calculations by hundredths.
2. Per cent means by hundredths.
3. Any per cent may be expressed in three ways: *a*, as a common fraction,  $\frac{1}{100}$ ,  $\frac{1}{100}$ ; *b*, as a decimal fraction, .05, .005; *c*, with the symbol, 5%,  $\frac{1}{200}$  %.
4. Express the following as decimals and common fractions: 6%,  $23\frac{1}{2}$  %, 215 %,  $\frac{3}{4}$  %,  $\frac{2}{3}$  %,  $\frac{1}{5}$  %,  $\frac{1}{4}$  %,  $212\frac{1}{2}$  %,  $37\frac{1}{2}$  %.
5. Express as decimals and with the symbol:  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{10}$ ,  $\frac{1}{8}$ ,  $\frac{3}{4}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{12}$ ,  $\frac{5}{8}$ .
6. The *base* is a whole of which a part is taken as a percentage.
7. The *rate per cent* is the number of hundredths taken of the base.
8. The *percentage* is the number obtained by taking a number of hundredths of the base.
9. These three terms are closely related, and any two being given the other may be found.
10. 

|                   |                                                                                                                                                                                      |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| \$20 Base         | Here the three terms are used as an example in multiplication. The <i>base</i> is the multiplicand, the <i>per cent</i> is the multiplier, and the <i>percentage</i> is the product. |
| .05 Per cent      |                                                                                                                                                                                      |
| \$1.00 Percentage |                                                                                                                                                                                      |
11. From our knowledge of the principles of multiplication three rules may be derived corresponding to the three cases of percentage.
12. Letting *p* stand for percentage, *r* for rate per cent, and *b* for base, we may express these three rules by three formulas:  

$$p = br; r = p \div b; b = p \div r.$$
13. State these formulas as rules.  
*To find any per cent of a number.*
14. Find 9% of 346.  

|       |                                                                                                                           |
|-------|---------------------------------------------------------------------------------------------------------------------------|
| 346   | $p = br.$ The percentage is found by multiplying the base by the rate expressed either as a decimal or a common fraction. |
| .09   |                                                                                                                           |
| 31.14 |                                                                                                                           |
- Find: 15. 20 % of \$600.                      16.  $37\frac{1}{2}$  % of \$240.
17.  $16\frac{2}{3}$  % of \$437.                      18. 98 % of \$845.

*To find what per cent one number is of another : —*

1. What per cent of 36 is 12?

12 is  $1\frac{1}{3}$  or  $\frac{1}{3}$  of 36  $r = \frac{p}{b}$ . Divide the percentage by the base to find the  
 $\frac{1}{3} = 33\frac{1}{3}\%$  rate. This division may be expressed as a common fraction, and then changed to hundredths, or divide as in division of decimals.

2. 378 is what per cent of 1800?

$$\begin{array}{r} .21 \\ 1800 \overline{) 378.00} \\ \underline{3600} \\ 1800 \\ \underline{1800} \end{array}$$

3. What per cent of  $61\frac{1}{2}$  is  $12\frac{1}{2}$ ?  
 4. What per cent of \$210 is \$42?  
 5. What per cent of \$1842 is \$73.68?  
 6. What per cent of \$540 is \$40?  
 7. 150 is what per cent of 1875?

8. \$261 is what per cent of \$348?

*To find a number when some per cent of it is given : —*

1. 50 is 10 % of what number?

(a)  $10\% = 50$

$1\% = \frac{1}{10}$  of 50 = 5

$100\% = 100 \times 5 = 500$

or

500.

First find 1% and then 100%;

or :

Divide the percentage by the rate.

(b)  $\begin{array}{r} 10 \overline{) 50.00} \\ \underline{50} \end{array}$

2. \$16.50 is 6% of how many dollars?  
 3. 644 yd. are 35% of how many yards?  
 4. \$83.10 is  $66\frac{2}{3}\%$  of what sum?  
 5.  $\frac{3}{4}$  lb. is  $\frac{1}{2}\%$  of how many pounds?  
 6. \$2.35 is  $16\frac{2}{3}\%$  of what sum?  
 7.  $\frac{1}{12}$  is  $33\frac{1}{3}\%$  of what fractional number?  
 8. 375 is 125% of what?  
 9. Find the number of which 12 is  $8\frac{1}{3}\%$ .  
 10. Find the number of which 216 is  $44\frac{1}{3}\%$ .  
 11. Find the number of which 175 is  $31\frac{1}{4}\%$ .  
 12. 120 is  $88\frac{1}{3}\%$  of what number?  
 13. 960 is  $12\frac{1}{2}\%$  of what number?  
 14. \$200 is  $\frac{1}{2}\%$  of what number?

SUMMARY OF PERCENTAGE — PROFIT AND LOSS. 287

1. *Cost* is the sum paid for an article.
2. *Selling Price* is the amount paid by the buyer to the seller. It is always equal to the cost plus the gain, or to the cost minus the loss.
3. *Profit* is the excess of the selling price over the cost.
4. Loss is the excess of the cost over the selling price.
5. All formulas and rules in Percentage apply to Profit and Loss. (See pages 285, 286.)
6. A farm which cost \$4750 was sold so as to gain 18%. What was received for it?
7. A lot of goods was bought for \$6124.50, and sold at a loss of 4%. What was the selling price?
8. A farmer sold a cow for \$32.20 and thereby lost 8%. Find the cost.
9. A house was sold for \$3203.20 at a gain of 12%. What was the cost?
10. A dealer sold a piano for \$270.90, and lost 10%. At what price should he have sold the piano to gain 10%?
11. A farm was sold at a loss of 20%. If the loss was \$516, what was the cost of the farm?
12. A merchant, who sells goods at a gain of 15%, clears \$315. Find the cost.
13. A farm was sold for \$6000, which was at a gain of 12%. What would have been the loss per cent had it been sold for \$4000?
14. If I buy goods at 20% below cost, and sell at 20% above cost, what per cent do I gain?
15. A man lost 20% of his money, and then gained 10% of what he had left. If he then had \$352, how much had he at first?
16. A man drew out  $33\frac{1}{3}\%$  of his bank deposit, and paid 25% of it for a house worth \$1800. What was his original bank deposit?

1. *Commission* is the sum paid to an agent for the transaction of business.

2. The one for whom the business is transacted is called the *principal* or *employer*.

3. The one performing the service is called *agent*, *factor*, *broker*, *collector*, or *commission merchant*.

4. Agents receive money for their employers by collecting debts or selling property, and their commission is some per cent of the money received.

5. Agents also expend money for their employers, and their commission is some per cent of the money paid out.

6. The *Net Proceeds* is the amount returned to the employer after deducting commission and other charges.

7. All rules and formulas in Percentage apply to Commission. See pages 285 and 286.

8. An agent's commission at 3% is \$414. What amount of goods did he sell?

9. I sent my agent \$6150 to invest in flour after deducting his commission at  $2\frac{1}{2}\%$ . What amount did he invest?

10. An agent bought for me 800 bbl. of flour at \$4.75 a barrel. If his commission was 2% and other charges \$31.50, what was the entire cost to me?

11. An agent received \$60 for selling potatoes at 50 cents a bushel on a commission of 4%. How many bushels did he sell?

12. An agent sold goods for \$4500, and remitted to his employer \$4349.50. What was the rate of his commission?

13. After selling goods an agent deducted \$112 commission, and sent his employer \$5488. What rate of commission did he receive?

14. An agent returned to his employer, as net proceeds of a sale, \$6165.75 after deducting \$93.75 for expenses and a commission of  $2\frac{1}{2}\%$ . What was the amount of the sale?

1. *Insurance* is a contract by which the insurer promises to pay the insured for any loss resulting from certain events, like fire, flood, storm, accident, or death.

2. From these different causes arise the different kinds of insurance, as fire insurance, accident insurance, life insurance, etc.

3. The *policy* is the written agreement between the insurance company and the person insured.

4. The *premium* is the sum paid for insurance. It is usually a certain per cent of the amount insured.

5. All rules and formulas in Percentage apply to Insurance. See pages 285 and 286.

6. Mr. Clark insured his house for \$4200 at  $\frac{3}{4}\%$ . What was the premium?

7. Mr. Ingham paid \$42 for insuring his house for \$2800. What was the rate of insurance?

8. Mr. Brown paid \$57.30 to insure his house at  $1\frac{1}{4}\%$ . What sum was named in the policy?

9. A building is insured for  $\frac{2}{3}$  of its value at 1%. What is the value of the building if the premium is \$72.36?

10. C's house worth \$12,000 is insured for  $\frac{3}{4}$  of its value. What is the rate of premium, if he pays \$96 for the insurance?

11. A building which cost \$40,000 is insured for  $\frac{3}{4}$  of its value at  $2\frac{1}{2}\%$ . If it should be totally consumed by fire, what would be the loss to the owner? To the insurance company?

12. Find the value of the property when the premium at  $\frac{1}{3}\%$  is \$30.00.

13. What is the rate of insurance when a policy for \$130,000 costs \$2487.70 premium?

14. A man 25 years old takes out a \$5000 life-insurance policy payable in 20 years. If he pays an annual premium of \$45.50 a \$1000, how much will his insurance cost him if he lives till the policy falls due?

(Review pages 144, 145.)

1. To pay the expenses of a town, city, county, or state government money is collected from the citizens.
2. A *tax* is the money levied upon persons or property for public purposes.
3. A *poll tax* is a tax levied upon each voter, without regard to the amount of property that he owns.
4. The *property tax* is a tax on property, and is usually a certain per cent of the assessed valuation of the property.
5. Property may be either *Personal* or *Real*.
6. Assessors are persons chosen to estimate the value of each piece of real estate.
7. From the *whole* tax subtract the poll-tax, if any, the result will be the *property tax*. Divide the property tax by the assessed value of the property to find the *tax rate*. Multiply each man's property by the tax rate to find his tax.
8. The whole tax of a town is \$16,020 and the taxable property is \$784,750. The number of polls is 260, each assessed \$1.25. If A's property is assessed at \$10,500, what is his tax?
9. A tax of \$29,692 is to be assessed in a certain town, the property of which is valued at \$530,000. There are 4246 polls at \$2 each. What is Mr. C's tax, whose property is assessed at \$8,400?
10. A tax of \$485.25 was paid when the rate of taxation was \$.00 $\frac{1}{2}$ . Find the value of the property.
11. A tax of \$673.50 was paid on property valued at \$44,900. What was the rate?
12. My property, which cost me \$15,600, is taxed at  $\frac{3}{4}$  of its value. What is the rate of taxation, if my tax is \$31.20?
13. In a certain town the valuation of the property amounts to \$1,720,000. The town raises \$28,780 by taxation. There are 840 persons upon whom a poll-tax of \$1.50 each is assessed. Find the rate of taxation on \$1,000.

1. Manufacturers and wholesale dealers issue price-lists of their goods.

2. A discount from this list-price is usually made. This discount varies as the condition of the market varies.

3. Frequently in trade several discounts are made, as 20% and 10% off. This means that first a discount of 20% is made, and then a discount of 10% from the remainder is made.

4. What is due on a bill of \$500, subject to a discount of 20% and 10%?

|                                   |                                       |
|-----------------------------------|---------------------------------------|
| (a) $100\% - 20\% = 80\%.$        | (b) $20\% \text{ of } \$500 = \$100.$ |
| $10\% \text{ of } 80\% = 8\%.$    | $\$500 - \$100 = \$400.$              |
| $80\% - 8\% = 72\%.$              | $10\% \text{ of } \$400 = \$40.$      |
| $72\% \text{ of } \$500 = \$360.$ | $\$400 - \$40 = \$360.$               |

5. If the price of an organ is \$108 after discounts of 20% and 10%, what is the list-price?

6. I bought some goods which were listed at \$800, at 20% below the list-price, and sold them at 10% below the list-price. How much did I gain?

7. What per cent did I gain in Ex. 6?

8. On a bill of goods amounting to \$1200, which is better for the purchaser, and how much better, 55% discount or two successive discounts of 50% and 5%?

9. The net price of an invoice of goods was \$4074, the purchaser having been allowed 30% and 3% off. What was the list-price?

10. Find the net amount of a bill of \$1250, discounts being 25% and 4%. Find a single discount equivalent to these two discounts.

11. An invoice of goods was listed at \$12,000. A merchant bought the goods at discounts of 20%, 10%, and 5%, and sold them at 35% above net cost prices. At what price did he sell? At what per cent below list-price did he sell?



## 292 SUMMARY OF PERCENTAGE — STOCKS AND BONDS.

1. For definitions see pages 196 and 197.
2. The cost, the selling price, the dividend, the brokerage, each is some % of the par value.
3. Let  $c$  = cost;  $s$  = selling price;  $d$  = rate of dividend;  $b$  = brokerage;  $n$  = number of shares;  $p$  = market value of one share;  $i$  = income;  $v$  = par value;  $d'$  = rate of investment.

$$c = n(p + b). \quad s = n(p - b). \quad n = \frac{c}{p + b}. \quad i = nvd.$$

$$n = \frac{i}{vd}. \quad d = \frac{i}{nv}. \quad n = \frac{s}{p - b}. \quad d' = \frac{vd}{p}.$$

Write these formulas as rules.

4. What is the cost of 75 shares of stock at 92, brokerage  $\frac{1}{2}\%$ ?
5. What are the proceeds from the sale of 110 shares of state bonds at 104, brokerage  $\frac{1}{2}\%$ ?
6. How many shares of mining stock at  $126\frac{1}{2}$  can be bought for \$31,750, brokerage  $\frac{1}{2}\%$ ?
7. If I own 76 shares of 5% stock, what will be my annual income?
8. How much must be invested in U.S. 4's at  $121\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ , to realize an income of \$3600?
9. Find the rate per cent of dividend when 81 shares of stock yield an annual income of \$155.
10. C sold stock at  $97\frac{1}{2}$ , brokerage  $\frac{1}{2}\%$ , receiving \$12,464. How many shares did he sell?
11. If I buy 6% stock at 80, what per cent shall I make on my investment?
12. What must be the price of stock when \$4200 worth of stock is bought for \$3570?
13. If I invest \$8976 in U.S. 4's at 102, what is my annual income?
14. How many shares of R.R. stock at 91, and brokerage  $\frac{1}{2}\%$ , can be bought for \$16949.25?

1. *Interest* is the price paid for the use of money.
2. *The principal* is the sum of money for the use of which interest is paid,
3. *The amount* is the sum of the principal and interest.
4. *The rate of interest* is the interest on one dollar for one year. It is always a specified number of hundredths of the principal.
5. Interest is the percentage; the principal is the base; and the rate is the rate of interest.
6. Interest is the most common of the applications of percentage. Because of the element of time that is involved it is the most difficult of the applications.
7. Find the interest on \$720 for 1 yr. 3 mo. 26 d. at 5%.

\$ 7.20 = int. for 2 mo. or 60 d.

43.20 = int. for 1 yr.

3.60 = int. for 1 mo.

2.40 = int. for 20 d.

.72 = int. for 6 d.

\$57.12 = int. for 1 yr. 3 mo. 26 d. at 6%.

9 52 = int. for 1 yr. 3 mo. 26 d. at 1%.

\$47.60 = int. for 1 yr. 3 mo. 26 d. at 5%.

Point off two places or move the decimal point two places to the left to find the interest for 2 mo. or 60 d. Take such multiples and parts of this sum as will give the interest for the required time at 6%. Add to or subtract from this sum such parts as will give the interest at the required rate.

8. Find the interest on \$820 for 3 yr. 2 mo. 13 d. at 7%.
9. Find the interest on \$474.60 for 2 yr. 8 mo. 6 d. at 6%.
10. Find the interest on \$4128 for 3 yr. 7 mo. 20 d. at 7½%.
11. Find the interest on \$274.80 for 1 yr. 5 mo. 18 d. at 5%.
12. Find the interest on \$378.20 for 2 yr. 3 mo. 25 d. at 4½%.
13. Find the interest on \$26745 for 4 yr. 8 mo. 21 d. at 5½%.
14. Find the interest on \$304.86 for 1 yr. 7 mo. 9 d. at 6%.
15. Find the interest on \$65392 for 2 yr. 3 mo. 10 d. at 6½%.
16. Find the interest on \$960.70 for 1 yr. 6 mo. 20 d. at 5%.
17. Find the interest on \$78805 for 2 yr. 8 mo. 22 d. at 4½%.
18. Find the interest on \$8615.50 for 3 yr. 10 mo. 16 d. at 6%.

294      *SUMMARY OF PERCENTAGE — INTEREST.*

1. Find the interest on \$240 from June 6, 1900, to Aug. 12, 1902.

From June 6, 1900, to June 6, 1902 = 2 yr.

From June 6, 1902, to Aug. 6, 1902 = 2 mo.

From Aug. 6, 1902, to Aug. 12, 1902 = 12 d.

\$ 2.40 = int. for 2 mo.

14.40 = int. for 1 yr.

14.40 = int. for 1 yr.

.48 = int. for 12 d.

\$31.68 = int. for 2 yr. 2 mo. 12 d.

This method of subtracting dates is the one now generally used. It is not necessary to have the work written out as in the example. Let the pupils do the work mentally, and write only the result.

Find the amount of: —

2. \$246.75 from Aug. 10, 1898, to June 8, 1902, at  $5\frac{1}{2}\%$ .
3. \$408.90 from June 18, 1899, to Oct. 20, 1901, at  $6\%$ .
4. \$540.50 from Jan. 8, 1899, to Feb. 2, 1903, at  $6\frac{1}{2}\%$ .
5. \$124.84 from Nov. 10, 1898, to Nov. 16, 1902, at  $6\%$ .
6. \$264.60 from Feb. 16, 1899, to Aug. 2, 1903, at  $5\frac{1}{2}\%$ .
7. \$647.28 from Dec. 15, 1898, to Nov. 5, 1902, at  $6\%$ .
8. \$124.40 from June 8, 1899, to Sept. 14, 1901, at  $4\frac{1}{2}\%$ .
9. \$762.40 from Dec. 3, 1900, to Feb. 9, 1903, at  $6\%$ .
10. \$345.60 from March 25, 1899, to July 11, 1902, at  $5\%$ .
11. \$465.70 from Jan. 9, 1900, to Sept. 29, 1902, at  $6\%$ .
12. \$567.80 from Oct. 1, 1902, to March 26, 1904, at  $4\%$ .
13. \$678.90 from Nov. 17, 1901, to April 27, 1903, at  $6\%$ .
14. \$789.10 from Dec. 6, 1901, to May 18, 1903, at  $5\frac{1}{2}\%$ .
15. \$891.20 from July 5, 1900, to Feb. 26, 1902, at  $6\%$ .
16. \$912.30 from Aug. 11, 1902, to March 2, 1904, at  $5\%$ .
17. \$123.40 from Sept. 1, 1902, to April 21, 1903, at  $6\%$ .
18. \$234.50 from Oct. 7, 1901, to May 2, 1903, at  $4\frac{1}{2}\%$ .
19. \$345.60 from Nov. 15, 1902, to June 25, 1903, at  $6\%$ .
20. \$636.20 from Dec. 7, 1901, to Sept. 19, 1903, at  $4\%$ .
21. \$751.26 from Nov. 2, 1901, to Aug. 17, 1903, at  $6\%$ .
22. \$467.85 from May 2, 1903, to Nov. 17, 1905.
23. \$761.07 from Aug. 19, 1902, to April 5, 1904.

(Review pages 171-175.)

|                                        |                |               |
|----------------------------------------|----------------|---------------|
| \$ 460.00                              | Boston, Mass., | March 10 1903 |
| For value received I promise to pay to |                |               |
| William S. Miller                      |                | or order,     |
| Four Hundred Sixty                     |                | Dollars,      |
| on demand with interest at 6%.         |                |               |
| H. S. Fiske                            |                |               |

1. A *promissory note* is a written promise of one person to pay another person or any one to whom he may order it paid a certain sum of money.

2. The *payee* is the person to whom the money is to be paid.

3. The *maker* is the person who promises to pay the money.

4. *Partial payments* are payments in part on notes.

5. An *indorsement* is a record of a partial payment, with the date of payment, made upon the back of the note.

6. A note of \$400 was dated Apr. 21, 1901. The indorsements were: June 27, 1902, \$125; Dec. 9, 1902, \$200. What was due Oct. 9, 1903?

|                 |                         |
|-----------------|-------------------------|
| \$400           | = Principal             |
| 28.40           | = Int. to June 27, 1902 |
| <u>\$428.40</u> | = Amt. to June 27, 1902 |
| 125.00          | = 1st payment           |
| <u>\$303.40</u> | = New principal         |
| 8.19            | = Int. to Dec. 9, 1902  |
| <u>\$311.59</u> | = Amt. to Dec. 9, 1902  |
| 200.00          | = Payment               |
| <u>\$111.59</u> | = New principal         |
| 15.58           | = Int. to Oct. 9, 1903  |
| <u>\$127.17</u> | = Amt. due Oct. 9, 1903 |

Find the amount of the principal to the time of the first payment. If the payment equals or exceeds the interest, subtract the payment from the amount and regard the remainder as a new principal, and proceed in the same manner with the remaining payments.

If the payment is less than the interest, find the amount of the principal to a time when the sum of the payments equals or exceeds the interest due.

## 296 SUMMARY OF PERCENTAGE — BANK DISCOUNT.

(For note see 297, Ex. 2. Review pages 185-188.)

1. *Bank discount* is the interest kept by a bank for advancing money on a promissory note, draft, or bill of exchange before it becomes due.

2. The *proceeds*, *avails*, or *cash value* of a note is the face of the note less the discount.

3. If a note is written so as to be payable a certain number of months after date, calendar months are to be understood. When the time specified is a certain number of days, use that exact number of days in finding the date of maturity.

4. The time from the day of discount to the date of maturity is called the *term of discount*.

5. *Days of grace* have been abolished by statute in many of the States. Use them or not according to the custom of the place in which you live.

6. Find the proceeds of a note for \$500, dated May 9, 1902, due in 60 days and discounted June 3, 1902.

|                         |              |                |          |
|-------------------------|--------------|----------------|----------|
| May 9 + 60 d. = July 8. |              | \$5.00 = 60 d. | \$500    |
| May 22                  | June 27      | 2.50 = 30 d.   | 2.92     |
| June 30                 | July 8       | .416 = 5 d.    | \$497.08 |
| July 8                  | 35 = term of | \$2.916 = Bank | Proceeds |
| 60                      | discount     | Discount       |          |

7. Add the time of the note to the date of the note to find the date of maturity. Find the number of days from the day of discount to the date of maturity to find the term of discount. Compute the interest on the face of the note for the term of discount at the given rate, to find the discount. Subtract the discount from the face of the note to find the proceeds. If the note is an interest-bearing note, first find the amount of the note at maturity, and use this amount as the basis for discount.

Find the proceeds of notes as follows: —

|     | FACE. | DATE.   | TIME.   | DAY OF DISCOUNT. | RATE. |          |
|-----|-------|---------|---------|------------------|-------|----------|
| 8.  | \$260 | March 1 | 2 mo.   | March 22         | 6%.   | INTEREST |
| 9.  | \$364 | April 4 | 60 days | May 1            | 6%.   | RATE OF. |
| 10. | \$586 | June 17 | 90 days | July 15          | 6%.   | 5%.      |
| 11. | \$697 | July 20 | 70 days | Aug. 11          | 6%.   | 6%.      |

1.

## CHECK.

New Haven, Conn., \_\_\_\_\_ 190\_\_ No \_\_\_\_\_

City Bank of New Haven

Pay to the order of \_\_\_\_\_ \$ \_\_\_\_\_

\_\_\_\_\_ Dollars.

\_\_\_\_\_

2.

## NOTE — INDIVIDUAL — TIME — NEGOTIABLE.

\$ \_\_\_\_\_ Boston, Mass., \_\_\_\_\_ 19\_\_

\_\_\_\_\_ after date I promise to pay to

\_\_\_\_\_ or order,

\_\_\_\_\_ Dollars

at the First National Bank.

Value received. \_\_\_\_\_

No. \_\_\_\_\_ Due \_\_\_\_\_

3.

## NOTE — JOINT AND SEVERAL — INTEREST BEARING.

\$ \_\_\_\_\_ Springfield, \_\_\_\_\_ 19\_\_

For value received we, jointly and sever-

ally, promise to pay to \_\_\_\_\_

or order \_\_\_\_\_ Dollars,

with interest at 6%. \_\_\_\_\_

\_\_\_\_\_

1.

NOTE — DEMAND — JOINT — NON-NEGOTIABLE.

|                                |                  |                |
|--------------------------------|------------------|----------------|
| \$ _____                       | New Haven, _____ | 19 _____       |
| On demand we promise to pay to |                  |                |
|                                |                  | _____ Dollars. |
| Value received                 | _____            |                |
|                                | _____            |                |

2.

RECEIPT IN FULL.

|                                |               |                |
|--------------------------------|---------------|----------------|
| \$ _____                       | Boston, _____ | 19 _____       |
| Received of _____              |               |                |
|                                |               | _____ Dollars, |
| in full of all demands to date |               |                |
| _____                          |               |                |

3.

RECEIPT FOR PART PAYMENT.

|                   |                 |               |
|-------------------|-----------------|---------------|
| \$ _____          | New Haven _____ | 19 _____      |
| Received of _____ |                 |               |
|                   |                 | _____ Dollars |
| on account.       |                 |               |
| _____             |                 |               |

(Review Involution.)

1. On page 211, we learned that  $(25)^2 = (20 + 5)^2 = (20)^2 + 2(20 \times 5) + 5^2$ . To cube a number we must multiply the square of the number by the number itself.

$$\begin{array}{r}
 (20)^2 + 2(20 \times 5) + 5^2 \\
 20 \quad + \quad 5 \\
 \hline
 (20)^3 + 2(20^2 \times 5) + (20 \times 5^2) \\
 \quad \quad (20^2 \times 5) + 2(20 \times 5^2) + 5^3 \\
 \hline
 (20)^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3.
 \end{array}$$

Substituting  $t$  for 20 and  $u$  for 5, we have the formula  $t^3 + 3t^2u + 3tu^2 + u^3$ ; i.e., every perfect cube consists of four parts, viz., the tens figure cubed, plus three times the tens figure squared times the unit figure, plus three times the tens figure times the units figure squared, plus the units figure cubed.

2. Using this formula, write the cube of:

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 36 | 74 | 48 | 63 | 85 | 72 | 28 |
| 17 | 58 | 62 | 84 | 95 | 38 | 49 |

3. To extract the cube root of a number is to find one of three equal factors.

4. Extract the cube root of 15,625.

$$\begin{array}{r}
 t^3 + 3t^2u + \left. \begin{array}{l} 3tu^2 + u^3 \end{array} \right\} = \overset{2}{15},\overset{5}{625} \\
 t^3 = 8 \\
 (3t^2) = 12 \overline{) 76} \\
 3t^2u = 60 \\
 \quad \quad 162 \\
 3tu^2 = 150 \\
 \quad \quad 125 \\
 u^3 = 125
 \end{array}$$

First point off the number into periods of three figures each, to find how many figures we are to have in our root. What is the greatest number whose cube is not more than 15? Place the 2 above the tens period. Cube the tens figure, and subtract it from the tens period. What is the remainder? Place beside it the first figure of the next period. The next part of the formula is  $3t^2u$ ; of this only  $3t^2$  is known. This is called the *trial divisor*. How many times is 12 contained in 76? Notice that 12 is only a trial divisor, and allowance must be made for the rest of the formula. Place the 5 units over units period. Find the value of  $3t^2u$ , and subtract. What is the remainder? Place the next figure of the power beside the remainder. Find the value of  $3tu^2$ , and subtract. What is the remainder? Place the next figure of the power beside it. Find the value of  $u^3$ , and subtract. Is there any remainder? What is the cube root of 15,625?

\* The remaining pages of this book may (without loss) be entirely omitted unless required by the course of study.



1. Find the cube root of 15,625.

Fig. 1.

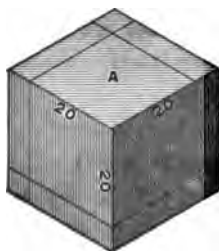


Fig. 2.

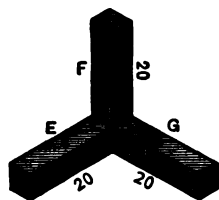
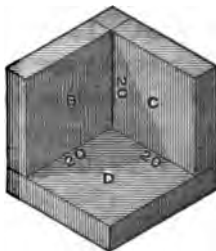


Fig. 3.

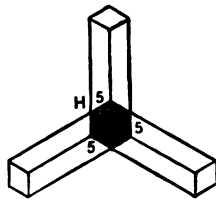


Fig. 4.

The entire cube in Fig. 1 represents 15,625.

The part marked A represents the largest tens figure cubed. Fig. 2 represents the part left after the tens cube has been removed. B, C, and D are three solids, each as long and wide as the tens cube.  $3 \times (20)^2 \times 5 =$  contents of B, C, and D. Fig. 3 represents what is left after B, C, and D have been taken away from Fig. 2. How long, wide, and thick are E, F, and G? What, then, are their cubic contents? Fig. 4 represents the part of Fig. 3 that is left after E, F, and G are removed. This is a little cube. What are its dimensions? What is its cubic contents? What part of the formula represents each of the eight parts of the cut-up cube?

2. Find the cube root of:

|        |           |           |            |
|--------|-----------|-----------|------------|
| 54,872 | 804,357   | 157,464   | 438,976.   |
| 32,768 | 941,192   | 912,673   | 274,625.   |
| 29,791 | 110,592   | 753,571   | 614,125.   |
| 42,875 | 147,649   | 405,224   | 185,193.   |
| 39,304 | 3,048,625 | 6,539,203 | 7,880,599. |

NOTE. — When there are more than two figures in the root, let  $t$  of the formula represent all of the root known, and begin again with  $3t^2u$ , and repeat.

1. Find the cube root of 34,328,125.

$$\begin{array}{r}
 34 \cdot 328 \cdot 125 \\
 27 = t^3 \\
 3t^2 = 27 \overline{)73} \\
 \underline{54} = 3t^2u \\
 192 \\
 \underline{36} = 3tu^2 \\
 1568 \\
 8 = u^3 \\
 3072 = \left. \begin{array}{l} \\ \\ \end{array} \right\} \overline{)15601} \\
 (3 \times 32^2) \left. \begin{array}{l} \\ \\ \end{array} \right\} \overline{)15360} = 3t^2u \\
 \underline{2412} \\
 2400 = 3tu^2 \\
 \underline{125} \\
 125 = u^3
 \end{array}$$

2. Find the depth of a cubical box whose volume is 175,616 cubic inches.

3. A cubical block of stone contains 857,375 cubic inches. What is the area of one side?

4. A hall in the form of a cube contains 357,911 cubic feet. At \$.90 a square yard, how much will it cost to carpet the floor?

5. How long, wide, and high is a cubical pile of wood containing 32 cords?

6. A rectangular solid is 343 ft. long, 216 ft. wide, and 729 ft.

high. Find the edge of a cube containing an equal number of cubical units.

7. A cube measures 5 in. on an edge. A second cube has 8 times the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube?

8. A cubical block of stone contains 50,653 cubic feet. What is its surface area?

9. What is the edge of a cube which contains as much as a solid 7 ft. long, 3 ft. 6 in. wide, and  $1\frac{1}{4}$  ft. high?

10. What is the number of square inches in one face of a cubical block whose contents are 74,088 cubic inches?

11. Find the cube root of:

|            |             |                 |
|------------|-------------|-----------------|
| 43,614,208 | 130,323,843 | 354,894,912.    |
| 41,063,625 | 303,464,448 | 751,089,429.    |
| 14,348,907 | 258,474,853 | 27,081,081,027. |
| 96,071,912 | 307,546,875 | 1,371,380,681.  |

If William Andrews of Boston owes John Blackmer of Chicago a sum of money, he can pay the debt in several ways :

1. He can buy a *post-office order* at the Boston post-office payable to Mr. Blackmer at the post-office at Chicago.

2. He can buy an express order at the office of an express company payable to Mr. Blackmer at any express-office in Chicago of the same company.

3. If he has money deposited at any bank, he can write a check, and send it to Mr. Blackmer. (See Lesson 31.)

4. He can buy a draft at a bank payable to Mr. Blackmer in Chicago.

Copy the following draft, and explain each item :

*\$2,500.*

*Boston, Jan. 10, 1899.*

*Ten days after date, pay to  
the order of William Andrews*

*Twenty-five Hundred Dollars.*

*Value received and charge the same to the account of  
Merchants National Bank.*

*William Jones, Cashier.*

*To the First National Bank,  
Chicago.*

Mr. Andrews writes on the back :

*Pay to the order of John Blackmer.*

*William Andrews.*

He then sends the draft to Mr. Blackmer in Chicago, who takes it to the First National Bank for acceptance, which is done by the cashier writing the word "Accepted," and his name underneath, across the face.

Sometimes the words "At sight" are written before "Pay to." These are called sight drafts, and are payable on presentation.

Drafts may be used for collecting debts as well as for paying them. Exchange is thus seen to be a method of making payments in distant places by means of drafts.

1. The following are the rates charged for express orders to any part of the United States or Canada:

\$5.00, 5¢.    \$20.00, 10¢.    \$40.00, 18¢.    \$75.00, 25¢.  
 10.00, 8¢.    30.00, 15¢.    60.00, 20¢.    100.00, 30¢.

2. The following are the rates for post-office money orders:

\$5.00, 5¢.    \$20.00, 10¢.    \$40.00, 15¢.    \$75.00, 25¢.  
 10.00, 8¢.    30.00, 12¢.    50.00, 18¢.    100.00, 30¢.

3. The cost of a draft varies. In the draft in Lesson 129, if the Boston banks have but little money on deposit in Chicago, they will charge Mr. Andrews a certain per cent for the draft. On the other hand, if they have large sums of money there that they want at home, they will gladly sell Mr. A. the draft at a discount.

4. Calling the rate of premium  $\frac{1}{2}\%$ , find the cost for sending the following sums of money by Post-office Order, by Express Order, and by Draft: \$25, \$50, \$65, \$80, \$100.

5. Find the cost of a draft on New York for \$800, when exchange is  $\frac{1}{2}\%$  premium.

The premium =  $\frac{1}{2}\%$  of \$800 = \$1.00.

$\therefore$  the cost = \$800 + \$1.00 = \$801.00. *Ans.*

6. How large a sight draft on Chicago can be purchased for \$4,010, when the exchange is  $\frac{1}{2}\%$  premium?

The cost of \$1.00 = \$1.0025.

\$4,010  $\div$  \$1.0025 = \$4,000. *Ans.*

7. What will be the cost of a 3 mo. time draft for \$3,000 at  $\frac{1}{2}\%$  premium?

The premium =  $\frac{1}{2}\%$  of \$3,000 = \$15.00.

The discount of \$3,000 for 3 mo. = \$45.00.

$\therefore$  the cost = \$3,000 + \$15.00 - \$45.00 = \$2,970. *Ans.*

Find the cost of the following drafts:

8. \$700, premium  $\frac{1}{2}\%$ , payable at sight.

9. \$1,200, discount  $\frac{1}{2}\%$ , payable in 90 days at 6%.

10. \$2000, premium  $\frac{1}{2}\%$ , payable in 30 days at 6%.

A square, used in shingling, etc., is 100 sq. ft.

A hand is 4 in., used in measuring horses.

A size is  $\frac{1}{2}$  in., used by shoemakers.

A span is 9 in., a fathom 6 ft. used by sailors.

A pace is 3 ft., used in estimating distances.

A league is 3 miles, used in measuring distances at sea.

A load is one cubic yard of earth.

A perch is  $24\frac{1}{2}$  cubic feet, used in measuring stone and masonry.

A long ton is 2240 lb., used in buying coal at the mines, and by custom-house officers in collecting duties.

A barrel of flour weighs 196 lb. ; a barrel of beef or pork, 200 lb. ; a quintal of fish, 100 lb. ; a keg of nails, 100 lb.

A bushel of oats weighs 32 lb. ; barley, 48 lb. ; rye or corn, 56 lb. ; wheat or potatoes, 60 lb. ; a firkin of butter, 56 lb.

A gallon is 231 cu. in., or  $7\frac{1}{4}$  gal. \* fill a cubic foot.

One bushel, even measure, contains 2,150.42 cu. in. or  $1\frac{1}{4}$  cu. ft.\*

One bushel, heaped measure, contains 2,688 cu. in. or  $1\frac{1}{4}$  cu. ft.\*

A chain is 66 ft., used by surveyors.

A bundle of paper contains 2 reams ; 5 bundles, a bale.

A folio is paper folded in 2 leaves for a book ; a quarto or 4to, 4 leaves ; an octavo or 8vo, 8 leaves ; a duodecimo or 12mo, 12 leaves.

Shingles are packed in bunches. 4 bunches make 1,000. The price is always given by the thousand.

1,000 shingles, laid 4 in. to the weather, will cover a square, or 100 sq. ft. ; 900 shingles when laid  $4\frac{1}{2}$  in.

A lath is 4 ft. long, and  $1\frac{1}{2}$  in. wide. 50 or 100 laths make a bunch. 1 bunch of 50 will cover 3 sq. yd., allowing for waste.

A section of land is one mile square, or 320 rd.  $\times$  320 rd.

A brick is 8 in. long, 4 in. wide, and 2 in. thick. 22 bricks make 1 cu. ft. of wall.

Wall-paper is 18 in. wide, and 24 ft. long, a single roll.

\* Approximately.

**STANDARD UNITS.**

Meter (m.).  
 Square meter (sq. m.).  
 Cubic meter (cu. m.).  
 Liter (l.).  
 Gram (g.).

**VOLUME.**

1,000 cu. mm. = 1 cu. cm.  
 1,000 cu. cm. = 1 cu. dm.  
 1,000 cu. dm. = 1 cu. m.  
 1 cu. m. = 1 ster (st.) of wood.

**LENGTH.**

10 mm. = 1 cm.  
 10 cm. = 1 dm.  
 10 dm. = 1 m.  
 10 m. = 1 Dm.  
 10 Dm. = 1 Hm.  
 10 Hm. = 1 Km.

**SURFACE.**

100 sq. mm. = 1 sq. cm.  
 100 sq. cm. = 1 sq. dm.  
 100 sq. dm. = 1 sq. m or centar (ca.).  
 100 sq. m. = 1 sq. Dm. or ar (a.).  
 100 sq. Dm. = 1 sq. Hm. or hektar (Ha.).  
 100 sq. Hm. = 1 sq. Km.

**CAPACITY.**

10 ml. = 1 cl.  
 10 cl. = 1 dl.  
 10 dl. = 1 l.  
 10 l. = 1 Dl.  
 10 Dl. = 1 Hl.  
 10 Hl. = 1 Kl.

**WEIGHT.**

10 mg. = 1 cg.  
 10 cg. = 1 dg.  
 10 dg. = 1 g.  
 10 g. = 1 Dg.  
 10 Dg. = 1 Hg.  
 10 Hg. = 1 Kg.

1,000 Kg. = 1 ton (T.).

**PREFIX****ABBRE-  
VIATION.****RATIO.**

|        |      |   |         |
|--------|------|---|---------|
| Milli- | (m.) | = | .001.   |
| Centi- | (c.) | = | .01     |
| Deci-  | (d.) | = | .1      |
| —      |      | = | 1.      |
| Deka-  | (D.) | = | 10.     |
| Hekto- | (H.) | = | 100.    |
| Kilo-  | (K.) | = | 1,000.  |
| Myria- | (M.) | = | 10,000. |

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**LINEAR MEASURE.**

|                 |                 |                                       |
|-----------------|-----------------|---------------------------------------|
| 12 inches (in.) | = 1 foot (ft.). | 5½ yards, or 16½ feet = 1 rod (rd.).  |
| 3 feet          | = 1 yard (yd.). | 320 rods, or 5280 feet = 1 mile (m.). |

**SQUARE MEASURE.**

|                                          |                             |
|------------------------------------------|-----------------------------|
| 144 square inches (sq. in.)              | = 1 square foot (sq. ft.).  |
| 9 square feet                            | = 1 square yard (sq. yd.).  |
| 30½ square yards, or<br>272½ square feet | } = 1 square rod (sq. rd.). |
| 160 square rods                          |                             |
| 640 acres                                | = 1 acre (a.).              |
|                                          | = 1 square mile (sq. m.).   |

**SOLID OR CUBIC MEASURE.**

|                             |                           |
|-----------------------------|---------------------------|
| 1728 cubic inches (cu. in.) | = 1 cubic foot (cu. ft.). |
| 27 cubic feet               | = 1 cubic yard (cu. yd.). |

**WOOD MEASURE.**

|                                   |                          |
|-----------------------------------|--------------------------|
| 16 cubic feet                     | = 1 cord foot (cd. ft.). |
| 8 cord feet, or<br>128 cubic feet | } = 1 cord (cd.).        |
|                                   |                          |

**LIQUID MEASURE.**

|               |                     |
|---------------|---------------------|
| 4 gills (gi.) | = 1 pint (pt.).     |
| 2 pints       | = 1 quart (qt.).    |
| 4 quarts      | = 1 gallon (gal.).  |
| 1 gal.        | = 231 cubic inches. |

**DRY MEASURE.**

|               |                         |
|---------------|-------------------------|
| 2 pints (pt.) | = 1 quart (qt.).        |
| 8 quarts      | = 1 peck (pk.).         |
| 4 pecks       | = 1 bushel (bush.).     |
| 1 bushel      | = 2150.42 cubic inches. |

**AVOIRDUPOIS WEIGHT.**

|                 |                  |
|-----------------|------------------|
| 16 ounces (oz.) | = 1 pound (lb.). |
| 2000 pounds     | = 1 ton (t.).    |
| 2240 pounds     | = 1 long ton     |

**CIRCULAR MEASURE.**

|                |                            |
|----------------|----------------------------|
| 60 seconds (") | = 1 minute (').            |
| 60 minutes     | = 1 degree (°).            |
| 360 degrees    | = 1 circumference (circ.). |

**MISCELLANEOUS TABLE.**

|           |                  |
|-----------|------------------|
| 12 units  | = 1 dozen.       |
| 12 dozen  | = 1 gross.       |
| 12 gross  | = 1 great gross. |
| 20 units  | = 1 score.       |
| 24 sheets | = 1 quire.       |
| 20 quires | = 1 ream.        |

**TIME MEASURE.**

|                   |                           |
|-------------------|---------------------------|
| 60 seconds (sec.) | = 1 minute (m.).          |
| 60 minutes        | = 1 hour (h.).            |
| 24 hours          | = 1 day (d.).             |
| 7 days            | = 1 week (wk.).           |
| 365 days          | = 1 common year (c. yr.). |
| 366 days          | = 1 leap year (l. yr.).   |
| 100 years         | = 1 century (C.).         |



















6-

Times.





